McGill Desautels Faculty of Management
Daniel Andrei, Financial Derivatives FINE 448 (BCom), Fall 2018

Midterm

October 2018

Date: ____________________

Your Name: ____________________

Your Signature:[1] ____________________

- This exam is open book, open notes. You can use a calculator or a computer, **but be sure to show or explain your work**.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period.
- You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- **If you are stuck on something, make an assumption, tell me what it is, and do the best you can.** I give partial credit if you provide enough correct information.

TIME LIMIT: 1 hour

TOTAL POINTS: 100

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By signing the exam: (i) you certify your presence, and (ii) you state that you neither gave nor received help during the examination.

1 (26 points) The McPoutine's restaurant chain has become, in spite of itself, one of the symbols of globalization. The value of its shares is currently (in period 0) \$1000 and may, in the next period, be either \$1100 or \$900, depending on whether there is a riot or not (the highest value naturally corresponding to the case when there is no riot). There are call and put options on McPoutine's shares with an exercise price of 1000, maturing in period 1. The risk-free interest rate is 1% per annum. McPoutine's does not pay dividends.

a. (8 points) What is the risk-neutral probability of a riot?

Risk-neutral probability

The government suddenly announces to general surprise (and during period 0) that in case of riot, it will let it go. The stock price of McPoutine's then drops to \$950.

b. (8 points) How much do investors now estimate the risk-neutral probability of a riot?

Risk-neutral probability

c. (***10 points) Assuming that you knew what the government was going to announce and the other investors did not know, what would you have done? Describe any option or underlying strategies that allow you to take advantage of your inside information. (Assume you can buy or sell the different titles in period 0 right before and right after the announcement.)

In each case, describe, for **one unit** purchased or sold,

(i) what you buy or sell, when, and at what price;

(ii) the effective return of your strategy (in percentage terms).

Which of these possibilities gives you the highest leverage?

1 Solution

a. The risk-neutral probability of not having a riot is

$$p^* = \frac{e^r - d}{u - d} = \frac{e^{0.01} - 900/1000}{1100/1000 - 900/1000} = 0.5503 \tag{1}$$

Thus, the risk-neutral probability of a riot is 44.97%

b. The risk-neutral probability of not having a riot is now

$$p^* = \frac{e^r - d}{u - d} = \frac{e^{0.01} - 900/950}{1100/950 - 900/950} = 0.2977 \tag{2}$$

Thus, the risk-neutral probability of a riot is 70.23%

c. The 3 possibilities are:

(i) Short sell the underlying. That is, selling it for \$1000, before the information becomes public, and buy it back for \$950. The effective return of this strategy is 5%.

(ii) Buy a put option. The price of the put option before the information becomes public is

$$P = \frac{0.4497 \times 100}{e^{0.01}} = \$44.53 \tag{3}$$

and after the announcement

$$P = \frac{0.7023 \times 100}{e^{0.01}} = \$69.53 \tag{4}$$

The effective return of this strategy is 56.15%.

(iii) Write a call option (or short-sell an existing call option). The price of the call option before the information becomes public is

$$C = \frac{0.5503 \times 100}{e^{0.01}} = \$54.48 \tag{5}$$

and after the announcement

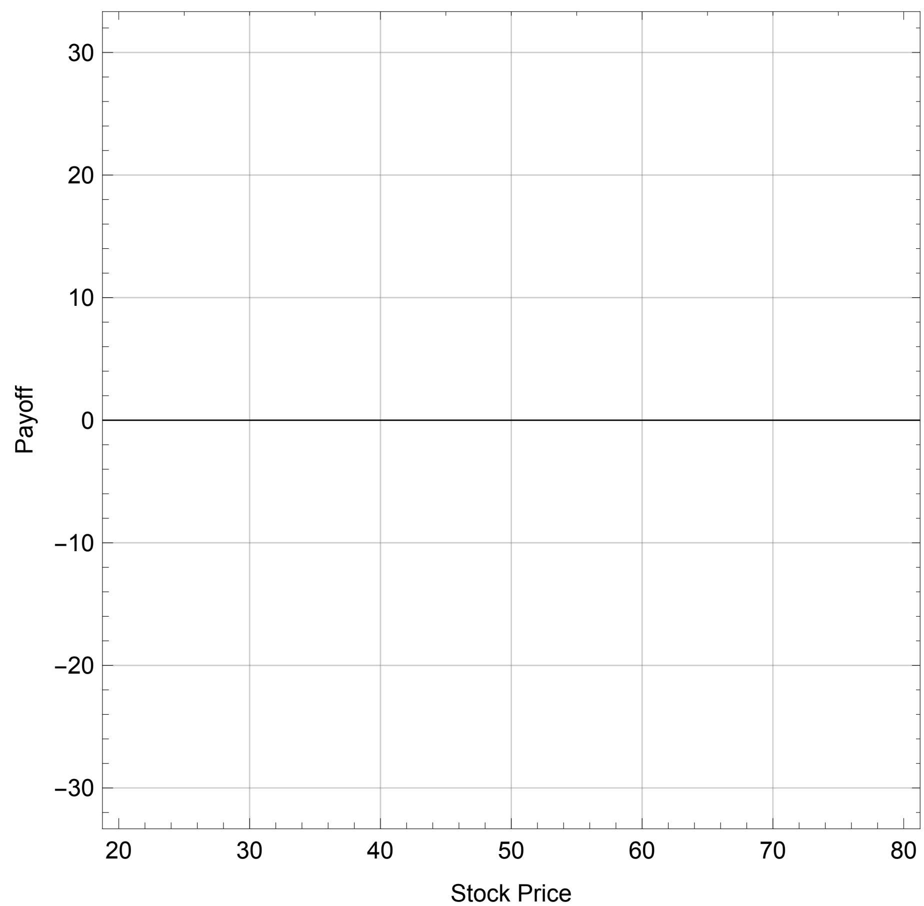
$$P = \frac{0.2977 \times 100}{e^{0.01}} = \$29.48 \tag{6}$$

The effective return of this strategy is 45.89%.

The strategy using the put is the one that yields in the highest effective return, so it is the one that has the highest leverage.

2 (24 points) You bought a put option with an exercise price of 30 and two call options with an exercise price of 50, and sold short a call option with an exercise price of 30. All these options mature in two years. The underlying stock is currently \$40 and its volatility is 25% per annum. The continuously compounded risk-free rate is 3%.

a. (8 points) Plot your **payoff** for this strategy. (Do not consider the initial premiums.)



b. (8 points) Describe a portfolio consisting of only two options and one position in the risk-free assets that is equivalent to the 4-option portfolio described above. (Assume that all the options you need are available.)

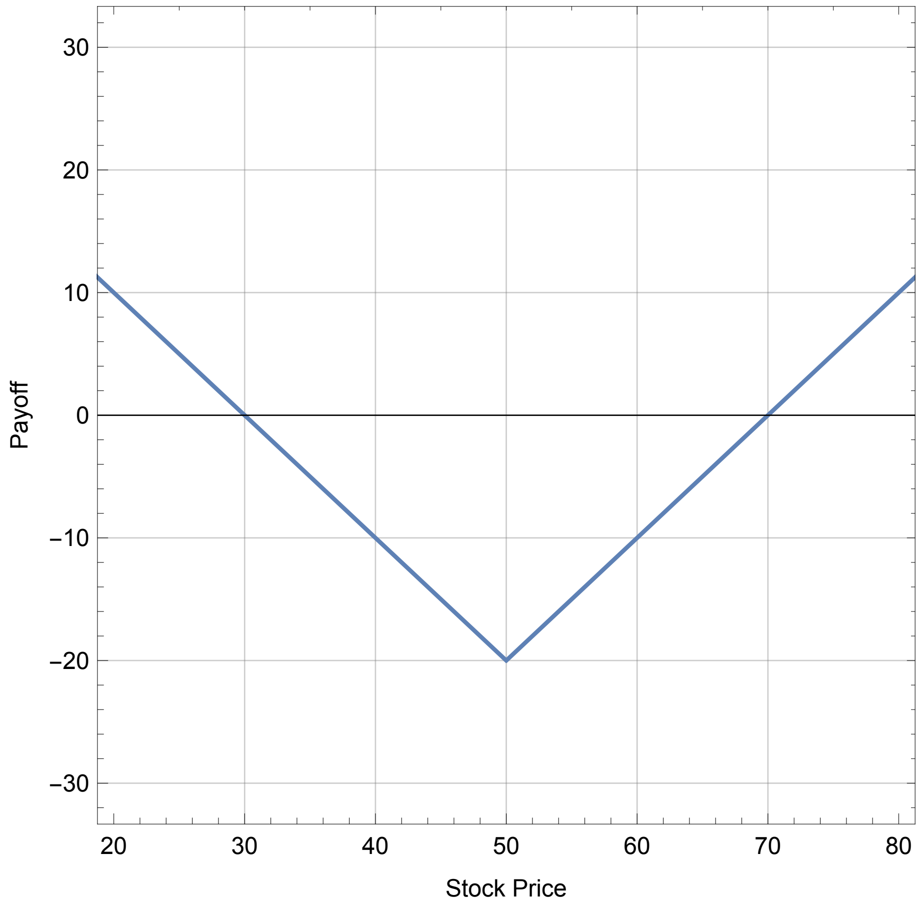
c. (8 points) Consider the option prices (strikes are in parentheses):

Option	Call(30)	Call(50)	Call(60)	Put(30)	Put(50)	Put(60)
Price	–	\$3.20	\$1.45	\$1.01	\$10.28	–

Calculate the value of the strategy at the initial time (that is, determine how much you have to pay or receive when you put it in place).

2 Solution

a. The payoff of the strategy is



b. An equivalent position can be obtained by purchasing a call option and a put option (each with a strike price of 50) and borrowing the present value of \$20 at the risk-free rate.

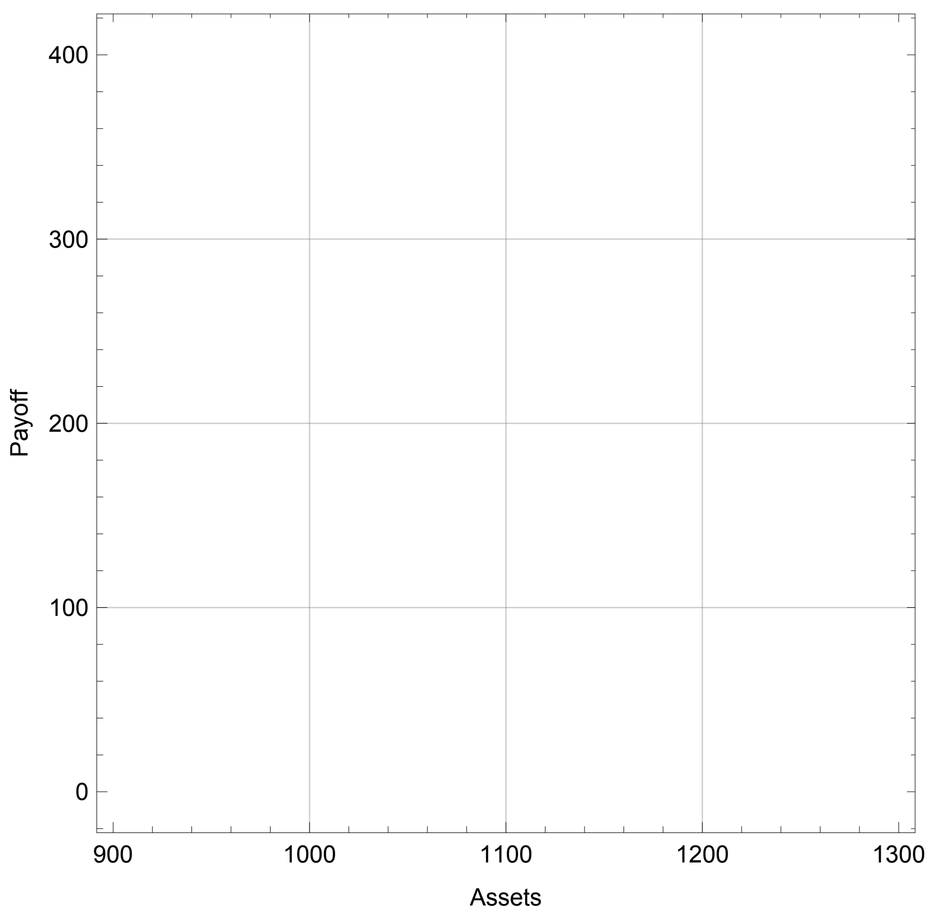
c. The total value cannot be calculated by using the initial 4 options, because the price of the call option with strike of 30 is missing. We will therefore use the position described at the previous point. The total price of the strategy is the sum of its different components:

$$\text{Price} = 3.20 + 10.28 - 20 \times e^{-0.03\times 2} = -\$5.3538 \tag{7}$$

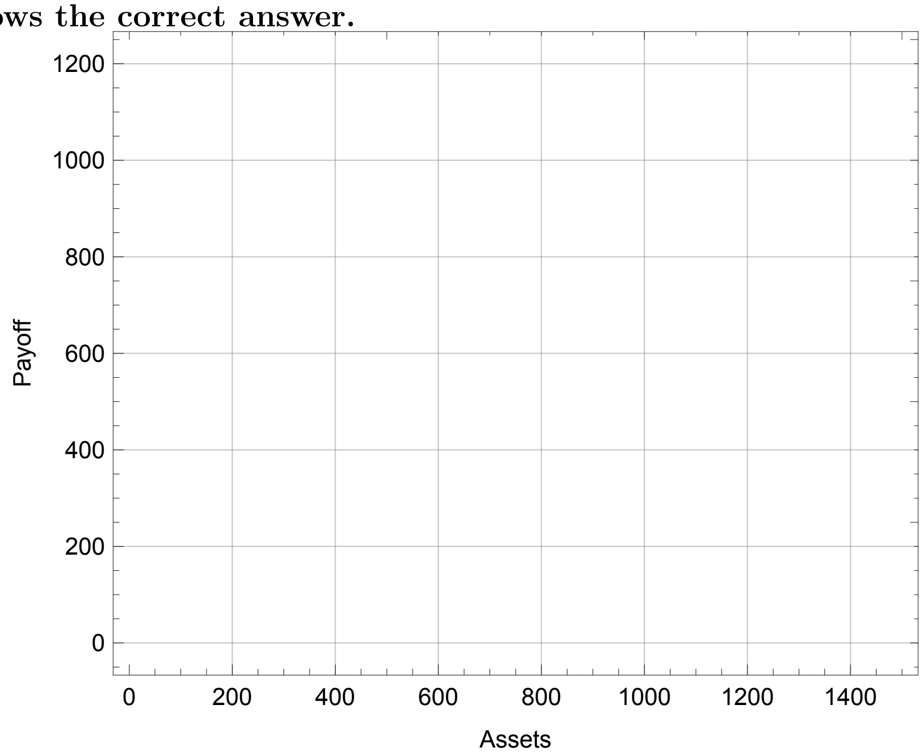
3 (34 points) A company has assets that are currently worth \$1000. It has issued debt whose total amount to be repaid at maturity in two and a half years is \$1100 (there are no payments before this date). Asset risk (i.e., the volatility of the company's assets) is currently 15% per annum, and the continuously compounded interest rate is 4% per annum. The company pays no dividends.

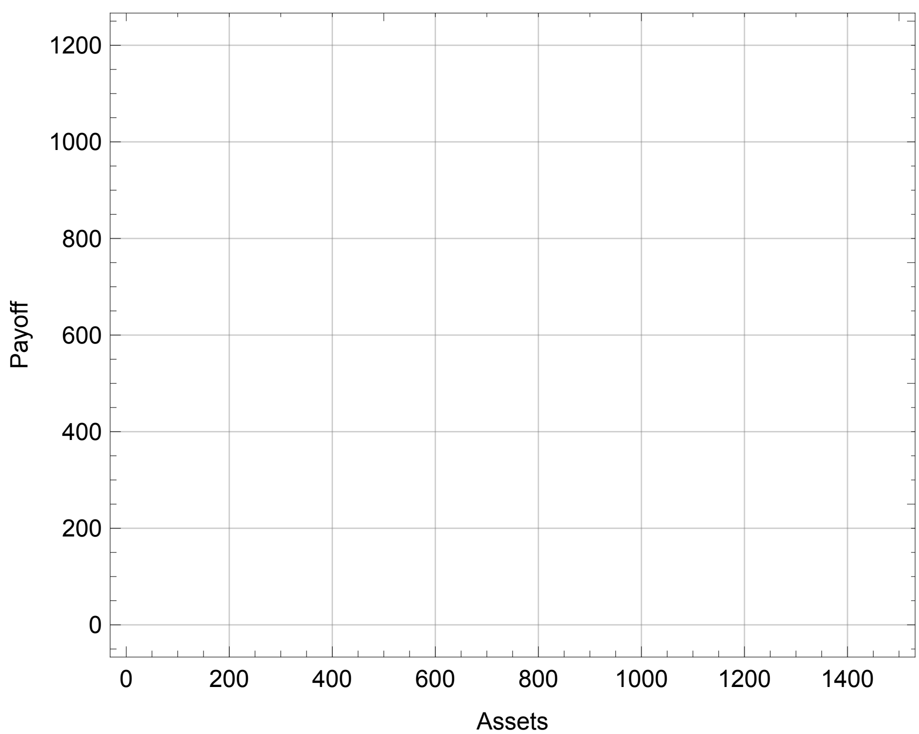
In two and a half years, if the value of the company's assets is larger than the face value of debt, it will pay off that face value, otherwise it will declare bankruptcy. In case of bankruptcy, the debt holders will own the assets of the company, **minus bankruptcy costs**. Bankruptcy costs are 3/11 of the value of the assets in two and a half years.

a. (8 points) In the **payoff** diagram below, plot the payoff of an equity holder who is the sole owner of the company.



b. (8 points) In the **payoff** diagram below, plot the payoff of debt holders of the company. **Do not forget the bankruptcy costs**. I include two diagrams in case you need one to practice. **Please indicate which of the two diagrams shows the correct answer.**





c. (8 points) Denote by A_T the liquidation value of the company's assets in two and a half years. Consider an European **Binary Option** with maturity two and a half years that pays:

$$BO_T = \begin{cases} \$100 & \text{if the company declares bankruptcy (if } A_T < \$1100) \\ \$0 & \text{otherwise} \end{cases}$$

Express the position of the debt holders as a portfolio of the following securities:

(i) a zero-coupon bond with face value \$1100,

(ii) a put option on the assets of the company, with maturity two and a half years and strike \$1100,

(iii) the binary option defined above

Position in the zero-coupon bond

Position in the put option

Position in the binary option

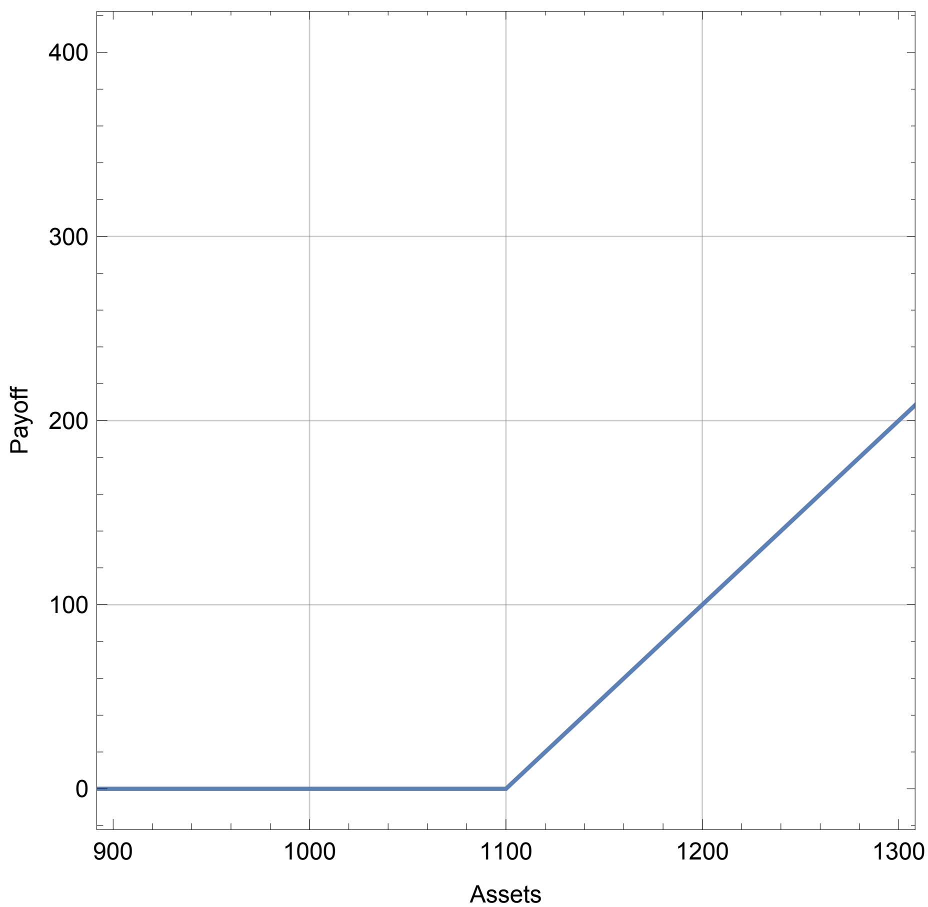
d. (***10 points) Evaluate the value of the portfolio built at point (c) using a three-period binomial tree. For this, fill the table below with the risk-neutral probabilities (I have already filled in the final payoffs). Once the table is filled, it is straightforward to find the value of the portfolio, that is, the value of the debt today.

Risk-neutral probability	Debt final payoff
	\$1100.00
	\$1100.00
	\$700.90
	\$532.99

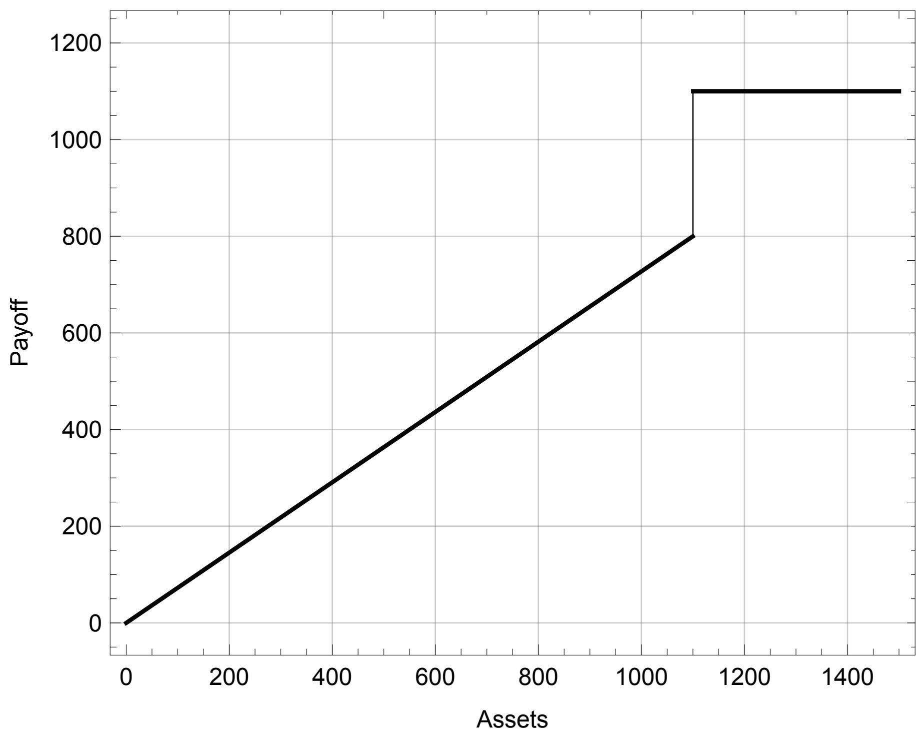
Value of debt

3 Solution

a. The payoff of the sole equity holder is



b. The payoff of debt holders is



c. To replicate the debt payoff, the positions are:

(i) Long one zero-coupon bond with face value \$100

(ii) Short 8/11 put options on the assets of the company, with strike \$1100

(iii) Short 3 binary options (the size of the "jump" in the previous figure)

d. Solve first for u and d:

$$u = e^{0.04\times 0.8333+0.15\sqrt{0.8333}} = 1.1856 \tag{8}$$

$$d = e^{0.04\times 0.8333-0.15\sqrt{0.8333}} = 0.9016 \tag{9}$$

Given this, we find $p^* = 0.466$. Thus, the risk-neutral probabilities for the final payoffs are in the first column of the table below:

Risk-neutral probability	Debt final payoff
0.1011	\$1100.00
0.3477	\$1100.00
0.3988	\$700.90
0.1524	\$532.99

The value of the debt today is then:

$$\text{Debt value} = e^{-0.04\times 2.5} 854.43 = \$773.12 \tag{10}$$

4 (16 points) Put options are strictly convex functions of the strike price. That is, if we were to plot the value of a put option as a function of different strikes, the resultant line would be convex.

a. (8 points) Two put options with the same maturity and on the same underlying, one with strike 30 and the other with strike 20, are trading for \$6 and \$4, respectively. Is there an arbitrage opportunity? Why? (Here you will earn points **only** if you justify your answer. **Hint:** What is the price of a put option with strike 0?)

b. (8 points) Which of the two options above is overpriced relative to the other one? Describe your arbitrage strategy. (Here you are required only to tell me what would be the positions you take, without actually computing the arbitrage profits.)

4 Solution

a. Since the value of a put option with strike 0 is \$0, we in fact know the prices of put options with three different strikes, i.e,

$$P(30) = 6; \quad P(20) = 4; \quad P(0) = 0 \tag{11}$$

In the plane (K,P(K)), these option values are on a line $P(K) = \frac{2}{3}K$. This contradicts the fact that put options are strictly convex functions of strike prices, and creates an arbitrage opportunity.

The arbitrage comes from the fact that the put with strike 20 is overpriced.

b. Using a "buy low, sell high" strategy, we should buy one put option with strike 30 and sell 1.5 put options with strike 20 (any combination that gives zero initial investment would work).