

Internet Appendix for “Why Did the q Theory of Investment Start Working?”

Appendix B. Additional results

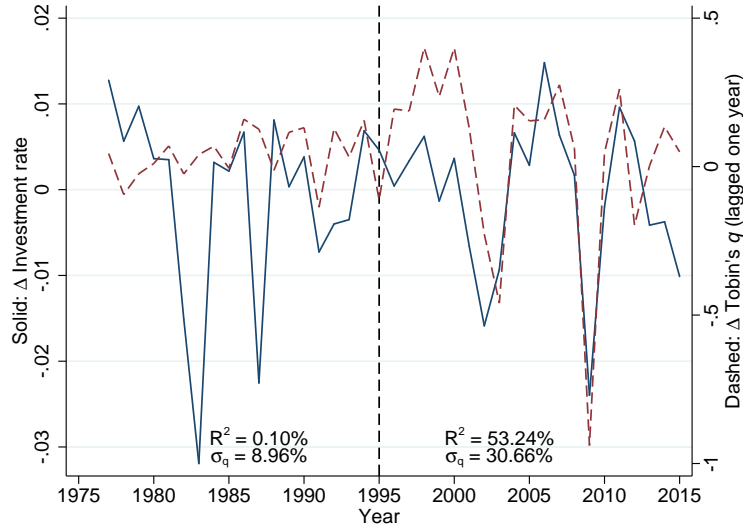


Fig. B.12.

First-differences of aggregate investment rate and lagged Tobin's q from annual Compustat data, 1975–2015.

The solid blue series represents the aggregate investment rate, defined as total annual capital expenditures in Compustat scaled by the previous year's total capital stock in Compustat. The dashed red series represents a one-year lag of aggregate Tobin's q , defined as the total market value of equity, plus book value of debt, minus current assets, divided by the total capital stock in Compustat. In both cases, the capital stock is measured as the aggregate gross stock of property, plant, and equipment. Both series are plotted in differences. As in Figure 2, the 1995–2015 subsample of Figure B.12 exhibits both greater volatility of Tobin's q , and a better fit of the aggregate investment- q relationship, than the 1975–1995 subsample. The volatility of Tobin's q rises from about 9% to about 31%, and the R^2 of the aggregate regression rises from 0.1% to 53%.

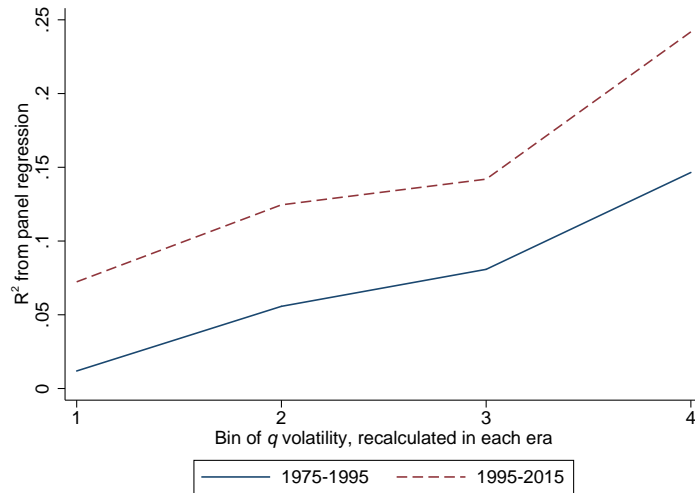


Fig. B.13.

R^2 of the panel regression across four bins of within-firm volatility of Tobin's q .

The bins are recalculated separately for the two subperiods 1975–1995 and 1995–2015.

	(1)	(2)	(3)	(4)
	I_t/K_{t-1}	I_t/K_{t-1}	I_t/K_{t-1}	I_t/K_{t-1}
L.q	0.0749*** (0.00447)	0.0608*** (0.00248)	0.0352*** (0.00183)	0.00425*** (0.000813)
Sample	Bin 1	Bin 2	Bin 3	Bin 4
Firm FE?	Yes	Yes	Yes	Yes
Obs.	33252	34418	34380	33733
R^2	0.0306	0.0680	0.0711	0.101

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.5.

As with Table 2, but not winsorized.

This table performs panel regressions of investment on lagged Tobin's q , using annual data from Compustat. Firms are sorted into bins based on the within-firm volatility of Tobin's q , with Bin 4 as the highest volatility. Standard errors are clustered by firm, and the table reports the within-firm R^2 of the regression.

	(1)	(2)	(3)	(4)
	$I_{tot;t}/K_{tot;t-1}$	$I_{tot;t}/K_{tot;t-1}$	$I_{tot;t}/K_{tot;t-1}$	$I_{tot;t}/K_{tot;t-1}$
$q_{tot,t-1}$	0.0670*** (0.00459)	0.0692*** (0.00297)	0.0683*** (0.00192)	0.0463*** (0.000879)
Sample	Bin 1	Bin 2	Bin 3	Bin 4
Firm FE?	Yes	Yes	Yes	Yes
Obs.	32982	34508	34424	33823
R^2	0.0216	0.0666	0.149	0.313

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.6.

As with Table 2, but using the total investment and q measures of [Peters and Taylor \(2017\)](#).

This table performs panel regressions of total investment on lagged total q , using annual data from Compustat. Firms are sorted into bins based on the within-firm volatility of total q , with Bin 4 as the highest volatility. Total investment and total q are winsorized at the 1st and 99th percentiles. Standard errors are clustered by firm, and the table reports the within-firm R^2 of the regression.

	(1)	(2)	(3)	(4)
	I_t/K_{t-1}	I_t/K_{t-1}	I_t/K_{t-1}	I_t/K_{t-1}
q_{t-1}	0.00789*** (0.00252)	0.00378*** (0.000846)	0.0204*** (0.00234)	0.0233*** (0.00222)
Sample	Non-high-tech	High-tech	Non-high-tech, pre-1995	High-tech, pre-1995
Firm FE?	Yes	Yes	Yes	Yes
Obs.	103959	31824	41215	8099
R^2	0.0407	0.124	0.0373	0.150

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.7.

As with Table 3, but not winsorized.

This table performs panel regressions of investment on lagged Tobin's q , using annual data from Compustat. "High-tech" refers to SIC codes 283, 357, 366, 367, 382, 384, and 737, following [Brown et al. \(2009\)](#). The data are annual Compustat from 1975-2015. Columns 3 and 4 restrict to pre-1995 firm-years. Standard errors are clustered by firm, and the table reports the within-firm R^2 of the regression.

	(1)	(2)	(3)	(4)
	$I_{tot;t}/K_{tot;t-1}$	$I_{tot;t}/K_{tot;t-1}$	$I_{tot;t}/K_{tot;t-1}$	$I_{tot;t}/K_{tot;t-1}$
L.q.tot	0.0529*** (0.00120)	0.0452*** (0.00106)	0.0528*** (0.00239)	0.0586*** (0.00325)
Sample	Non-high-tech	High-tech	Non-high-tech, pre-1995	High-tech, pre-1995
Firm FE?	Yes	Yes	Yes	Yes
Obs.	103946	31824	41209	8099
R^2	0.165	0.294	0.0898	0.239

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table B.8.

As with Table 3, but using the total investment and q measures of [Peters and Taylor \(2017\)](#).

This table performs panel regressions of total investment on lagged total q , using annual data from Compustat. "High-tech" refers to SIC codes 283, 357, 366, 367, 382, 384, and 737, following [Brown et al. \(2009\)](#). The data are annual Compustat from 1975-2015. Columns 3 and 4 restrict to pre-1995 firm-years. Total investment and total q are winsorized at the 1st and 99th percentiles. Standard errors are clustered by firm, and the table reports the within-firm R^2 of the regression.

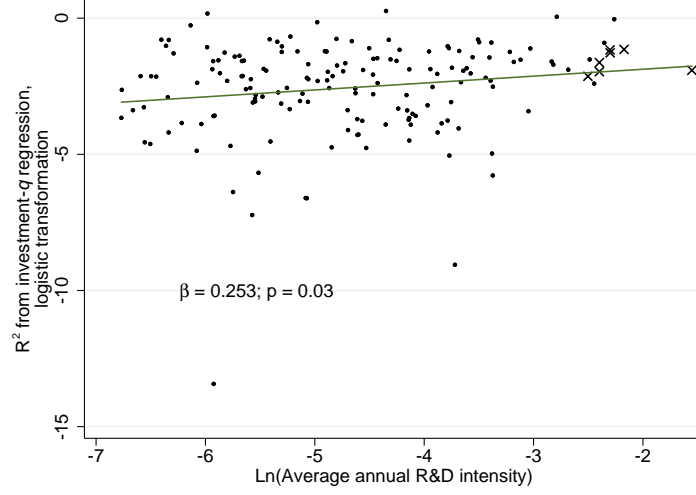


Fig. B.14. Industry-level investment- q correlations and R&D intensity.
As in Figure 8, but R^2 values are transformed via the function $\ln(R^2) - \ln(1 - R^2)$.

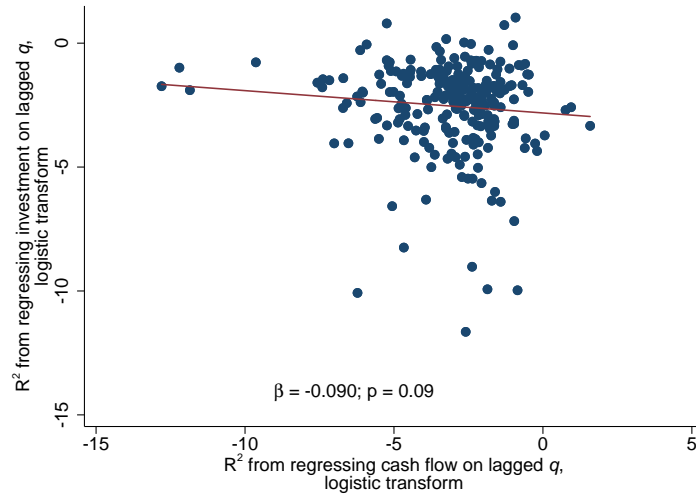


Fig. B.15. R^2 of investment- q regressions and cash-flow- q regressions by industry.
As in Figure 10 in the main paper, but R^2 values are transformed via the function $\ln(R^2) - \ln(1 - R^2)$.

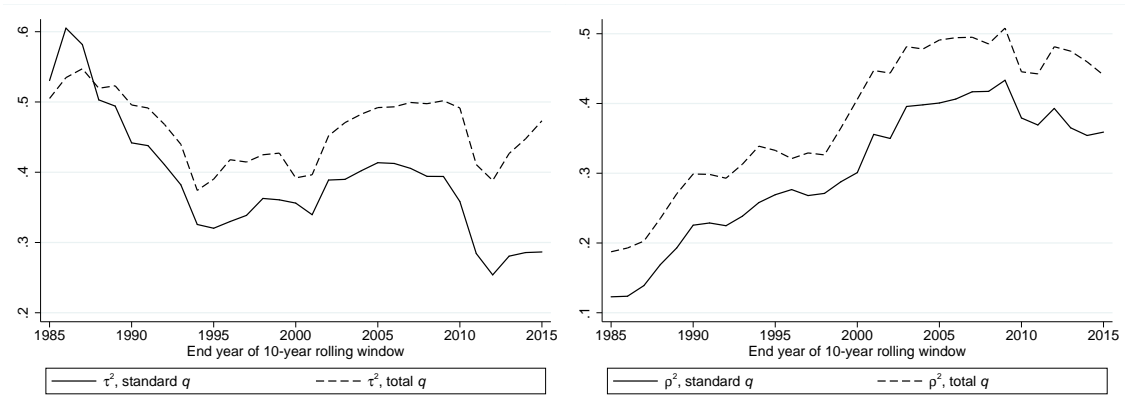


Fig. B.16. Examines trends in the ρ^2 and τ^2 statistics of Erickson et al. (2014), using ten year windows of data and three cumulants to identify the system. (This reproduces Figure 11 in the paper.)

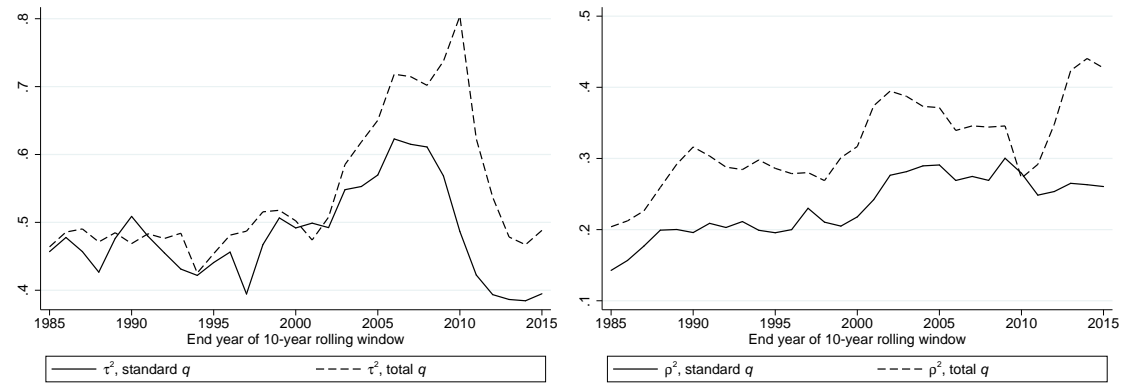


Fig. B.17. Ten year windows, five cumulants.

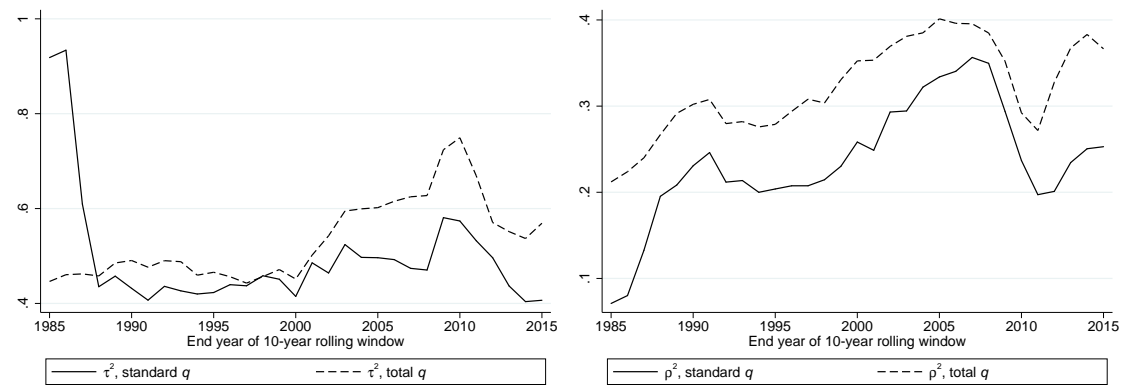


Fig. B.18. Ten year windows, seven cumulants.

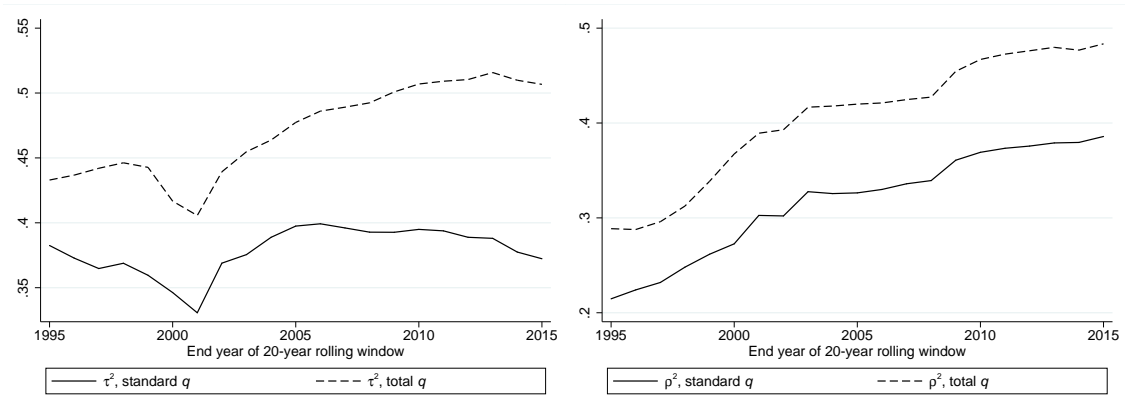


Fig. B.19. Twenty year windows, three cumulants.

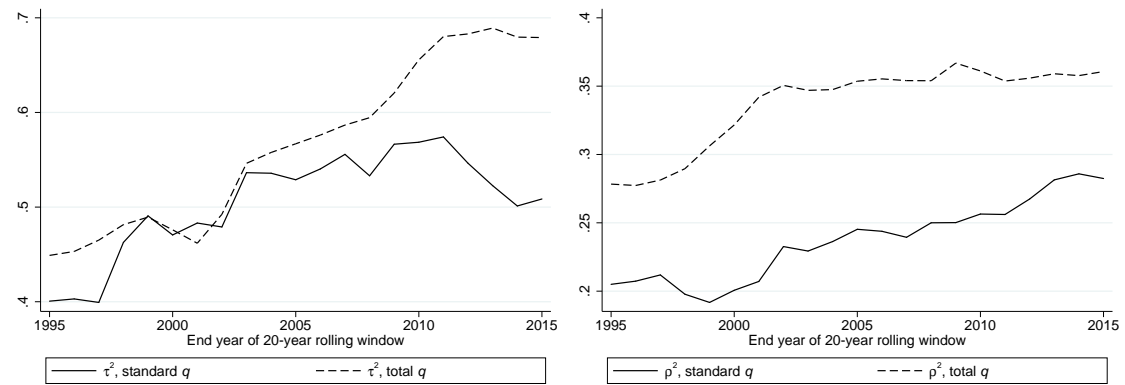


Fig. B.20. Twenty year windows, five cumulants.

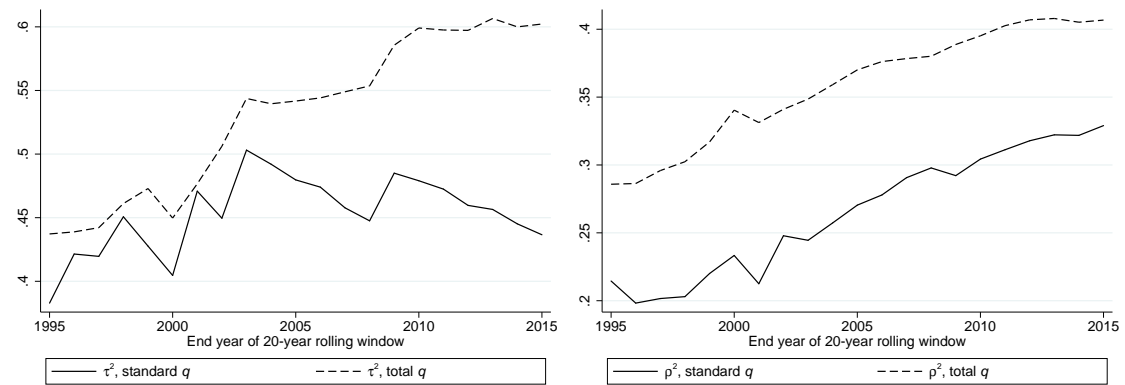


Fig. B.21. Twenty year windows, seven cumulants.

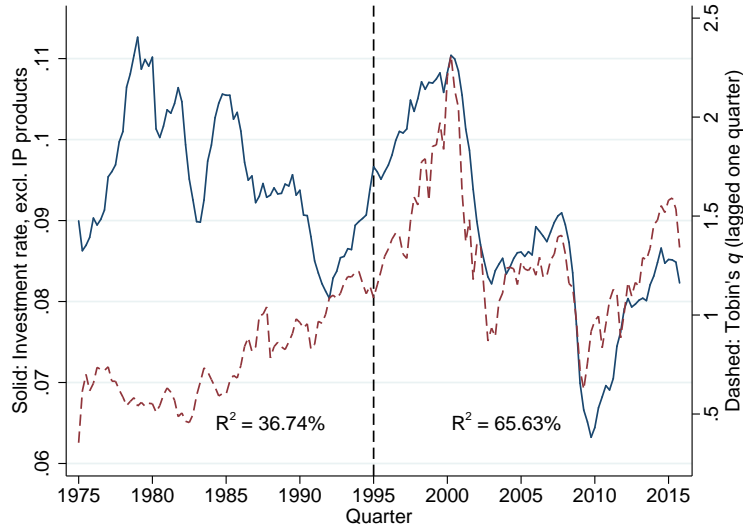


Fig. B.22. Repeats the analysis of Figure 1 in the paper, but the blue series is adjusted to exclude the stock of intellectual property products from the denominator, and to exclude spending on intellectual property products from the numerator. In the left half of the figure, the R^2 of the regression rises to 37% compared to the 6.5% in Figure 1. However, the estimated slope in the left half of the figure is negative ($\beta = -0.02$), while in the right half of the figure it is positive ($\beta = 0.03$).

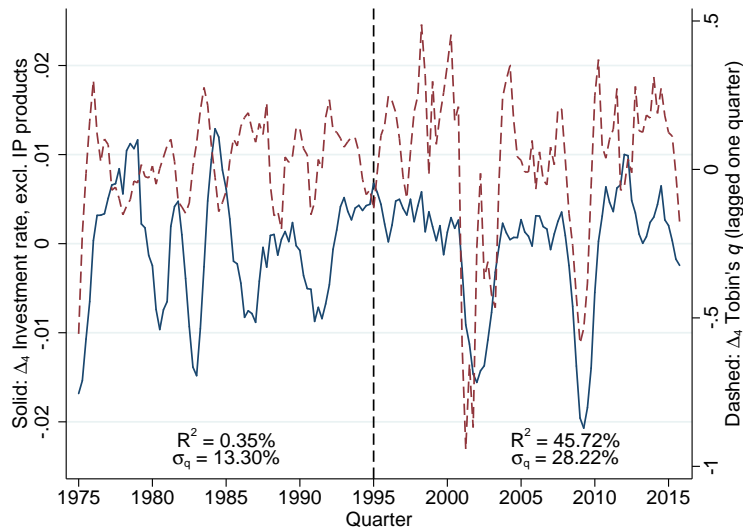


Fig. B.23. Repeats the analysis of Figure 2 in the paper, but the blue series is adjusted to exclude the stock of intellectual property products from the denominator, and to exclude spending on intellectual property products from the numerator.

Appendix C. Endogenous decision to invest in innovation and learning

This Appendix provides numerical results for Section 3.5 in the paper. The left panel in Figure C.24 plots the second derivative $q_{\Phi J}$ as a function of Φ , approximated using finite difference (this second derivative has the same sign as $\tilde{V}_{\Phi J}$). Each line in the plot corresponds to a different value of $J \in \{0.01, 0.025, 0.05\}$. The right panel plots the second derivative $q_{\Phi \sigma_\mu}$ as a function of Φ , approximated using finite difference. Each line in the plot corresponds to a different value of $\sigma_\mu \in \{0.02, 0.08, 0.14\}$. In these plots, the state variables are fixed at $\theta_t = \hat{\mu}_t = \bar{\mu}$ and $\nu_t = 0$ (the results remain the same with different values for the state variables). In both cases, the second derivative is positive at all times, consistent with the optimal purchase of research Φ^* increasing in J and in σ_μ .

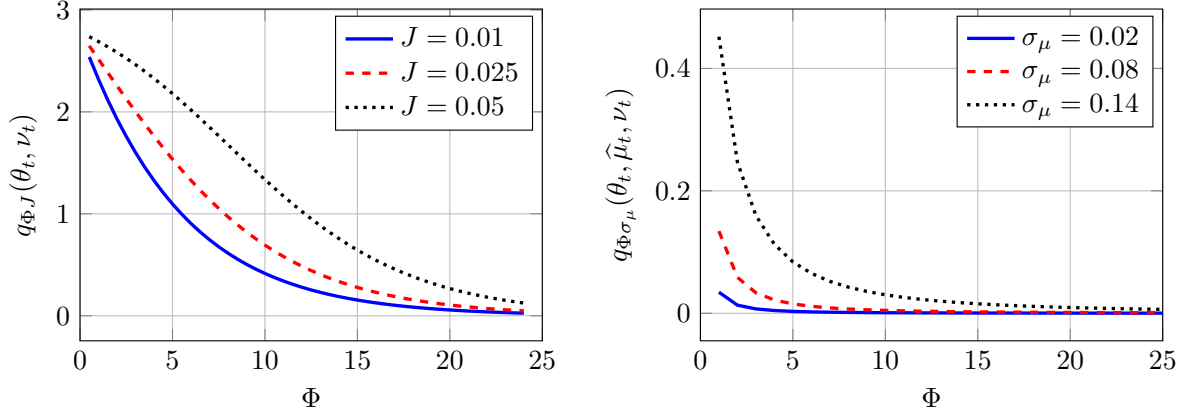


Fig. C.24. The left panel plots $q_{\Phi J}$ as a function of Φ in the model with innovation jumps. The right panel plots $q_{\Phi \sigma_\mu}$ as a function of Φ in the model with learning. These second derivatives are computed using the finite difference method. Each line in the left panel corresponds to a different value of $J \in \{0.01, 0.025, 0.05\}$. Each line in the right panel corresponds to a different value of $\sigma_\mu \in \{0.02, 0.08, 0.14\}$. The state variables are fixed at $\theta_t = \hat{\mu}_t = \bar{\mu}$ and $\nu_t = 0$. The rest of the calibration is given in Section 3.2 for the innovation model and in Section 3.3 for the learning model.

Appendix C.1. Two possible explanations for the falling investment- q regression slope among more R&D-intensive firms

This appendix discusses the decreasing slope across bins in Table 2 and Figure 4. Perhaps the simplest explanation for this effect is that R&D-intensive firms have more stringent adjustment costs. Panel A of Figure C.25 illustrates this effect. It depicts the relationship between investment and average q for two simulated samples in the model with learning (results are qualitatively similar in the model with innovation jumps). The blue triangles are simulated from a model without learning and with the adjustment cost parameter fixed at $a = 16$ (as in the baseline calibration). The red crosses are simulated from a model with learning ($\Phi = 20$) but with a higher adjustment cost parameter of $a = 25$. Each of the two models is simulated at yearly frequency for 100 years, and all variables are demeaned. Increasing the adjustment cost parameter lowers the slope of the regression (which equals $1/a$ in both cases). At the same time, learning improves the investment- q relation (the average R^2 over 5,000 simulations is 18% for the low adjustment cost case and 47% for the high adjustment cost case).

Another explanation for the same pattern is that successful R&D yields market power. We extend the setting of Section 3 to allow for market power, which we represent through decreasing returns to scale in the profit function (Cooper and Ejarque, 2003). Suppose the profit function is as follows:

$$\Pi(K_t, \theta_t) = \theta_t K_t^\alpha. \quad (\text{C.1})$$

If $\alpha < 1$, the Hayashi (1982) conditions are violated and marginal q no longer equals average q , so that the use of average q in the investment- q regression induces measurement error.

The firm's objective function leads to a linear relationship between investment and marginal q ,

$$\frac{I}{K} = -\frac{1}{a} + \frac{1}{a} V_K(K, x) + \frac{1}{a} \nu, \quad (\text{C.2})$$

Model	Market power	Calibration	(1) Avg(R^2)	(2) Avg(slope)	(3) Avg(σ_q)
Panel A: Model with innovation jumps					
(a) No innovation jumps	No ($\alpha = 1$)	$J = 0, \Phi = 0$	0.104	0.063	0.136
(b) Innovation jumps	Yes ($\alpha = 0.90$)	$J = 0.05, \Phi = 20$	0.150	0.046	0.172
Panel B: Model with learning					
(i) No learning	No ($\alpha = 1$)	$\sigma_\mu = 0, \Phi = 0$	0.179	0.063	0.184
(ii) Learning	Yes ($\alpha = 0.90$)	$\sigma_\mu = 0.15, \Phi = 20$	0.402	0.038	0.614

Table C.9.

Simulations with varying degrees of market power.

Panel A reports simulation results for the model with innovations. Row (1) considers a model without innovation and without market power ($J = 0, \alpha = 1$). In row (2), we increase the size of the innovations jumps J and firm's spending in research Φ , and decrease the returns to scale parameter α . Panel B reports simulation results for the model with learning. Row (i) considers a model without learning and with constant returns to scale. Row (ii) considers a model with learning and decreasing returns to scale. All other parameters are as in the baseline calibration (see Sections 3.2 and 3.3). Each simulation contains 100 yearly data points. The average R^2 coefficient and the average slope coefficient from 5,000 regressions of I/K on average q (V/K) are reported in columns (1) and (2). Column (3) reports the mean volatility of average q over the 5,000 simulations.

where $V_K(K, x)$ denotes marginal q , i.e., the shadow cost of capital, where $x = \{\theta, \nu\}$ in the innovation model, and $x = \{\theta, \hat{\mu}, \nu\}$ in the learning model.

With the innovation jumps of Section 3.2, we compare two specifications:

- (a) A model without innovation jumps and with constant returns to scale ($\alpha = 1$). This is a special case of the model analyzed in Section 3.2, in which marginal q equals average q .
- (b) A model with innovation jumps and decreasing returns to scale ($\alpha = 0.9$). In this specification, $J = 0.05$ and the firm's research spending is $\Phi = 20$.

With learning from Section 3.3, we compare two specifications:

- (i) A model without learning and with constant returns to scale ($\alpha = 1$). This is a special case of the model analyzed in Section 3.3, in which marginal q equals average q .
- (ii) A model with learning and decreasing returns to scale ($\alpha = 0.9$). In this specification, $\sigma_\mu = 0.15$ and the firm learns from cash-flow realizations and from the additional signal s_t , with $\Phi = 20$.

For all the above specifications, the calibration is otherwise the same as in Section 3. The values chosen for the parameter α are in line with calibrations used in the literature (e.g., Gomes, 2001). Each of the four models above is simulated 5,000 times at yearly frequency for 100 years. For each simulation, we run the investment- q regression using average q (which equals V/K) as a proxy for marginal q . Table C.9 presents the R^2 coefficient, the slope of the regression, and the volatility of average q , where all reported statistics are averaged over the 5,000 simulations.

Column (1) shows that the R^2 coefficient increases with Φ , which is our main result: firms that invest more in innovation and learning exhibit higher R^2 coefficients for the investment- q regression. Column (2) shows that the slope of the investment- q regression decreases with market power, both in the model with innovation jumps and in the model with learning. It might seem that this decreasing slope should also lead to a decrease in R^2 , but the offsetting impact of research leads to a net increase in R^2 . The reason for the increase in R^2 is the increase in the volatility of average q , as shown in column (3).

Panels B and C of Figure C.25 depict the relationship between investment and average q , where the blue triangles are generated from simulating models (a) and (i) in Table C.9 (i.e., without innovation or learning, and with constant returns to scale), and the red crosses are generated from simulating models (b) and (ii) in Table C.9 (i.e., with innovation or learning, and decreasing returns to scale). Both panels reproduce qualitatively the pattern of Figure 4, where the R^2 coefficients increase as the slope coefficients decrease. Indeed, while decreasing returns to scale dampen the slope coefficients, innovation and learning induce a higher volatility of average q , which ultimately leads to a higher R^2 . In our simulations, Tobin's q is on average higher with decreasing returns to scale, in line with the intuition from Lindenberg and Ross (1981)

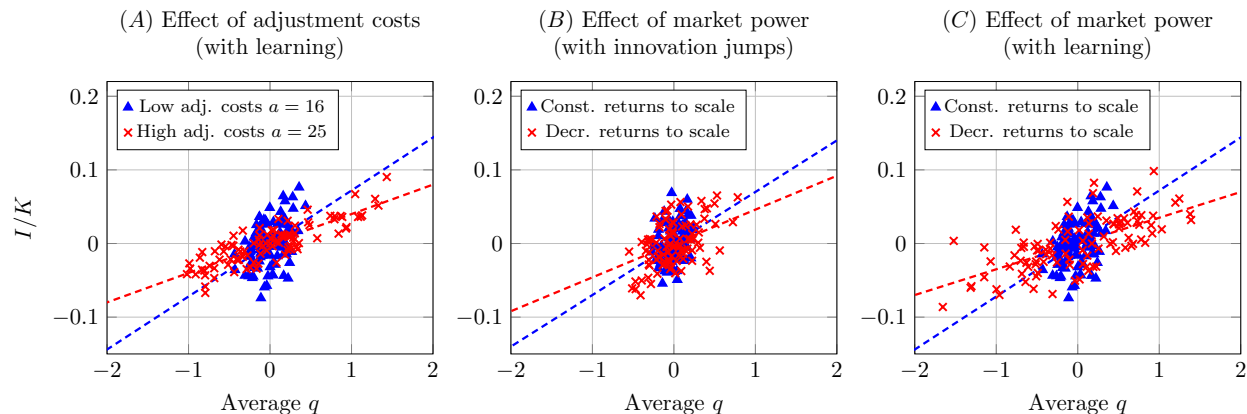


Fig. C.25. Two possible explanations for the pattern in Figure 4 (decreasing slopes along with increasing R^2 coefficients).

In panel (A), firms that learn more, and hence have a more volatile q , also have a higher adjustment cost parameter a . In panels (B) and (C), firms that innovate more or learn more also gain market power. In the three panels, the triangles correspond to a model without innovation and without learning. In panels (A) and (C), the crosses correspond to a model with learning and $\Phi = 20$. In panel (B) the crosses correspond to a model with innovation and $J = 0.05, \Phi = 20$. The lines depict the fitted relationships between investment and q . The simulations are performed over 100 years at an annual frequency.

that q should persist above one for firms with monopoly rents. This is not apparent from the figure, because the variables are demeaned, but it is worth mentioning.

Our findings in this section echo a result in [Abel and Eberly \(1994\)](#). In their Lemma 2, they show that marginal q and average q can be proportional even in the presence of decreasing returns to scale. The investment- q regression can therefore exhibit a low slope without necessarily leading to a poor fit of the regression. The slope and R^2 are indeed two different diagnostics for different features of the q theory of investment.¹

References

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¹We thank Andrew Abel for pointing out this implication to us.