

# Why Did the $q$ Theory of Investment Start Working?

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## Abstract

We show that the relationship between aggregate investment and Tobin's  $q$  has become remarkably tight in recent years, contrasting with earlier times. We connect this change with the growing empirical dispersion in Tobin's  $q$ , which we document both in the cross-section and the time-series. To study the source of this dispersion, we augment a standard investment model with two distinct mechanisms related to firms' research activities: innovations and learning. Both innovation jumps in cash flows and the frequent updating of beliefs about future cash flows endogenously amplify volatility in the firm's value function. Perhaps counterintuitively, the investment- $q$  regression works better for research-intensive industries, a growing segment of the economy, despite their greater stock of intangible assets. We confirm the model's predictions in the data, and we disentangle the results from measurement error in  $q$ .

Keywords: Investment, Tobin's  $q$ , research and development

JEL classification: E2, G3, O3

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## 1. Introduction

The  $q$  theory of investment predicts a strong relationship between corporations' market values and their investment rates. Hayashi (1982) provides justification for measuring marginal  $q$  with a valuation ratio, average  $q$  (also known as Tobin's  $q$ ), so that a simple regression of investment on Tobin's  $q$  should have a strong fit. Researchers have found that this regression in fact performs quite poorly. While the Hayashi model assumptions may not hold exactly in the data, the stark disconnect between investments and valuations has piqued the interest of financial economists. A large literature investigates the potential reasons why Tobin's  $q$  does not explain investment well in the data, pointing to the existence of financial constraints, decreasing returns to scale, inefficient equity-market valuations, and measurement problems, among other things.<sup>1</sup>

Curiously, even as this literature has continued to grow, the stylized fact has changed. Using data from the BEA's NIPA tables and the Fed's Flow of Funds, we document that the aggregate investment- $q$  regression has worked remarkably well in recent years. The simple regression achieves an  $R^2$  of 70% during 1995–2015, comparable to the empirical performances of the bond price  $q$  regression proposed in Philippon (2009) and the total tangible and intangible investment- $q$  regression in Peters and Taylor (2017).<sup>2</sup> If one were to test the simple theory using data from recent years, one would conclude that the  $q$  theory of investment is in fact an empirical success.

Yet this recent development only deepens the puzzle, as problems with  $q$  theory highlighted by the literature seem to have *worsened* in recent years. For example, Peters and Taylor (2017) focus on the failure to measure intangible assets, which have grown substantially in the aggregate, and Philippon (2009) focuses on measuring  $q$  via bond markets to avoid relying on equity market valuations, which are increasingly volatile and may seem unreliable. We show that, counterintuitively, it is precisely the growing volatility in valuations, *especially* in intangible-intensive industries, that has contributed to the revived empirical performance of the classic regression.

Before presenting the model, we establish several stylized facts. First, the volatility of aggregate  $q$  in the data is higher precisely during the years when the aggregate investment- $q$  regression performs better. Second, the within-firm variation of Tobin's  $q$  in Compustat has risen steeply since the late 1990s. Finally, the panel version of the investment- $q$  regression also fits much better when Tobin's  $q$  is more volatile. These stylized facts support our intuition: the empirical performance of the theory hinges critically on the amount

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<sup>1</sup>For examples, see Fazzari et al. (1988), Gilchrist and Himmelberg (1995), Kaplan and Zingales (1997), Erickson and Whited (2000), Gomes (2001), Cooper and Ejarque (2003), Moyen (2004), Philippon (2009), Abel and Eberly (2011), and Peters and Taylor (2017).

<sup>2</sup>This regression uses the BEA's current definition of fixed assets, which was restated retroactively in 2013 to include intellectual property products. In the online Internet Appendix, we show that even without this restatement the  $R^2$  remains at 65%. See further discussion in Section 2.

of endogenous variation that one finds in Tobin’s  $q$ .

To explain these recent developments, we propose two mechanisms related to firms’ research activities. In our first mechanism, we recognize that, although firms invest in research to increase their future cash flows, innovations may or may not materialize. The firm’s increased cash flow volatility from innovations directly feeds into the volatility of Tobin’s  $q$  valuations, generating a better fit in the investment- $q$  regression. In the alternative mechanism, we focus on the learning taking place within research-intensive firms. Research causes faster updating of beliefs about future cash flows, which endogenously produces more variation in  $q$ . This learning-induced variation is informative about the firm’s investment policy. With either mechanism, the main result is that the investment- $q$  regression works better when there is more endogenous variation in the regressor  $q$ . This provides a simple, yet previously unexplored explanation behind the poor fit of the regression. The culprit is the historically low variation in Tobin’s  $q$  relative to residual factors affecting investment.

Turning to the specific features of the model, we study a standard  $q$ -theoretic investment framework, most closely resembling the setting of [Abel \(2017\)](#). In the innovation-based model, we augment the setting by assuming that research expenses increase the probability of innovations. We represent innovations as jumps in the stochastic process for the mean-reverting cash flows. In the learning-based model, we assume that the long-term cash-flow mean evolves over time, and the firm can never fully learn. Research expenses allow the firm to acquire, at a cost, informative signals about the time-varying cash-flow mean. These features provide theoretical foundations for the stochastic variation in marginal  $q$ . We show that both innovations and learning endogenously amplify the volatility of marginal  $q$ , thereby improving the fit of the investment- $q$  regression.

An empirical implication is that firms spending more in research and development (R&D) should feature a tighter fit between investment and Tobin’s  $q$ . At first glance, this prediction seems counterintuitive because research creates an intangible asset, and therefore a measurement error when accounting only for tangible capital in Tobin’s  $q$  as discussed in [Peters and Taylor \(2017\)](#). Our model abstracts from this measurement error, and our empirical findings point to a large offsetting effect.

In the cross-section of firms in Compustat, industries featuring greater investment in R&D, higher rates of patenting, and greater intangibility have noticeably higher  $R^2$  values in their investment- $q$  panel regressions compared to those from the average industry. This stylized fact is documented in [Peters and Taylor \(2017\)](#) and earmarked as a puzzle. Our model provides an explanation, by predicting that research-intensive firms exhibit greater volatility in Tobin’s  $q$ . We confirm that the better fit in high-tech industries was present even before the aggregate regression fit began to improve, so it is not driven simply by the fact that these firms are more common later in the sample. Rather, as high-tech firms have become a larger segment of the

economy, their greater endogenous volatility in Tobin’s  $q$  has caused the aggregate regression to improve.

The correlation between cash flows and Tobin’s  $q$  allows us to assess the relative importance of innovation versus learning as mechanisms for our findings. With innovation jumps in the cash flow process, the cash flow volatility imparted by the jumps transfers directly to Tobin’s  $q$ , as there is a tight connection between cash flows and Tobin’s  $q$ . In contrast, the learning mechanism requires the firm and its investors to take into account more signals than just observing cash flow realizations. More learning therefore decreases the correlation between cash flows and Tobin’s  $q$ . While either mechanism increases the  $R^2$  of the investment- $q$  relationship, they have contrasting predictions for the correlation between cash flows and Tobin’s  $q$ . We find a negative relationship between the  $R^2$  of the investment- $q$  relationship and the  $R^2$  of the Tobin’s  $q$ -cash flow relationship, consistent with the learning mechanism. While innovation jumps may also be an important mechanism behind our findings, this evidence emphasizes the role of updating investment decisions and valuations based on multiple signals.

In sum, we find that the classic  $q$  theory of investment works surprisingly well in recent years, and counterintuitively it works best for firms with high volatilities in equity valuations, high levels of R&D investment, and low levels of tangibility. Our findings have several general implications. They suggest that models based on innovations and learning may be particularly well-suited to study corporate investment behavior. They also suggest that Tobin’s  $q$ , “arguably the most common regressor in corporate finance” (Erickson and Whited, 2012), may be a better empirical proxy for the firm’s investment opportunities than previously thought.

An empirical paper closely related to ours is Peters and Taylor (2017). They adjust the investment- $q$  regression variables to include intangible capital. While we report results using the classic definitions of investment and  $q$  in keeping with the previous literature, all results continue to hold when these quantities are adjusted for intangibles with the “total” investment and  $q$  series.<sup>3</sup>

Another related empirical paper is Gutiérrez and Philippon (2016). They highlight that aggregate investment has trended downward while aggregate Tobin’s  $q$  has trended upward, a divergence they attribute to weakened competition and governance in the US. Our analysis is mostly silent on the levels of investment and  $q$ , and focuses instead on the correlations, which have improved in recent years. However, in the online Internet Appendix, we investigate the effect of weakened competition. We extend our model to demonstrate how greater market power can generate a lower Tobin’s  $q$  slope and yet a higher investment- $q$  regression  $R^2$ . Lindenberg and Ross (1981) and Cooper and Ejarque (2003) also examine competition-related implications on Tobin’s  $q$ .

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<sup>3</sup>Results are available in Appendix B of the online Internet Appendix.

Our paper builds on a long theoretical literature investigating the  $q$  theory of investment.<sup>4</sup> The most closely related theory paper to ours is [Abel \(2017\)](#). For the innovation-based mechanism, we borrow insights from the endogenous growth literature, which analyzes the role of technological innovations in promoting economic growth.<sup>5</sup> Our paper is also related to the widely studied relationship between firms' R&D innovations and their growth since [Griliches \(1979\)](#).<sup>6</sup>

For the learning-based model, we simply assume that learning symmetrically and simultaneously occurs for the firm and the market alike as in [Alti \(2003\)](#), [Moyen and Platikanov \(2012\)](#), and [Hennessy and Radnaev \(2018\)](#). In contrast, many papers study settings in which managers are assumed to possess superior information compared to outsiders (e.g. [Myers and Majluf, 1984](#)), or the reverse channel in which managers extract information from their stock prices when making investment decisions.<sup>7</sup> We choose the simple, symmetric learning framework as it is powerful enough to provide the empirical prediction of interest, namely that the fit of the investment- $q$  regression improves when firms are more intensely engaged in research and learning.

Finally, our paper complements the recent literature showing that financial markets have become more informative in recent years. [Bai et al. \(2016\)](#) argue that the recent rise in price informativeness is due to greater information production in financial markets. [Chen et al. \(2007\)](#) and [Bakke and Whited \(2010\)](#) document a stronger relationship between stock prices and investment for firms with more informative stock prices, whereas [Dow et al. \(2017\)](#) demonstrate how the information production in financial markets can amplify business cycles. [Durnev et al. \(2004\)](#) show that greater firm-specific variation in stock returns is associated with greater measures of investment efficiency. The theoretical models of [Farboodi et al. \(2017\)](#) and [Begenau et al. \(2017\)](#) describe how this rise in price informativeness affects capital allocation in the economy. In line with this growing body of evidence, we document a remarkable improvement in the relationship between investment and  $q$  in recent years.

The rest of the paper is organized as follows: Section 2 establishes the motivating empirical facts related to the empirical dispersion in Tobin's  $q$  and the fit of the investment- $q$  regression. Section 3 builds an investment model with innovations and learning that endogenizes volatility in  $q$  and derives testable implications. Section 4 returns to the data and investigates the implications of the model. Section 5 concludes.

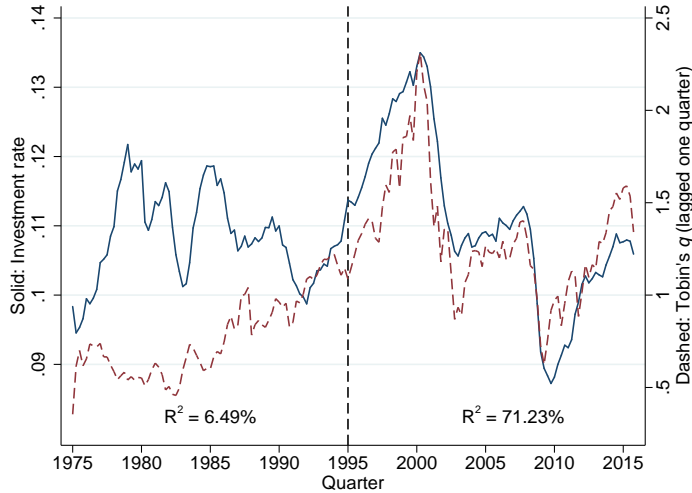
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<sup>4</sup>Foundational contributions to Tobin's  $q$  theory originate from [Keynes \(1936\)](#), [Brainard and Tobin \(1968\)](#), [Tobin \(1969\)](#), [Mussa \(1977\)](#), [Lindenberg and Ross \(1981\)](#), [Abel \(1983\)](#), and [Salinger \(1984\)](#), among many others.

<sup>5</sup>For example, see the seminal papers of [Romer \(1990\)](#), [Grossman and Helpman \(1991\)](#), and [Aghion and Howitt \(1992\)](#).

<sup>6</sup>See the literature surveys of [Mairesse and Sassenou \(1991\)](#) and [Hall et al. \(2009\)](#), as well as the more recent contributions of [Aw et al. \(2011\)](#), [Bloom et al. \(2013\)](#), and [Doraszelski and Jaumandreu \(2013\)](#).

<sup>7</sup>See [Subrahmanyam and Titman \(1999\)](#), [Bresnahan et al. \(1992\)](#), [Dow and Gorton \(1997\)](#), [Goldstein and Guembel \(2008\)](#), and [Edmans et al. \(2015\)](#), among others.



**Fig. 1.** Aggregate quarterly investment rate and lagged Tobin's  $q$ .

The solid blue line plots private nonresidential fixed investment divided by the lagged stock of private nonresidential fixed assets. The dashed red line plots the lagged value of Tobin's  $q$ , calculated as the value of corporate equity and liabilities, less inventories and financial assets, divided by the stock of corporate fixed assets. See [Appendix A.1](#) for details about the construction of the series, which follows [Hall \(2001\)](#).

## 2. Stylized empirical facts

### 2.1. Improved fit of the aggregate regression

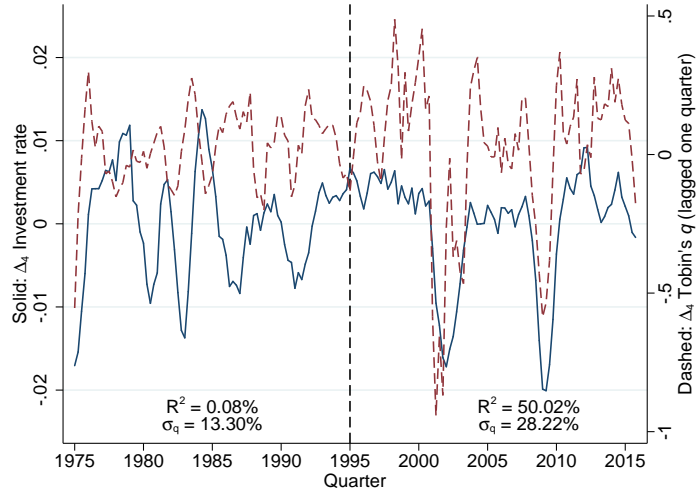
We first document that the aggregate investment- $q$  regression has performed much better in recent years. To do so, in [Figure 1](#) we plot and compare aggregate investment, and lagged aggregate Tobin's  $q$ , from 1975 to 2015. To construct these series, we use quarterly data from the BEA's NIPA tables and the Fed's Flow of Funds, following the steps described in [Hall \(2001\)](#). Details related to the construction of these series are provided in [Appendix A.1](#).

The investment- $q$  regression is specified as

$$\frac{I_{t+1}}{K_t} = \alpha + \beta q_t + \varepsilon_t, \quad (1)$$

where  $t$  indexes quarters,  $I$  is investment in fixed assets,  $K$  is the stock of fixed assets, and  $q$  is measured as the ratio of corporate financial value less financial assets and inventories to the stock of corporate fixed assets. The  $R^2$  from regression (1) has been the primary focus of the empirical literature assessing the performance of the  $q$  theory of investment. In [Figure 1](#) we plot the investment and Tobin's  $q$  time series behind this regression in order to assess its performance.

The figure is divided into two subperiods of 20 years each. At the bottom of each subperiod is the  $R^2$  value that would be obtained from the standard regression of aggregate investment rate on lagged  $q$  using only the data from that subperiod. During the first subperiod, 1975-1995, the relationship between aggregate investment and Tobin's  $q$  is disappointingly weak, and the standard regression achieves an  $R^2$  of only 6.5%.



**Fig. 2.** Year-over-year differences of aggregate quarterly investment rate and lagged Tobin's  $q$ . The series are constructed as in Figure 1.

This fact has been widely confirmed, e.g., Philippon (2009), Table III (top panel, second column). As a result, modern empirical research often describes the investment- $q$  regression as an empirical failure. Many papers attempt to improve the classic regression in various ways, as discussed above. But in the second subperiod, 1995-2015, the investment- $q$  regression performs much better. From 1995-2015, the  $R^2$  is over 70%. Looking only at the more recent past, one would conclude that the simple regression implementation of  $q$  theory is in fact a resounding success.<sup>8</sup>

Figure 2 performs a similar analysis in differences. The solid blue and dashed red series are the year-over-year differences of the series from Figure 1. The  $R^2$  values from the regression within the two 20-year subperiods are listed at the bottom of the figure, and they suggest the same conclusion as in Figure 1. The  $R^2$  of the investment- $q$  regression rose from less than 1% in 1975-1995 to roughly 50% in 1995-2015.

Also listed at the bottom of each subperiod in Figure 2 are the volatilities of the explanatory variable in the regression, differenced Tobin's  $q$ . These figures provide motivating evidence for our core mechanism. The volatility of Tobin's  $q$  is lower during the subperiod in which the investment- $q$  regression performs worse, and it is higher during the subperiod in which the regression performs better.<sup>9</sup>

Under the null hypothesis that the model is true, the investment- $q$  regression should yield a higher  $R^2$

<sup>8</sup>Note that private nonresidential fixed assets were redefined in 2013 to include intellectual property products, which partially reflects the Peters and Taylor (2017) concept of total investment that includes R&D expense (although they also include spending on organizational capital measured through SG&A expense). Figures B.22 and B.23 the online Internet Appendix repeat the analysis excluding the 2013 adjustment, and show that the conclusions of Figures 1 and 2 remain the same. Intuitively, this category of spending has trended upward in recent years but is very stable (similar to corporate R&D), so that it has little effect on the  $R^2$  of the aggregate regressions.

<sup>9</sup>Figure B.12 in the online Internet Appendix demonstrates patterns similar to Figure 2 using annual Compustat data, which is the data source for the cross-sectional empirical analysis in this paper.

when there is more dispersion in the key explanatory variable, Tobin’s  $q$ . Thus, based on our results, one possible explanation for the improved fit of the aggregate regression is that the theory has always been “true,” but that Tobin’s  $q$  has become more volatile relative to the model’s residuals. In the next section, we use panel data from Compustat to examine more closely the empirical dispersion in Tobin’s  $q$  and establish this pattern.

## 2.2. Increased dispersion in Tobin’s $q$

Shifting our focus from the aggregate series discussed above, we next reconstruct the series of investment and valuation at the firm level. Using annual panel data on publicly-traded firms from Compustat, we confirm and explore the growing empirical dispersion in Tobin’s  $q$ .

To construct the dataset for this analysis, we accessed the annual Compustat database on September 14, 2017. We retain observations from 1975 to 2015, and drop financial firms (SIC codes 6000–6999), utilities (4900–4999), and public administration firms (9000–9999). We remove the in-process component of R&D by adding in Compustat variable *rdip*. Following [Peters and Taylor \(2017\)](#), we drop any observations with less than \$5 million in gross property, plant, and equipment (PP&E), as well as any observations with negative or missing total assets or sales.

We measure the market value of equity as  $prcc\_f \times csho$ . We define the numerator of Tobin’s  $q$  as the market value of equity plus book value of total debt minus current assets; and its denominator as gross PP&E. Physical investment is capital expenditures divided by lagged gross PP&E. “Total” investment is capital expenditures plus R&D and 30% of SG&A, divided by the lagged sum of gross PP&E and intangible capital. The intangible capital series is downloaded from the online resources for [Peters and Taylor \(2017\)](#), from which we also obtain the series of total  $q$  for some of our analysis. Cash flow is measured as income before extraordinary items plus depreciation divided by gross PP&E.

Finally, we drop observations with missing values of investment, Tobin’s  $q$ , or cash flow, and we winsorize cash flow,  $q$  (both standard and total), and investment (both physical and total) at the 1st and 99th percentiles. Table 1 displays summary statistics for this sample.

In Figure 3, we investigate how Compustat firms have changed over time by examining the within-firm volatility. To create this figure, we proceed in two steps. First, we calculate for each Compustat firm the within-firm volatility of Tobin’s  $q$  during its entire lifetime in Compustat. This creates volatility measures of valuation that are fixed at the firm level. Next, for each year, we average these fixed volatility numbers across all firms that are present in Compustat that year. The series is thus driven by changes in the composition of Compustat firms. Finally, the series is smoothed evenly over a five-year lag.

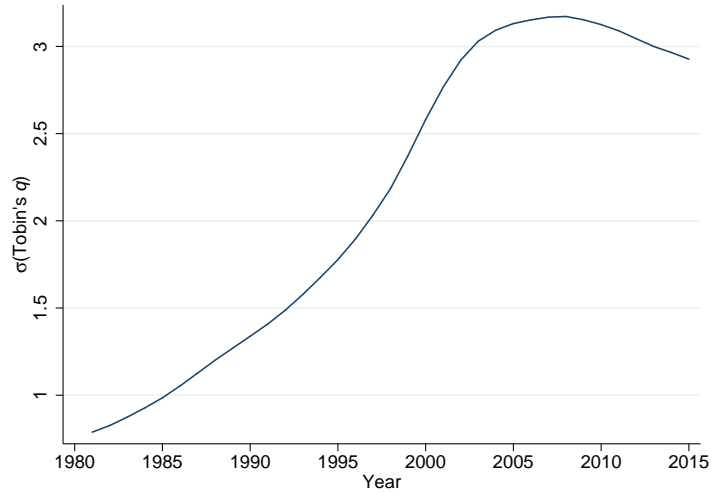
Figure 3 reveals that within-firm volatilities of Tobin’s  $q$  have greatly increased relative to their 1980



	Mean	Median	Std. dev.
Capital expenditures	176.67	10.54	1080.13
Property, plant, and equipment (gross)	1819.43	89.79	11310.32
Investment rate	0.18	0.11	0.21
Total investment rate	0.21	0.16	0.18
Tobin's $q$	2.93	0.96	6.13
Total $q$	0.99	0.57	1.51
Volatility of Tobin's $q$	2.26	0.94	3.24
Volatility of total $q$	0.76	0.46	0.79
Cash flow	0.09	0.13	0.55

**Table 1.**

This table displays summary statistics from the annual Compustat sample, 1975–2015. Investment rate is capital expenditures divided by lagged gross property, plant, and equipment (PP&E). Total investment rate is capital expenditures plus research and development expense plus 30% of selling, general, and administrative expense, scaled by lagged total capital (gross PP&E plus intangible capital, where the latter is provided in [Peters and Taylor, 2017](#)). Tobin's  $q$  is the market value of equity plus book value of debt minus current assets, divided by gross PP&E. Total  $q$  is provided in [Peters and Taylor, 2017](#). Volatilities of Tobin's  $q$  and total  $q$  are calculated within-firm. Cash flow is income before extraordinary items plus depreciation, divided by gross PP&E. All ratios are winsorized at the 1st and 99th percentiles.



**Fig. 3.** Within-firm dispersion in Tobin's  $q$ , 1980-2015.

For each firm in Compustat, we calculate the within-firm volatility of Tobin's  $q$  during that firm's entire lifetime in Compustat. We then average that firm-level measure across all firms in Compustat for each year, and drop any firms for which this average is zero. The series is thus driven by changes in the composition of Compustat firms. Finally, the series is smoothed evenly over a five-year lag.

	(1)	(2)	(3)	(4)
	$I_t/K_{t-1}$	$I_t/K_{t-1}$	$I_t/K_{t-1}$	$I_t/K_{t-1}$
$q_{t-1}$	0.0739*** (0.00408)	0.0575*** (0.00215)	0.0329*** (0.00105)	0.0116*** (0.000298)
Sample	Bin 1	Bin 2	Bin 3	Bin 4
Firm FE?	Yes	Yes	Yes	Yes
Obs.	33190	34424	34386	33783
$R^2$	0.0348	0.0832	0.121	0.199

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

**Table 2.**

This table performs panel regressions of investment on lagged Tobin’s  $q$ , using annual data from Compustat.

Firms are sorted into bins based on the within-firm volatility of Tobin’s  $q$ , with Bin 4 as the highest volatility. Standard errors are clustered by firm, and the table reports the within-firm  $R^2$  of the regression. Investment and Tobin’s  $q$  are winsorized at the 1st and 99th percentiles.

values. Tobin’s  $q$  volatility has more than tripled. The increase in Tobin’s  $q$  volatility is especially noticeable in the late 1990s and early 2000s. The series thus show that the composition of Compustat has shifted towards firms with higher volatilities.

### 2.3. Better performance for firms with more volatile $q$

Motivated by the Compustat evidence above which shows that firms are exhibiting greater volatility of  $q$ , we next demonstrate that the investment- $q$  regression works better where this volatility is greater.

The first point to make is that the within-firm volatility of Tobin’s  $q$  varies by orders of magnitude across firms. We sort Compustat firms into four bins of within-firm  $q$  volatility, and find that the average volatility in the lowest bin is 0.24, while in the highest bin it is 6.6.<sup>10</sup>

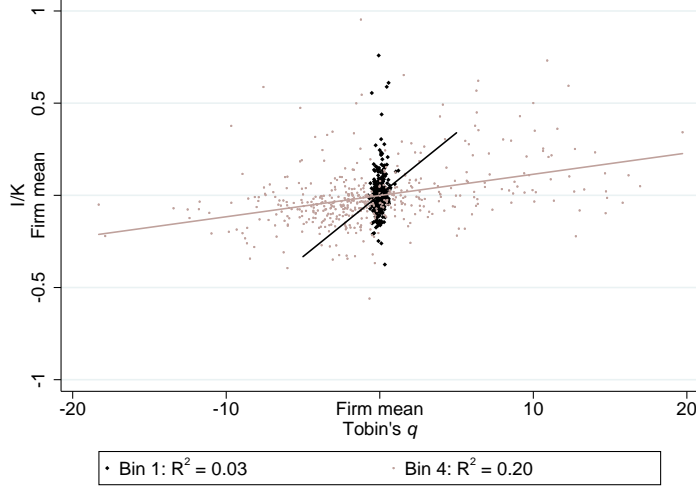
The bin with the highest-volatility firms is where we find that the investment- $q$  relationship is the tightest. To show this, we estimate standard panel regressions of investment on lagged Tobin’s  $q$ :

$$\frac{I_{i,t+1}}{K_{it}} = \alpha_i + \beta q_{it} + \varepsilon_{it}, \quad (2)$$

where  $i$  indexes firms,  $t$  indexes years,  $I$  is capital expenditures,  $K$  is gross property, plant, and equipment,  $q$  is defined as  $\frac{V}{K}$ , where  $V$  is the market value of equity plus book value of debt minus current assets. All of these definitions are taken from [Peters and Taylor \(2017\)](#).

Table 2 performs this regression separately across four bins, with Bin 1 as the lowest within-firm volatility in Tobin’s  $q$  and Bin 4 as the highest. The table confirms that the regression fit improves when Tobin’s  $q$  is more volatile: The  $R^2$  value, which in all cases is calculated after taking out the firm fixed effects, is monotonically increasing across the bins of volatility in Tobin’s  $q$ .

<sup>10</sup>Related, [Erickson and Whited \(2000\)](#) observe that Tobin’s  $q$  is highly skewed in the data, which aids the identification of their strategy based on higher-order moments.



**Fig. 4.** Within-firm correlations of Tobin’s  $q$  and investment.

The scatter plot corresponds to Table 2. Each dot represents a firm-year observation, of which 500 are sampled from the lowest and highest bins of within-firm volatility in Tobin’s  $q$ . The x-axis measures Tobin’s  $q$ , and the y-axis measures the investment rate. Both numbers are demeaned within-firm to remove the firm fixed effect, so zeros on the axes correspond to the firm mean values. For each bin, the best-fit line of the same color reflects the regression in Table 2.

Figure 4 visually illustrates how the volatility in the data gives rise to these results. It samples 500 observations randomly from each of the lowest and the highest bins of volatility, and plots the investment rate against the value of Tobin’s  $q$  for each observation, along with regression lines with slopes that correspond to the coefficients in Table 2. To relate to the fixed-effects regression, investment rate and Tobin’s  $q$  are first demeaned within-firm. The lowest-volatility bin shows no particular relationship between  $q$  and the investment rate, while the highest-volatility bin illustrates a fairly tight relationship.

In Table 2 and Figure 4, it may seem puzzling that the slope of the regression falls across bins of Tobin’s  $q$  volatility, even as the  $R^2$  increases. Mathematically, to reconcile these findings, one needs an offsetting source of higher volatility in Tobin’s  $q$ . Our analysis shows that innovation and learning are mechanisms for generating that volatility. However, why the slope should fall across bins of volatility is a separate and interesting question on its own.

As we will show later, high-tech firms exhibit more volatility in  $q$  than other firms. This suggests several potential, related explanations for the pattern of falling slopes. One simple explanation could be that the omission of intangible investment in Figure 4 is more important for higher-volatility firms. A second explanation could be that the same firms have more convex adjustment costs, since the standard Tobin’s  $q$  model (presented below) specifies an inverse relationship between the adjustment cost convexity and the slope. This would be consistent with the suggestion in Peters and Taylor (2017) that intangible firms face more convex adjustment cost for physical investment. A third could be that the same firms have more market power, which causes the slope of the regression to fall, as argued in Cooper and Ejarque (2003). This

last explanation applies naturally to R&D-intensive firms, which are often characterized by market power gained from innovation.

In the online Internet Appendix, we consider each of these explanations: First, we show that the pattern of falling slopes across volatility bins largely disappears when using the total investment and  $q$  measures of [Peters and Taylor \(2017\)](#). Second, in an extension to our main model, we study firms with heterogeneous adjustments costs and decreasing returns to scale, and in both cases we are able to reproduce the patterns in Table 2 and Figure 4. However, our main focus is on mechanisms behind the volatility of Tobin's  $q$ , not behind the slope of the regression. Therefore, in the main analysis we study firms without any of these features, in which case the slope of the investment- $q$  regression should theoretically be the same for all firms.

If the large volatility in  $q$  is meaningless for investment, the improving fit of the investment- $q$  regression should not obtain. Greater variation in  $q$  provides the opportunity for the investment- $q$  regression to work, but does not force it to do so. Instead, our findings suggest that the information reflected in equity market valuations is tightly connected to investment policies, and this relationship becomes the clearest when valuations move the most.

In untabulated results, we observe that the pattern is also robust to adding year fixed effects; to excluding all fixed effects; and to sorting on the stock price volatility instead of Tobin's  $q$  volatility, confirming that higher volatility comes from the numerator of  $q$ , not its denominator. In Figure B.13 in the online Internet Appendix, we also connect these cross-sectional patterns with the aggregate time-series patterns from the earlier figures, by showing that the fit of the regression increases across bins of volatility in Tobin's  $q$  in both the 1975–1995 and 1995–2015 subsamples. These results suggest that there have always been some firms for which the investment- $q$  regression was tighter due to greater dispersion in Tobin's  $q$ , and that these firms have become more important in the aggregate in recent years.

In sum, the stylized facts discussed in this section demonstrate that the investment- $q$  regression works better in settings with more dispersion in Tobin's  $q$ , both in the cross-section and in the time-series. The volatility is not merely noise, but rather is statistically informative about the firm's investment policy. In Section 3 below, we rationalize these facts using two mechanisms explaining why R&D-intensive firms appearing in the data in more recent years are likely to exhibit a tighter relationship between their investments and valuations.

### 3. Model

We develop a model of firm investment, extending the setup analyzed by [Abel \(2017\)](#) to account for R&D innovations and, in turn, learning about the expected long-term growth in cash flows.

### 3.1. Setup

Consider a competitive firm with capital  $K_t$  at time  $t$ , which accumulates according to

$$dK_t = (I_t - \delta K_t)dt, \quad (3)$$

where  $I_t$  denotes the firm's investment decision.

Similar to [Erickson and Whited \(2000\)](#), adjustments to the capital stock are linear homogeneous in  $I$  and  $K$

$$\psi(I_t, K_t, \nu_t) = \frac{a}{2} \left( \frac{I_t}{K_t} \right)^2 K_t + \nu_t I_t, \quad (4)$$

where  $a$  is a positive constant so that the adjustment cost function is strictly convex. The term  $\nu_t$  represents a shock to the purchase price of capital. It follows a stochastic process with zero mean

$$d\nu_t = -\kappa \nu_t dt + \sigma_\nu dW_t^\nu, \quad (5)$$

where  $W_t^\nu$  is a standard Brownian motion. While the firm knows the current value of  $\nu_t$ , the econometrician does not. For the econometrician,  $\nu_t$  is noise.

The firm produces cash flows according to a technology with constant returns to scale

$$\Pi(K_t, \theta_t) = \theta_t K_t, \quad (6)$$

where we use the output price as numéraire. Without loss of generality, we abstract from describing the flexible labor decision.<sup>11</sup>

### 3.2. Innovations

Firms spend funds on research in the hopes of increasing their future profitability. These endeavors are often viewed as risky. For example, either the pharmaceutical research generates a breakthrough drug or it does not. To represent cash flows in this setting, we borrow from the literature on term structure models with jump-enhanced stochastic processes.<sup>12</sup> In particular, we model innovation jumps as part of the stochastic process for cash flows in a manner similar to [Das \(2002\)](#). Alternatively, the increased volatility from innovations can be obtained in other ways (e.g., through a simple increase of the Gaussian volatility parameter), but innovation jumps are just as plausible.

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<sup>11</sup>We can equivalently write the firm's problem to include a labor decision. In this case, the firm produces according to a Cobb-Douglas production function  $A_t L_t^\alpha K_t^{1-\alpha}$ , where  $0 < \alpha < 1$  and  $A_t > 0$ . It pays a constant wage rate  $w$  per unit of labor, set to 1 for simplicity. The instantaneous cash flow of the firm is  $\max_{L_t} [A_t L_t^\alpha K_t^{1-\alpha} - L_t] = \underbrace{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} A_t^{\frac{1}{1-\alpha}}}_{\equiv \theta_t} K_t \equiv \Pi(K_t, \theta_t)$ .

<sup>12</sup>For example, see [Duffie et al. \(2000\)](#), [Johannes \(2004\)](#), and [Piazzesi \(2010\)](#).

We assume that innovations occur randomly. Once the firm innovates, its cash flow per unit of capital jumps by a positive amount  $J$ , observed by the firm. The mean-reverting process for cash flows is:

$$d\theta_t = \lambda(\bar{\mu} - \theta_t)dt + \sigma_\theta dW_t^\theta + JdN_t, \quad (7)$$

where the mean profitability  $\bar{\mu}$  is a known constant. The two Brownian motions ( $W_t^\nu$  and  $W_t^\theta$ ) are independent. The last term embodies the innovation jump  $J$ , whose random arrival is governed by a Poisson process with frequency given by

$$h(\Phi) = \iota_1 (1 - e^{-\iota_2 \Phi}). \quad (8)$$

Consistent with [Thompson \(2001\)](#), [Klette and Kortum \(2004\)](#), [Aghion et al. \(2005\)](#), and [Varusawitharana \(2015\)](#), we assume that the research cost  $\Phi$  increases the probability of an innovation. In fact, the parametrization (8) implies that the success rate is increasing and concave in the cost  $\Phi$ , where parameters  $\iota_1$  and  $\iota_2$  are assumed to be positive. The success rate equals zero for  $\Phi = 0$ , and the maximum success rate,  $\iota_1$ , is attained in the limit  $\Phi \rightarrow \infty$ .

For now we consider the research cost  $\Phi$  as exogenously given, and [Section 3.5](#) below endogenizes the choice of  $\Phi$ . For simplicity, our model considers an R&D decision made one time in the firm's lifetime, rather than a dynamic decision. This can be interpreted as the firm committing to a fixed level of R&D expenditure. It allows us to connect our model to the cross-sectional distribution of firm-level R&D intensity, which is relatively stable, without introducing a second capital accumulation decision into the firm's problem. Our motivation is the fact that R&D spending is much more stable within-firm than capital expenditures (e.g. [Hall, 2002](#)), so that the most important variation is across firms rather than within firms. As a result,  $\Phi$  is not a decision variable in [equation \(10\)](#) below, because it is chosen at the beginning of the firm's life.

Because the cash flow process  $\theta_t$  is persistent, innovation jumps  $J$  carry forward into future cash flows. Thus, in this model research spending affects future cash flows in two ways: it increases their unconditional expectation and their riskiness. In the presence of jumps, the unconditional mean and the unconditional variance of  $\theta_t$  are

$$\mathbb{E}[\theta_t] = \bar{\mu} + \frac{h(\Phi)J}{\lambda}, \quad \text{Var}[\theta_t] = \frac{\sigma_\theta^2 + h(\Phi)J^2}{2\lambda}. \quad (9)$$

Research spending increases expected future cash flows in proportion to the size of the innovation jump  $J$  and the success rate of innovations,  $h(\Phi)$ . If the cash flow process (7) is more persistent, i.e., if  $\lambda$  is lower, innovations will carry forward longer into the future, further increasing the expected cash flows. Moreover, innovations also increase the variance through the jump  $J$ , a higher success rate  $h(\Phi)$ , and a higher persistence (lower  $\lambda$ ).

The firm's objective is to maximize the expected discounted sum of future cash flows, net of investment costs,

$$V(K_t, \theta_t, \nu_t) = \max_I \mathbb{E}_t \left[ \int_t^\infty e^{-r(s-t)} \{ \theta_s K_s - I_s - \psi(I_s, K_s, \nu_s) \} ds \right], \quad (10)$$

subject to equations (3) and (4), where  $r$  is the interest rate. The information set of the firm at time  $t$  is summarized by the capital stock  $K_t$ , the cash flow  $\theta_t$ , and the shock to the purchase price of capital  $\nu_t$ .

The Hamilton-Jacobi-Bellman equation associated with problem (10) is

$$rV = \max_I \{ \theta K - I - \psi(I, K, \nu) + \mathcal{D}V(K, \theta, \nu) \}, \quad (11)$$

where  $\mathcal{D}$  is the differential operator. This leads to the first order condition for investment,

$$0 = V_K(K, \theta, \nu) - 1 - \psi_I(I, K, \nu). \quad (12)$$

In our model as in Hayashi (1982), the shadow cost of capital, marginal  $q$ , is equal to average  $q$ ,  $\frac{V}{K}$ ,

$$V(K, \theta, \nu) = q(\theta, \nu)K. \quad (13)$$

Replacing the adjustment cost function (4) yields the following relationship between the rate of investment and  $q$ :

$$\frac{I_t}{K_t} = -\frac{1}{a} + \frac{1}{a}q(\theta_t, \nu_t) - \frac{1}{a}\nu_t. \quad (14)$$

When the firm's cash flows  $\theta_t$  become more volatile, it feeds directly into the volatility of  $q(\theta_t, \nu_t)$ . Research spending therefore increases the volatility of both cash flows and  $q$ . As shown in Section 3.4 below, the  $R^2$  of the investment- $q$  regression increases with the variance of  $q$ . Thus, this innovation jump model delivers our main result.

Using equation (13) and solving for the optimal investment, we obtain the following partial differential equation for  $q$  (where  $q_x$  denotes the partial derivative of  $q$  with respect to the state variable  $x$ ):

$$\begin{aligned} 0 = & \theta_t + \frac{(1 + \nu_t)^2}{2a} - \frac{1 + a(r + \delta) + \nu_t}{a}q + \lambda(\bar{\mu} - \theta_t)q_\theta - \kappa\nu_t q_\nu \\ & + \frac{\sigma_\theta^2}{2}q_{\theta\theta} + \frac{\sigma_\nu^2}{2}q_{\nu\nu} + \frac{1}{2a}q^2 + \iota_1(1 - e^{-\iota_2\Phi})[q(\theta_t + J, \nu_t) - q(\theta_t, \nu_t)]. \end{aligned} \quad (15)$$

The last term is due to jumps.<sup>13</sup> We solve this equation numerically by approximating  $q(\theta, \nu)$  with Chebyshev polynomials.<sup>14</sup> For the simulated data in Section 3.4, we use the following calibration:  $a = 16$ ,

<sup>13</sup>See Merton (1971, p. 396) for a description of Itô's lemma for Poisson processes.

<sup>14</sup>Since  $\theta$  and  $\nu$  are both mean-reverting, we define a grid that is centered on  $\{\bar{\mu}, 0\}$ . The algorithm yields a very accurate solution, with an approximation error of magnitude  $10^{-22}$  obtained with six polynomials in each dimension. For a similar

$r = 3\%$ ,  $\delta = 10\%$ ,  $\lambda = 0.5$ ,  $\sigma_\theta = 0.087$ ,  $\kappa = 5000$ ,  $\sigma_\nu = 40$ ,  $\bar{\mu} = 0.15$ ,  $\iota_1 = 1$ ,  $\iota_2 = 0.2$ , and  $(J, \Phi) \in \{(0, 0), (0.025, 5), (0.05, 20)\}$ .

### 3.3. Learning

While firms do invest in the possibility of innovations and future increased profits, in this section we focus on another aspect of research. Research-intensive firms are in the business of learning. The updating of beliefs about cash flows through time can, by itself, generate volatile valuations. To allow for clear comparison between this model and the innovation model above, we shut down innovation jumps so that the research spending has no effect on future cash flows. Instead, we view research as the purchase of a signal about the firm's future mean profitability.

In this setting, the cash flow per unit of capital  $\theta_t$  follows the mean reverting process

$$d\theta_t = \lambda(\mu_t - \theta_t)dt + \sigma_\theta dW_t^\theta. \quad (16)$$

While the instantaneous cash flow  $\theta_t$  is observable, its long-term mean  $\mu_t$  is not. The firm forms expectations over its future stream of cash flows, but cannot perfectly infer the process driving cash flows from past realizations because the unobservable long-term mean  $\mu_t$  evolves stochastically.

The long-term mean  $\mu_t$  also follows a mean-reverting process

$$d\mu_t = \eta(\bar{\mu} - \mu_t)dt + \sigma_\mu dW_t^\mu. \quad (17)$$

The firm learns about the long-term mean from two sources. The first source is free. The firm uses information from past cash-flow realizations in order to infer the long-term mean  $\mu_t$  in the process (16). The second source is costly. The firm may purchase a signal  $s_t$  that is informative about changes in the long-term mean  $dW_t^\mu$ ,

$$ds_t = dW_t^\mu + \frac{1}{\sqrt{\Phi}} dW_t^s, \quad (18)$$

where all Brownian motions ( $W_t^\nu$ ,  $W_t^\theta$ ,  $W_t^\mu$ , and  $W_t^s$ ) are independent. The parameter  $\Phi \geq 0$  dictates the informativeness of the signal. For now we consider  $\Phi$  as exogenously given, and Section 3.5 below discusses how the signal informativeness  $\Phi$  is optimally chosen ex ante by the firm.

Denote by  $\mathcal{F}_t$  the information set of the firm at time  $t$ . Conditional on this information set, the firm forms beliefs about the unobservable long-term mean  $\mu_t$ . We refer to the posterior mean of  $\mu_t$ ,  $\hat{\mu}_t \equiv \mathbb{E}[\mu_t | \mathcal{F}_t]$ , as the *filter*, and to the posterior variance of  $\mu_t$ ,  $\zeta_t \equiv \mathbb{E}[(\mu_t - \hat{\mu}_t)^2 | \mathcal{F}_t]$ , as the *Bayesian uncertainty*. Standard filtering theory (Liptser and Shiryaev, 2001) implies that the distribution of  $\mu_t$  conditional on  $\mathcal{F}_t$  is Gaussian

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approach, see Alti (2003).



with mean  $\hat{\mu}_t$  and variance  $\zeta_t$ :

$$\mu_t \sim \mathcal{N}(\hat{\mu}_t, \zeta_t). \quad (19)$$

The following proposition and its corollary obtain from filtering theory (theorem 12.7, p. 36 of [Liptser and Shiryaev, 2001](#)), with the proof provided in [Appendix A.2](#).

**Proposition 1. (*Learning*)** *The filter  $\hat{\mu}_t$  evolves according to*

$$d\hat{\mu}_t = \eta(\bar{\mu} - \hat{\mu}_t)dt + \frac{\lambda\zeta_t}{\sigma_\theta} d\widehat{W}_t^\theta + \sigma_\mu \sqrt{\frac{\Phi}{1+\Phi}} d\widehat{W}_t^s, \quad (20)$$

where  $d\widehat{W}_t^\theta \equiv dW_t^\theta + \frac{\lambda}{\sigma_\theta}(\mu_t - \hat{\mu}_t)dt$  represents the “surprise” component of the change in cash flows per unit of capital and  $d\widehat{W}_t^s \equiv \sqrt{\frac{\Phi}{1+\Phi}}ds_t$  is a scaled version of the signal in equation (18), such that  $\widehat{W}_t^s$  is a standard Brownian motion. The Bayesian uncertainty  $\zeta_t$  follows the locally deterministic process

$$\frac{d\zeta_t}{dt} = \frac{\sigma_\mu^2}{1+\Phi} - 2\eta\zeta_t - \frac{\lambda^2\zeta_t^2}{\sigma_\theta^2}. \quad (21)$$

The standard Brownian motion  $d\widehat{W}_t^\theta$  arises as follows. The firm expects a change in cash flows per unit of capital of  $\lambda(\hat{\mu}_t - \theta_t)dt$ , but instead observes the realization  $d\theta_t$ . The difference,  $d\theta_t - \lambda(\hat{\mu}_t - \theta_t)dt$ , represents the unexpected change, i.e., the “surprise.” Dividing this difference by  $\sigma_\theta$  yields the standard Brownian motion  $d\widehat{W}_t^\theta$ . This Brownian motion is distinct from the true cash-flow shock  $dW_t^\theta$  which is unobservable by the firm, because it incorporates firm’s expectations of future cash-flow growth (see [Appendix A.2](#)).

We characterize the stationary solution to this learning problem. We assume that enough time has passed such that the Bayesian uncertainty has reached a steady state. This is a common assumption in the literature on incomplete information (e.g., [Scheinkman and Xiong, 2003](#); [Dumas et al., 2009](#)), and it fits well in our model with infinite horizon. The steady-state value for  $\zeta$ ,  $\bar{\zeta}$ , is obtained by setting the right-hand side of equation (21) to zero. This yields a quadratic equation with only one positive root:

$$\bar{\zeta} = \frac{\sigma_\theta^2}{\lambda^2} \left( \sqrt{\eta^2 + \frac{1}{1+\Phi} \frac{\lambda^2\sigma_\mu^2}{\sigma_\theta^2}} - \eta \right). \quad (22)$$

If  $\sigma_\mu > 0$ , learning is constantly regenerated and the steady-state Bayesian uncertainty is positive. It is increasing in  $\sigma_\mu$ , decreasing in  $\Phi$ , and goes to zero only in the limiting case as  $\Phi \rightarrow \infty$  (when  $\mu_t$  becomes perfectly observable).

If  $\sigma_\mu = 0$ , the long-term mean is a constant,  $\mu_t = \bar{\mu}, \forall t$ . In this case, we assume that there has been a sufficiently long period of learning for the firm to observe  $\bar{\mu}$ . Accordingly, the Bayesian uncertainty decays to zero. We refer to this case as a model without learning.

Replacing the steady state uncertainty in the diffusion of  $\hat{\mu}_t$  in Proposition 1 allows us to compute the stationary solution for the instantaneous variance of the filter. This variance, which we characterize in the

following corollary, measures the intensity with which the firm is learning and updating its beliefs about the long-term mean  $\mu_t$ .

**Corollary 1.1.** *The instantaneous variance of the filter,*

$$\text{Var}_t[d\hat{\mu}_t] = \sigma_\mu^2 - 2\eta\bar{\zeta}, \quad (23)$$

*is strictly increasing in both  $\sigma_\mu$  and  $\Phi$ .*

According to Corollary 1.1, the variance of the filter increases when there is more uncertainty about the long-term mean  $\mu_t$  or when the firm acquires information through a more informative signal  $\Phi$ . Although the filtered long-term mean  $\hat{\mu}_t$  is less volatile than the truth  $\mu_t$  (because the filter is a projection of  $\mu_t$  on the observation filtration of the firm), Corollary 1.1 shows that learning with a more informative signal  $\Phi$  strictly increases the variance of the filter. In the limit when  $\Phi \rightarrow \infty$ , the firm perfectly observes  $\mu_t$ , the Bayesian uncertainty  $\bar{\zeta}$  collapses to zero, and the variance of the filter reaches the instantaneous variance of the unobserved process,  $\sigma_\mu^2$ .

Furthermore, Corollary 1.1 shows that learning affects the instantaneous variance of the filter and the Bayesian uncertainty in opposite ways: learning with more informative signals (higher  $\Phi$ ) increases the variance of the filter  $\text{Var}_t[d\hat{\mu}_t]$  in equation (23), but it decreases the Bayesian uncertainty  $\bar{\zeta}$  in equation (22). The instantaneous variance of the filter is not be confused with the Bayesian uncertainty. The former is a measure of time variation in firm's beliefs. The latter is a measure of uncertainty about  $\mu_t$  conditional on the information set  $\mathcal{F}_t$ . In other words, although the firm decreases uncertainty through learning at any moment in time, its beliefs become more volatile as they are more aggressively updated from one moment to the other.

Two key results arise from Proposition 1 and its Corollary, reflecting the two sources of information from which firms learn. We refer to  $d\widehat{W}_t^\theta$  as cash-flow shocks and to  $d\widehat{W}_t^s$  as information shocks. The first result is that learning from cash-flow realizations induces a positive correlation between the filter  $\hat{\mu}_t$  and cash flows  $\theta_t$ , through cash-flow shocks  $d\widehat{W}_t^\theta$ . This extrapolative feature of learning (Brennan, 1998) amplifies the impact of cash-flow shocks. Second, learning from the signal  $s_t$  causes the firm's estimate of the long-term cash-flow mean  $\hat{\mu}_t$  to respond to information shocks  $d\widehat{W}_t^s$ . This increases the variance of  $\hat{\mu}_t$ .

We note that the learning taking place does not change the instantaneous volatility of the cash-flow process (16) itself, which remains constant for any level of  $\sigma_\mu$ . In fact, the long-term mean of  $\theta_t$ , its conditional variance, and its unconditional variance do not depend on the intensity of firm's learning. These values are  $\bar{\mu}$ ,  $\sigma_\theta$ , and  $\sqrt{\sigma_\theta^2 + \frac{\lambda^2 \sigma_\mu^2}{\eta(\eta+\lambda)}}/\sqrt{2\lambda}$ , respectively. Learning, however, does increase the volatility of the filter through the continuous updating of the long-term cash-flow mean  $\hat{\mu}_t$ .

Similarly as before, the firm's objective is to maximize the expected discounted sum of future cash flows,

net of investment costs,

$$V(K_t, \theta_t, \hat{\mu}_t, \nu_t) = \max_I \mathbb{E}_t \left[ \int_t^\infty e^{-r(s-t)} \{ \theta_s K_s - I_s - \psi(I_s, K_s, \nu_s) \} ds \right], \quad (24)$$

subject to equations (3) and (4). The difference with the innovation jump maximand in equation (10) is that the information set of the firm at time  $t$  also includes the conditional expectation of cash-flow growth  $\hat{\mu}_t$ . In the learning model, as in the innovation model, the Hayashi (1982) conditions hold, and the first order condition for investment can be written as:

$$\frac{I_t}{K_t} = -\frac{1}{a} + \frac{1}{a} q(\theta_t, \hat{\mu}_t, \nu_t) - \frac{1}{a} \nu_t. \quad (25)$$

Replacing this relationship in the HJB equation associated with problem (24) yields the following partial differential equation for  $q$ :

$$\begin{aligned} 0 = & \theta_t + \frac{(1 + \nu_t)^2}{2a} - \frac{1 + a(r + \delta) + \nu_t}{a} q + \lambda(\hat{\mu}_t - \theta_t) q_\theta + \eta(\bar{\mu} - \hat{\mu}_t) q_{\hat{\mu}} - \kappa \nu_t q_\nu \\ & + \frac{\sigma_\theta^2}{2} q_{\theta\theta} + \left( \frac{\sigma_\mu^2}{2} - \eta \bar{\zeta} \right) q_{\hat{\mu}\hat{\mu}} + \frac{\sigma_\nu^2}{2} q_{\nu\nu} + \lambda \bar{\zeta} q_{\theta\hat{\mu}} + \frac{1}{2a} q^2. \end{aligned} \quad (26)$$

We solve this equation numerically using the same method as before, with a state space grid centered on  $\{\bar{\mu}, \bar{\mu}, 0\}$ . For the simulations in Section 3.4, we use the following calibration:  $a = 16$ ,  $r = 3\%$ ,  $\delta = 10\%$ ,  $\lambda = 0.5$ ,  $\sigma_\theta = 0.1$ ,  $\kappa = 5000$ ,  $\sigma_\nu = 40$ ,  $\bar{\mu} = 0.25$ ,  $\eta = 0.5$ , and  $(\sigma_\mu, \Phi) \in \{(0, 0), (0.15, 0), (0.15, 20)\}$ . When we choose  $\sigma_\mu = 0.15$ , the calibration results in approximately the same unconditional moments for cash flows  $\theta$  as in the model with innovation jumps in which  $(J, \Phi) = (0.05, 20)$ .

### 3.4. The relationship between investment and $q$

Without  $\nu$ , the econometrician would observe a deterministic relationship between investment and  $q$  in equations (14) or (25) and, counterfactually, this relationship would always have an  $R^2$  of one. In both cases, the shock to the capital purchase price causes the  $R^2$  to be below one.

For ease of exposition, we let  $x_t$  denote the state vector at time  $t$ . In the model with innovations,  $x_t \equiv \{\theta_t, \nu_t\}$ ; in the one with learning,  $x_t \equiv \{\theta_t, \hat{\mu}_t, \nu_t\}$ . In both models, the  $R^2$  has the same analytical expression:

$$R^2 = \frac{\text{Var}[q(x_t)] \left( 1 - \frac{\text{Cov}[q(x_t), \nu_t]}{\text{Var}[q(x_t)]} \right)^2}{\text{Var}[q(x_t)] + \text{Var}[\nu_t] - 2 \text{Cov}[q(x_t), \nu_t]}. \quad (27)$$

The  $R^2$  coefficient increases with the variance of  $q$  as long as the covariance between  $q$  and  $\nu$  is negligible.<sup>15</sup>

<sup>15</sup>The  $R^2$  depends on the relationship between  $q_t$  and  $\nu_t$ . In our numerical calibration, we ensure that the covariance between  $q_t$  and  $\nu_t$  is virtually zero, i.e.,  $q_\nu \approx 0$ . This occurs for large values of  $\kappa$ , i.e., when the persistence of  $\nu_t$  is close to zero. A non-negligible persistence of  $\nu_t$  creates temporal dependence through which  $q_t$  depends on  $\nu_t$ . Even in this case, the covariance term in equation (27) is of small magnitude, and does not impact our main intuition.

Notice also that a stronger regression coefficient for  $q$  in (14) or (25) does not mechanically affect the  $R^2$ , since the adjustment cost parameter  $a$  simplifies away from (27).

The firm's innovations and learning affect the  $R^2$ . An application of Itô's lemma on the innovation-derived  $q(\theta_t, \nu_t)$  yields:

$$dq = \xi_1(\theta_t, \nu_t)dt + q_\theta \sigma_\theta dW_t^\theta + q_\nu \sigma_\nu dW_t^\nu + [q(\theta_t, \nu_t) - q(\theta_{t-}, \nu_t)] dN_t, \quad (28)$$

where  $\xi_1(\theta_t, \nu_t)$  denotes the drift (its specific form does not matter for our analysis). Research spending by the firm increases the probability of innovation jumps, and with it the volatility of Tobin's  $q$  through the last term above. According to equation (27), an increased volatility in  $q$  boosts the  $R^2$  of the investment- $q$  regression.

The firm's learning also increases the volatility of  $q(\theta_t, \hat{\mu}_t, \nu_t)$ , as shown in:

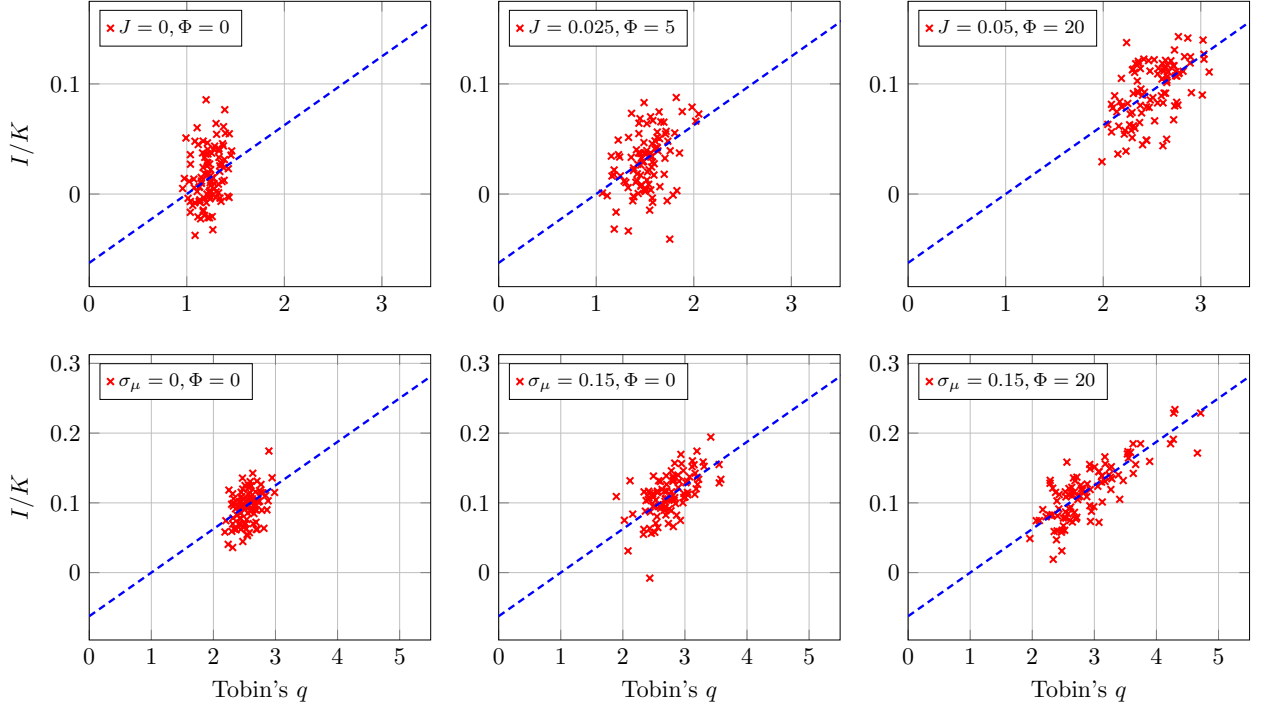
$$dq = \xi_2(\theta_t, \hat{\mu}_t, \nu_t)dt + \left( q_\theta \sigma_\theta + q_{\hat{\mu}} \frac{\lambda}{\sigma_\theta} \bar{\zeta} \right) d\widehat{W}_t^\theta + q_{\hat{\mu}} \sigma_{\hat{\mu}} \sqrt{\frac{\Phi}{1+\Phi}} d\widehat{W}_t^s + q_\nu \sigma_\nu dW_t^\nu, \quad (29)$$

When the firm learns about the unobservable productivity growth  $\mu_t$ ,  $q$  becomes more sensitive to cash-flow shocks  $d\widehat{W}_t^\theta$  through the second term in brackets above. Tobin's  $q$  also becomes sensitive to information shocks  $d\widehat{W}_t^s$  through the third term above. Both these effects increase the volatility of  $q(\theta_t, \hat{\mu}_t, \nu_t)$  and, according to equation (27), the  $R^2$  of the investment- $q$  regression.

We illustrate the impact of innovations and learning on the  $R^2$  by means of simulations. To this end, we implement a discretization of the continuous-time processes at a yearly frequency (see Appendix A.3). We then solve for the partial differential equation and compute  $q_t$  for each point of the state space. The resulting value for  $q_t$  can then be replaced in the first order condition for investment, yielding the investment rate  $I_t/K_t$ . This completes the dataset necessary for implementing investment- $q$  regressions.

Figure 5 plots as an example one simulation of 100 yearly observations. The three upper panels report simulations for the innovation model of Section 3.2, while the lower three report simulations for the learning model of Section 3.3. The horizontal axis in each panel is the marginal  $q$ . The vertical axis represents the optimal investment rate  $I_t/K_t$ .

In the case of the innovation model, we compare three types of firms. In the left panel, we consider the case of a firm that has no opportunity for innovations ( $J = 0$ ) and does not spend on research ( $\Phi = 0$ ). In the middle and the right panels we gradually increase  $J$  to 0.025 and then 0.05, and  $\Phi$  to 5 and then 20. The three panels show that research spending increases the volatility of Tobin's  $q$  and consequently the fit of the regression. The volatility of Tobin's  $q$  obtained from 5,000 such simulations averages 0.13 in the first panel, 0.15 in the second panel, and 0.18 in the third panel. The average  $R^2$  coefficient also increases from 10%, to 12%, and 17% respectively.



**Fig. 5.** Relationship between investment and  $q$  in the innovation model of Section 3.2 (upper panels) and in the learning model of Section 3.3 (lower panels).

The bins are recalculated separately for the two subperiods 1975–1995 and 1995–2015. In each model, we simulate 100 yearly data points for three different firms. Upper panels: in the left panel, the firm has no innovation opportunity ( $J = 0, \Phi = 0$ ); in the middle and right panels, we gradually increase  $J$  and  $\Phi$ . Lower panels: in the left panel, the firm does not learn about  $\mu_t$ , which is held constant at  $\bar{\mu}$ ; in the middle panel, the firm learns about  $\mu_t$  exclusively from the cash-flow process (16), i.e.,  $\Phi = 0$ ; in the right panel, the firm learns about  $\mu_t$  from the cash-flow process (16) and from the signal (18) with  $\Phi = 20$ . The rest of the calibration used for these simulations is given in Section 3.2 for the innovation model and in Section 3.3 for the learning model.

Turning to the learning model, the left panel corresponds to the model without learning, that is, the firm sets  $\mu_t = \bar{\mu}, \forall t$ . In the middle panel, the firm learns about  $\mu_t$ , but only using the observable process for  $\theta_t$ , i.e.,  $\Phi = 0$ . In the right panel, the firm also learns through the signal in equation (18), with  $\Phi = 20$ . Changes in  $\mu_t$  are not yet perfectly observed, but with  $\Phi = 20$  the signal in (18) is more informative relative to the cash-flow signal in equation (16). The three panels show that learning improves the fit of the regression. As elaborated above, this occurs through an increase in the volatility of the regressor  $q$ , from 0.18, to 0.39, and 0.45. The  $R^2$  coefficient increases from 18%, to 48%, and 56%.

Although innovation and learning improve the  $R^2$  of the investment- $q$  regression, they do not influence its slope, which remains equal to  $1/a$  across all models. This can be seen in Figure 5, where the fitted line remains the same in all panels. In contrast, in the data of Figure 4 the slope decreases across the bins. With a higher adjustment cost parameter  $a$ , firms would be characterized by lower slope coefficients  $1/a$ . Alternatively, with decreasing returns to scale ( $\alpha < 1$ ), the lower slope coefficients also obtain. A higher adjustment cost parameter among R&D firms, as well as decreasing returns to scale representing market power from R&D innovations, are two of the possible explanations we investigate in our online Internet Appendix for the falling slope among more R&D-intensive firms.

### 3.5. Endogenous decision to invest in innovation and learning

We endogenize the decision to invest in research. In the innovation-based model, more research (i.e., a higher  $\Phi$ ) increases the probability of a persistent jump in cash flows. In the learning-based model, it leads to better information from which to make investment decisions. As a trade-off to these benefits, R&D is costly (e.g., [Detemple and Kihlstrom, 1987](#)). We consider a static decision in which the firm makes a choice of  $\Phi$  at time 0 and maintains this capacity over its lifetime.

The firm value immediately after the choice of  $\Phi$  is defined as  $\tilde{V}(\cdot)$ , and its associated cost,  $c(\Phi)$ , is a strictly increasing and convex function with  $c'(0) = 0$ . With  $\Phi$  as a parameter in  $\tilde{V}(\cdot)$ , the problem is equivalent to the earlier models without an endogenous  $\Phi$ . The optimal  $\Phi^*$  is defined by the first-order condition  $\tilde{V}_\Phi(\cdot) = c'(\Phi^*)$ , and there is an interior solution if and only if  $\tilde{V}_{\Phi\Phi}(\cdot) - c''(\Phi^*) < 0$ .

Jumps  $J$  and the uncertainty of the cash-flow mean  $\sigma_\mu$  are central features of the innovation and learning mechanisms respectively. We are therefore interested in investigating whether the optimal purchase of research increases with the magnitude of the jump  $J$  and uncertainty  $\sigma_\mu$ . Differentiating the first-order condition with respect to  $J$  (in the model with innovation jumps) and with respect to  $\sigma_\mu$  (in the model with

learning) and rearranging, we get

$$\frac{d\Phi^*}{dJ} = \frac{\tilde{V}_{\Phi J}(\cdot)}{c''(\Phi^*) - \tilde{V}_{\Phi\Phi}(\cdot)} \quad (30)$$

$$\frac{d\Phi^*}{d\sigma_\mu} = \frac{\tilde{V}_{\Phi\sigma_\mu}(\cdot)}{c''(\Phi^*) - \tilde{V}_{\Phi\Phi}(\cdot)}. \quad (31)$$

In both cases, the denominator is positive if the problem has an interior solution. We see that the optimal amount of research  $\Phi^*$  increases in the innovation jump  $J$  or in the uncertainty  $\sigma_\mu$  if and only if an increase in the size of the innovation jump or in the uncertainty about  $\mu_t$  increases the marginal benefit of research ( $\tilde{V}_{\Phi J}(\cdot) > 0$  or  $\tilde{V}_{\Phi\sigma_\mu}(\cdot) > 0$ ).

The problem therefore reduces to showing that  $\tilde{V}_{\Phi J}(\cdot) > 0$  and  $\tilde{V}_{\Phi\sigma_\mu}(\cdot) > 0$ . While there is no closed-form proof of these inequalities, they can be checked numerically as  $\tilde{V}(\cdot)$  is just the value function from the problem without an endogenous research choice  $\Phi$ . Our numerical results demonstrate that this is indeed the case for both innovation and learning models.<sup>16</sup>

These results imply that firms operating in environments prone to large innovation jumps  $J$  or more uncertain environments, e.g., high-tech firms, optimally choose to invest more in research. In other words, firms commit to a higher research intensity  $\Phi$  if they expect large innovation jumps  $J$  or face more uncertainty  $\sigma_\mu$  in their cash-flow mean. This amplifies the direct effects that larger jumps and more uncertainty already have on the volatility of Tobin's  $q$  and hence on the  $R^2$  fit of the investment- $q$  regression, discussed in Sections 3.2, 3.3, and 3.4 above. Altogether, investments in innovation and learning generate a strong cross-sectional implication: the investment- $q$  regression performs better for firms that spend more on R&D.

#### 4. Empirical analysis of the model predictions

In this section, we dig deeper into the empirical predictions of the model.

##### 4.1. Better performance in high-tech industries

Section 3.5 contains the main prediction of the model, where the investment- $q$  regression performs better among firms that endogenously choose to expend greater resources on research. Empirically, we are interested in identifying groups of firms where this is most likely to take place. We focus on firms that decide to spend more on R&D. Our proposed innovation and learning mechanisms should cause the investment- $q$  regression to work better in industries featuring high R&D. This insight provides testable cross-sectional implications of the model.

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<sup>16</sup>The numerical results are available in Appendix C.

	(1)	(2)	(3)	(4)
	$I_t/K_{t-1}$	$I_t/K_{t-1}$	$I_t/K_{t-1}$	$I_t/K_{t-1}$
$q_{t-1}$	0.0167*** (0.000629)	0.0111*** (0.000335)	0.0226*** (0.00149)	0.0231*** (0.00172)
Sample	Non-high-tech	High-tech	Non-high-tech, pre-1995	High-tech, pre-1995
Firm FE?	Yes	Yes	Yes	Yes
Obs.	103959	31824	41215	8099
$R^2$	0.0913	0.174	0.0516	0.163

Standard errors in parentheses  
\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

**Table 3.**

This table performs panel regressions of investment on lagged Tobin’s  $q$  using annual data from Compustat.

The data are annual Compustat from 1975-2015. “High-tech” refers to SIC codes 283, 357, 366, 367, 382, 384, and 737, following [Brown et al. \(2009\)](#). Columns 3 and 4 restrict to pre-1995 firm-years. Investment and Tobin’s  $q$  are winsorized at the 1st and 99th percentiles. Standard errors are clustered by firm, and the table reports the within-firm  $R^2$  of the regression.

For an operational definition of a research-intensive industry, we use the following seven SIC codes: 283 (drugs), 357 (office and computing equipment), 366 (communications equipment), 367 (electronic components), 382 (scientific instruments), 384 (medical instruments), and 737 (software). We refer to these as “research-intensive” or “high-tech” industries for the remainder of this paper. The industry classification follows [Brown et al. \(2009\)](#), which shows that the seven industries account for nearly all the growth in aggregate R&D during the 1990s.

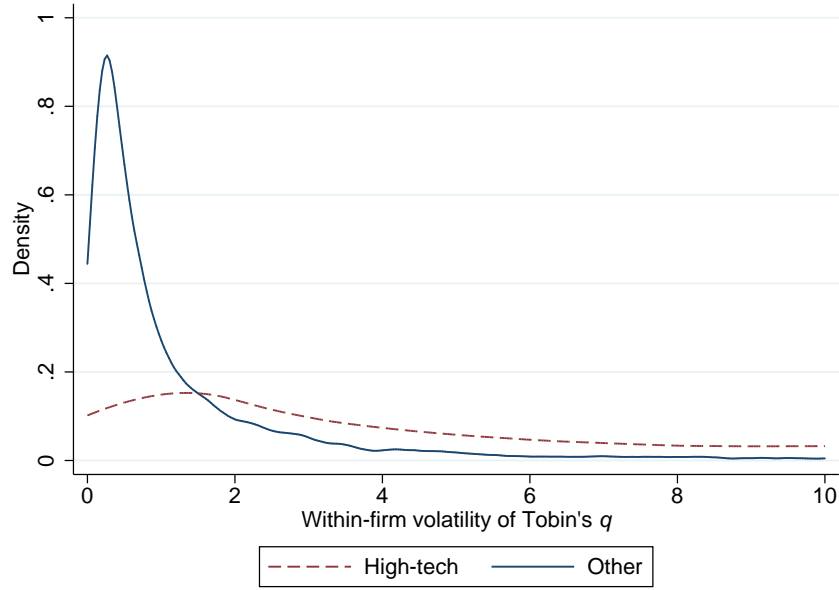
We build up our analysis of research-intensive industries in several layers. First, we examine the empirical distribution of Tobin’s  $q$  volatility in these industries compared to the average Compustat firm. Panel A of Figure 6 calculates the within-firm volatility of Tobin’s  $q$  for each firm, and plots the empirical density of this volatility, separating out high-tech firms from the other industries. This volatility follows a skewed distribution. It is higher on average for the high-tech firms than for the others. Finally, Panel B of Figure 6 repeats this analysis using the total  $q$  measure of [Peters and Taylor \(2017\)](#). As expected, total  $q$  is less volatile than standard  $q$ , but it is still more volatile for high-tech than other firms.

It is not surprising that market valuations of high-tech companies are particularly volatile. What is less clear is that these fluctuations are highly predictive of investment, as expected under the  $q$  theory of investment. This contrasts with the alternative view that market fluctuations arise from problems in measuring the firm’s capital stock or from difficulties outsiders face in valuing the firm, which would make these fluctuations simply exogenous noise with respect to the firm’s investment policy.

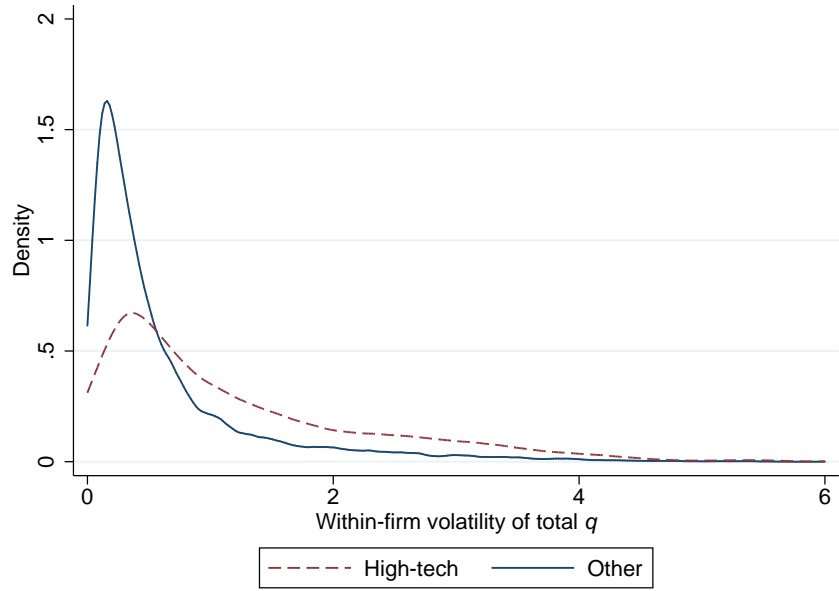
Tables 3 and 4 repeat the panel regressions of investment on lagged  $q$ , as specified earlier in equation (2) and implemented in Table 2, where various columns separate out high-tech from other industries.

Columns 1 and 2 of Table 3 show that the standard investment- $q$  panel regression fares better among high-tech firms: the  $R^2$  value from the regression almost doubles from 9% to 17% when we move from the





A. Using standard  $q$ .



B. Using total  $q$ .

**Fig. 6.** Empirical distribution of the within-firm volatility of Tobin's  $q$  in annual Compustat from 1975 to 2015.

The figure calculates the volatility for each firm, then plots the distribution of volatility for firms in high-tech industries separately from other industries. High-tech industries are defined as SIC codes 283, 357, 366, 367, 382, 384, and 737, following [Brown et al. \(2009\)](#). Panel A uses the standard measure of Tobin's  $q$ , Panel B uses the total  $q$  measure of [Peters and Taylor \(2017\)](#).

	(1)	(2)	(3)	(4)
	$I_t/K_{t-1}$	$I_t/K_{t-1}$	$I_t^{\text{tot}}/K_{t-1}$	$I_t^{\text{tot}}/K_{t-1}$
$q_{t-1}$	0.00879*** (0.000249)	0.0107*** (0.000310)	0.0116*** (0.000492)	0.00780*** (0.000284)
Sample	High-tech	High-tech	Non-high-tech	High-tech
Firm FE?	No	Yes	Yes	Yes
Year FE?	No	Yes	No	No
Obs.	31824	31824	103949	31824
$R^2$	0.180	0.261	0.0653	0.151

Standard errors in parentheses

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

**Table 4.**

This table performs panel regressions of investment on lagged Tobin’s  $q$  using annual data from Compustat.

The regressions are as in Table 3, except as noted in each column. Column 1 drops both firm and year fixed effects, and Column 2 includes both firm and year fixed effects as is done in Peters and Taylor (2017). In columns 3 and 4, R&D is added to capital expenditures as a measure of intangible investment. Both measure of investment, and Tobin’s  $q$ , are winsorized at the 1st and 99th percentiles. Standard errors are clustered by firm, and the table reports the within-firm  $R^2$  of the regression.

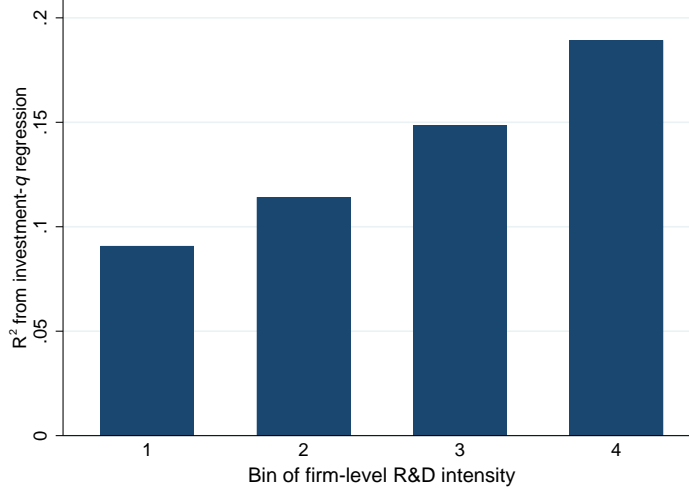
non-tech to the high-tech subsample.

One may object that, since we already have shown that the investment- $q$  regression works better in recent years, this comparison simply captures the increasing importance of high-tech firms towards the end of the sample. To check this, in Columns 3 and 4 we restrict the sample to years prior to 1995. The same discrepancy holds for these early years: the  $R^2$  of the panel regression increases from 5% for non-tech industries to 16% for high-tech, in accord with the investment- $q$  regression working better for high-tech industries.

Table 4 checks robustness to some alternative approaches. Column 1 shows that firm fixed effects are not driving the performance of the regression, as the (overall)  $R^2$  from the pooled regression is similar to the (within)  $R^2$  reported in Table 3. Column 2 shows that the fit of the regression improves even more when we add time fixed effects, as is done in some of the other papers in the Tobin’s  $q$  literature. In additional results (available in Appendix B of the online Internet Appendix) we document further robustness to various combinations of fixed effects and approaches to winsorizing.

Columns 3 and 4 return to our main panel specification with firm fixed effects but no time fixed effect, and adds in annual R&D expense plus 30% of annual SG&A expense as a measure of intangible investment, following Peters and Taylor (2017). The conclusion remains the same as before: the regression works better in high-tech industries ( $R^2 = 15\%$ ) than in other industries ( $R^2 = 7\%$ ).

To show that the results extend beyond the coarse high-tech proxy, we examine  $R^2$  values using more general measures of R&D intensity. In Figure 7, we examine evidence at the firm level. We sort firms in the Compustat panel into four bins based on their average R&D intensity (defined as the ratio of annual R&D to total assets) within their lifetime in Compustat, dropping any firms for which this average is zero. Within



**Fig. 7.**  $R^2$  values from the panel investment- $q$  regressions, performed separately for each bin of firm-level R&D intensity.

R&D intensity is annual R&D expense divided by book assets, assigning zero for missing R&D observations. It is calculated separately for each firm-year in Compustat from 1975-2015, then averaged within-firm.

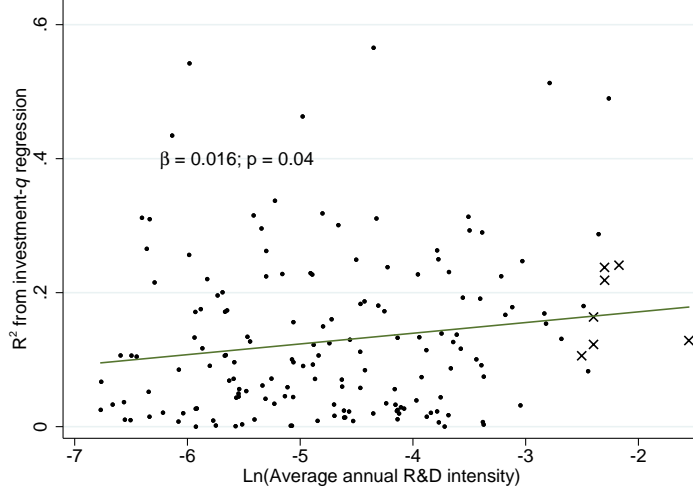
each bin, we estimate the panel investment- $q$  regression. The figure plots the  $R^2$  values obtained from the regression in each bin. These values show an increasing pattern, from 9% in the lowest bin to over 19% in the highest bin.

In Figure 8, we perform a similar exercise at the industry level. We perform the investment- $q$  regression separately within each 3-digit SIC industry in Compustat, and plot the resulting  $R^2$  values against average R&D intensity calculated across firm-years in that industry. In constructing this figure, we drop 56 industries (22% of the sample) for which average R&D intensity is less than 0.1% of total assets. We also drop five industries with fewer than five firm-year observations in Compustat.

Figure 8 plots the  $R^2$  values against the log of industry average R&D intensity. The figure shows a clear positive association. The seven high-tech industries identified in the previous analysis are marked with an “ $\times$ ” in the figure. They cluster near each other at high values of R&D intensity, and relatively high  $R^2$  values, although not the highest observed  $R^2$  across all industries.

The finding that the standard investment- $q$  regression works better in high-tech industries was previously established in Peters and Taylor (2017). In their Section 5.1, Peters and Taylor use several cross-sectional proxies beyond the simple industry classification to explore a number of explanations for this fact, but ultimately reject all of them. They conclude: “*Why the classic  $q$ -theory fits the data better in high-intangible settings is also an interesting open question.*” Our innovation and learning mechanisms provide plausible explanations for this finding.

The growth of high-tech industries is key to understanding the improved fit of the aggregate investment- $q$  relationship in recent years, and by extension the future empirical performance of the  $q$  theory of investment.



**Fig. 8.** Industry-level investment- $q$  correlations and R&D intensity.

Each dot corresponds to a 3-digit SIC industry. The  $y$ -axis plots  $R^2$  values from the panel investment- $q$  regression performed separately in each industry. The  $x$ -axis plots the log of industry-average R&D intensity. R&D intensity is defined as annual R&D expense divided by book assets, assigning zero for missing R&D data. It is calculated separately for each firm-year in Compustat from 1975-2015, then averaged across firm-years in each 3-digit SIC industry in Compustat. The figure depicts only industries where this average is at least 0.001, and for which there are at least five firm-year observations in Compustat. The “x” markers denote the R&D-intensive industries identified in [Brown et al. \(2009\)](#).

Figure 9 shows that the firms in the high-tech industry classification represent a growing fraction of the number of firms and of book assets in Compustat. Similarly, [Peters and Taylor \(2017\)](#) show that their measure of intangible capital, which capitalizes past intangible investments such as R&D and SG&A, also increases over time in both Compustat and the aggregate data from the Fed Flow of Funds.

In conjunction with our cross-sectional findings, these trends suggest that the  $q$  theory of investment may have been the right theory at the wrong time. While the theory has traditionally not fared well for the capital-intensive firms that dominated the economy when the theory was first developed, it turns out to be well-suited for the new research-intensive economy that features wider endogenous swings in valuations and investments.

#### 4.2. Innovation versus learning

We next explore a subtler implication of the model relating to the role of cash flows. Innovation jumps impart more volatility to cash flows, and therefore to Tobin’s  $q$ . In fact, the correlation between cash flows and  $q$  is nearly perfect, but for the negligible influence of  $\nu_t$ . With learning, however, the firm chooses to take into account other signals than contemporaneous cash flows. The correlation between cash flows and  $q$  is therefore lower with more learning. While more research through innovation or learning increases the  $R^2$  of the investment- $q$  relationship, the two mechanisms have opposite predictions for the correlation between cash flows and Tobin’s  $q$ . In contrast to innovation, more research through learning decreases the correlation between cash flows and Tobin’s  $q$ .



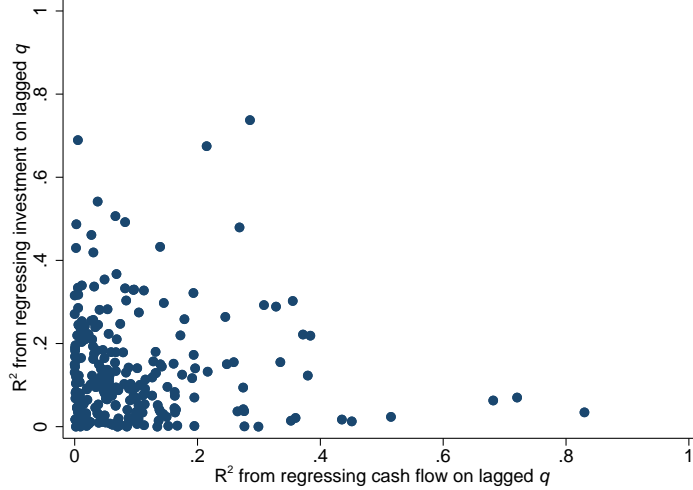
**Fig. 9.** Fraction of firms in Compustat each year that fall into our classification of high-tech industries.

The blue line is an equal-weighted average, while the red line weighs firms by their shareholders' equity. High-tech industries are defined as SIC codes 283, 357, 366, 367, 382, 384, and 737, following [Brown et al. \(2009\)](#).

Building on this intuition, we separate Compustat firms by industry at the 3-digit SIC code, and we investigate how the tightness of the fit between investment and  $q$  is related to the tightness of the fit between cash flow and Tobin's  $q$ . Within each industry, we estimate fixed-effects regressions of cash flow on lagged  $q$ , then of investment on lagged  $q$ . We save the  $R^2$  values from both of these regressions for each industry, and plot them in Figure 10, retaining only industries with at least five firm-years.

The pattern in Figure 10 is consistent with the learning mechanism. The industries with the tightest connection between  $q$  and investment (the highest values on the  $y$ -axis) are also the industries with the weakest connection between  $q$  and cash flow (the left-most values on the  $x$ -axis). Conversely, the industries with the tightest connection between  $q$  and cash flow are also the industries with the weakest connection between  $q$  and investment. The overall pattern is contrary to what we would expect with innovation jumps, suggesting that the investment- $q$  regression works better when  $q$  is less predictive of cash flows over a short horizon, which is consistent with the learning mechanism. While our findings do not indicate that there is no innovation jump, they do suggest empirical effects in line with the firm learning and updating with signals other than cash flows.

Both axes of Figure 10 measure  $R^2$  values, which are mechanically bounded between zero and one. For this reason, it may seem difficult to judge formally the relationship plotted in the figure. The same concern could apply to Figure 8, in which the vertical axis also plotted  $R^2$  values. To address this concern, Figures B.14 and B.15 in the online Internet Appendix repeat the analysis of Figures 8 and 10, but transform the  $R^2$  values via the negative logistic transformation  $\ln(R^2) - \ln(1 - R^2)$ , motivated by the analysis in [Durnev et al. \(2004\)](#). The transformed values capture the difference between the explained and unexplained variation in



**Fig. 10.**  $R^2$  of investment- $q$  regressions and cash-flow- $q$  regressions by industry.

Each dot corresponds to a 3-digit SIC industry classification. The  $x$ -axis plots, for each industry, the  $R^2$  value from a fixed-effects regression of cash flow on lagged Tobin's  $q$ . The  $y$ -axis plots, for the same industry, the  $R^2$  from a fixed-effects regression of investment on lagged Tobin's  $q$ . The data are annual Compustat from 1975-2015. The figure depicts only industries for which there are at least five firm-year observations in Compustat. Cash flow is defined as income before extraordinary items plus depreciation expense divided by gross property, plant, and equipment. Cash flow, investment, and  $q$  are all winsorized at the 1st and 99th percentiles.

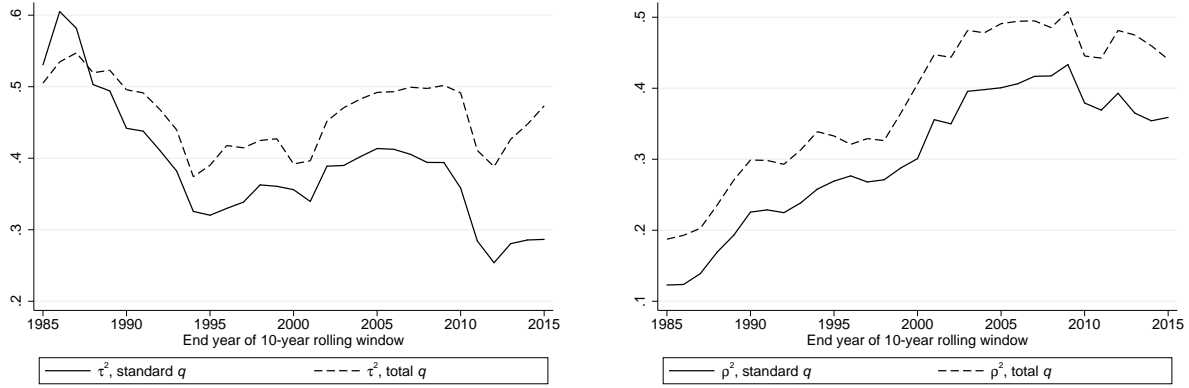
the model. Unlike the raw  $R^2$ s, the transformed values are not bounded between zero and one, making it more natural to model them as linear functions. The figures also fit lines through the transformed points. In both cases, the estimated linear associations are statistically significant, confirming the results seen in Figures 8 and 10.

#### 4.3. Accounting for measurement error in $q$

In our setting, marginal  $q$  is always equal to average  $q$ . Empirically, however, Tobin's average  $q$  may be a poor proxy for a number of reasons, such as measurement error in the firm's capital stock. In this section, we empirically account for measurement error.<sup>17</sup>

We draw on the large literature on measurement error in Tobin's  $q$ . Two contributions are especially relevant to our work. First, [Erickson and Whited \(2000\)](#) develop an estimator that is robust to measurement error by exploiting identifying information in the third- and higher-order moments of the empirical distribution of Tobin's  $q$ . [Erickson et al. \(2014\)](#) improve on this approach by focusing on cumulants rather than moments. These approaches yield, among other things, estimates of two population  $R^2$  values: first, the  $R^2$  from the measurement regression of Tobin's average  $q$  on "true" marginal  $q$ , labeled  $\tau^2$ ; and second, the  $R^2$  from the investment regression of investment rate on "true" marginal  $q$ , labeled  $\rho^2$ . We use these parameter

<sup>17</sup>Throughout this section, we refer to measurement error that satisfies the identifying assumptions in [Erickson and Whited \(2000\)](#).



A. Time-series of  $\tau^2$ .

B. Time-series of  $\rho^2$ .

**Fig. 11.** Measurement error and investment- $q$   $R^2$  over time.

This figure plots the time-series of  $\tau^2$  and  $\rho^2$  estimates recovered from the cumulant-estimator approach of [Erickson, Jiang, and Whited \(2014\)](#). Each estimate is calculated from a ten-year window ending in the year labeled on the axis. Solid lines use the standard definitions of investment and Tobin’s  $q$ , while dashed lines use the total- $q$  and total-investment measures defined in [Peters and Taylor \(2017\)](#).

estimates to quantify the cross-sectional importance of measurement error and the “true” performance of the  $q$  theory.

Second, [Peters and Taylor \(2017\)](#) focus on the role of intangibles, which are missing from the standard measurement of investment and average  $q$ . They propose to capitalize R&D and SG&A expenditures as intangible investments, and show that this approach improves the performance of the regression. The adjustment is largest for high-tech firms, for whom intangibles are relatively more important. We examine how the adjustment behaves in the cross-section.

As motivating evidence, we first examine the evolution of  $\tau^2$  and  $\rho^2$  through time with and without the adjustment for intangibles. Figure 11 displays the time-series of  $\tau^2$  and  $\rho^2$ . The estimators of [Erickson et al. \(2014\)](#) are applied to rolling ten-year windows of Compustat data, using three cumulants to exactly identify the system. The figure separately plots the series with (dashed lines) and without (solid lines) intangibles in the measures of investment and  $q$ .

First, consider the two series for  $\tau^2$ , which are displayed in the left panel of Figure 11. These capture the degree of measurement error driving a wedge between average  $q$  and marginal  $q$ . A higher value of  $\tau^2$  corresponds to a greater  $R^2$  in the measurement regression, and thus a lower degree of measurement error. In the early years, Figure 11 shows that the  $\tau^2$  values for total  $q$  and standard  $q$  are close together. This suggests that intangibles did not create a large amount of measurement error, consistent with the plots of aggregate intangible investment presented in [Peters and Taylor \(2017\)](#). In the later years, however, the two lines diverge, with the total- $q$  intangible adjustment yielding a consistently better proxy for marginal  $q$ . Since the late 1990s, the quality of the standard  $q$  proxy has worsened, while the quality of total  $q$  has improved.

As is well-known, intangibles are an increasingly important feature of the economy and accounting for them improves the measurement.

Second, consider the two series for  $\rho^2$ , which are displayed in the right panel of Figure 11. These capture the performance of the “true” investment regression, i.e., the regression of investment on the “true” marginal  $q$ . Using total investment, which includes R&D expenses and 30% of SG&A expenses as intangible investment, produces a consistently better fit than the standard investment with physical capital expenditures only. Assuming that the cumulant-estimator approach has addressed measurement error in Tobin’s  $q$ , the discrepancy between total investment and standard investment is not driven by measurement error. Rather, it suggests that  $q$  theory also applies to intangible investment, and accounting for intangibles improves the empirical performance of the  $q$  theory.

Most importantly for our purpose, the  $\rho^2$  values for both investment measures have trended upwards over time. Again, under the identifying assumptions of Erickson et al. (2014), this is not due to measurement error. Instead, it reflects the fact that the explanatory power of “true” marginal  $q$  on investment has improved, consistent with the motivating evidence from Figures 1 and 2. The improved fit of the regression is the main prediction of our model. Figure 11 empirically summarizes the importance of measurement error vis-à-vis the improving investment- $q$  relationship.

Figures B.16 through B.21 in Appendix B of the online Internet Appendix repeat this analysis using a greater number of cumulants to overidentify the system, and also using a twenty-year instead of a ten-year rolling window. The broad conclusion of all the figures is the same as in Figure 11: total  $q$  outperforms standard  $q$ , with the gap between the two widening in recent years; and the performance of the measurement-error-corrected investment- $q$  regression improves, regardless of whether total  $q$  or standard  $q$  is used.

In unreported results, we explore the  $\rho^2$  evidence cross-sectionally, between firms with above-median volatility of Tobin’s  $q$  and those with below-median volatility. The  $R^2$  fit remains much higher among firms with higher volatility of Tobin’s  $q$ , after correcting for the measurement error using Erickson et al. (2014). This pattern holds for both standard  $q$  (55%>20%) and total  $q$  (64%>28%). When firms are separated into high-tech and other industries, the fit of the  $q$  theory is higher among high-tech firms, regardless of whether standard  $q$  (51%>22%) or total  $q$  (56%>35%) is used to proxy for marginal  $q$ .

In sum, the investment- $q$  regression fits better among high-tech industries and those with high volatility in Tobin’s  $q$ , after adjusting for measurement error in Tobin’s  $q$  as in Erickson et al. (2014), and after accounting for intangibles as in Peters and Taylor (2017). This suggests that the empirical support for our model operates through a better fit of the true regression of investment on marginal  $q$ , not through differences in measurement error.



## 5. Conclusion

This paper is motivated by the empirical finding that the relationship between aggregate investment and Tobin's  $q$  has become remarkably tight in recent years. This observation stands in contrast to a large literature showing that this regression performed quite poorly in the past. We attribute the improvement in the empirical performance of the classic regression to an increase in the empirical variation in Tobin's  $q$  relative to residual factors affecting investment.

We rationalize these patterns with two possible mechanisms. On one hand, research has the potential of producing innovations leading to higher future cash flows, where the random arrival of the innovation jump introduces volatility in valuations. On the other hand, learning leads to more updating about investment decisions and valuations. Both mechanisms endogenously produce more variation in Tobin's  $q$ , improving the fit of the regression. Thus, the improved fit of the investment- $q$  relationship is related to the substantial growth in expenditures on research and other intangibles in the aggregate economy. We investigate the model's predictions in the cross-section of firms in Compustat, and find empirical support.

In conclusion, the  $q$  theory of investment can describe the data quite well, when given sufficient variation in the key regression variable. Counterintuitively, this variation arises in firms far different from the canonical capital-intensive firms for which the theory was initially developed. Our findings suggest that corporate innovations and learning may be important features to capture in investment models, and that Tobin's  $q$  may be a particularly effective proxy for investment opportunities in R&D industries. Most importantly, as research-intensive firms are a growing segment of the economy, the future of the investment- $q$  relationship looks increasingly bright.

- Abel, A., 1983. Optimal Investment under Uncertainty. *American Economic Review* 73, 228–233.
- Abel, A., 2017. The Effects of  $q$  and Cash Flow on Investment in the Presence of Measurement Error. *Journal of Financial Economics* (forthcoming).
- Abel, A. B., Eberly, J. C., 2011. How  $Q$  and Cash Flow Affect Investment without Frictions: An Analytic Explanation. *Review of Economic Studies* 78, 1179–1200.
- Aghion, P., Bloom, N., Blundell, R., Griffith, R., Howitt, P., 2005. Competition and Innovation: An Inverted-U Relationship. *The Quarterly Journal of Economics* 120, 701–728.
- Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. *Econometrica* 60, 323–351.
- Alti, A., 2003. How Sensitive Is Investment to Cash Flow when Financing Is Frictionless? *Journal of Finance* 58, 707–722.
- Aw, B. Y., Roberts, M. J., Xu, D. Y., 2011. R&D Investment, Exporting, and Productivity Dynamics. *American Economic Review* 101, 1312–1344.
- Bai, J., Philippon, T., Savov, A., 2016. Have Financial Markets Become More Informative? *Journal of Financial Economics* 122, 625–654.
- Bakke, T.-E., Whited, T. M., 2010. Which Firms Follow the Market? An Analysis of Corporate Investment Decisions. *Review of Financial Studies* 23, 1941–1980.
- Begenau, J., Farboodi, M., Veldkamp, L., 2017. Big Data in Finance and the Growth of Large Firms *Working Paper*.
- Bloom, N., Schankerman, M., Reenen, J. V., 2013. Identifying Technology Spillovers and Product Market Rivalry. *Econometrica* 81, 1347–1393.
- Brainard, W. C., Tobin, J., 1968. Pitfalls in Financial Model Building. *American Economic Review Papers and Proceedings* 58, 99–122.
- Brennan, M. J., 1998. The Role of Learning in Dynamic Portfolio Decisions. *European Finance Review* 1, 295–306.
- Bresnahan, T., Milgrom, P., Paul, J., 1992. The Real Output of the Stock Exchange. In: *Output Measurement in the Service Sectors*, National Bureau of Economic Research, Inc, pp. 195–216.
- Brown, J. R., Fazzari, S. M., Petersen, B. C., 2009. Financing Innovation and Growth: Cash Flow, External Equity, and the 1990s R&D Boom. *Journal of Finance* 44, 151–185.
- Chen, Q., Goldstein, I., Jiang, W., 2007. Price Informativeness and Investment Sensitivity to Stock Price. *Review of Financial Studies* 20, 619–650.
- Cooper, R., Ejarque, J., 2003. Financial Frictions and Investment: Requiem in  $q$ . *Review of Economic Dynamics* 6, 710–728.
- Das, S. R., 2002. The surprise element: Jumps in interest rates. *Journal of Econometrics* 106, 27–65.
- Detemple, J., Kihlstrom, R., 1987. Acquisition d’information dans un modèle intertemporel en temps continu. *L’Actualité économique* 63, 118–137.
- Doraszelski, U., Jaumandreu, J., 2013. R&D and Productivity: Estimating Endogenous Productivity. *The Review of Economic Studies* 80, 1338–1383.
- Dow, J., Goldstein, I., Guembel, A., 2017. Incentives for Information Production in Markets where Prices Affect Real Investment. *Journal of the European Economic Association* 15, 877–909.
- Dow, J., Gorton, G., 1997. Stock Market Efficiency and Economic Efficiency: Is There a Connection? *Journal of Finance* 52, 1087–1129.

- Duffie, D., Pan, J., Singleton, K., 2000. Transform analysis and asset pricing for affine jump-diffusions. *Econometrica* 68, 1343–1376.
- Dumas, B., Kurshev, A., Uppal, R., 2009. Equilibrium Portfolio Strategies in the Presence of Sentiment Risk and Excess Volatility. *The Journal of Finance* 64, 579–629.
- Durnev, A., Morck, R., Yeung, B., 2004. Value-enhancing capital budgeting and firm-specific stock return variation. *The Journal of Finance* 59, 65–105.
- Edmans, A., Goldstein, I., Jiang, W., 2015. Feedback Effects, Asymmetric Trading, and the Limits to Arbitrage. *American Economic Review* 105, 3766–3797.
- Erickson, T., Jiang, C., Whited, T., 2014. Minimum Distance Estimation of the Errors-in-Variables Model Using Linear Cumulant Equations. *Journal of Econometrics* 183, 211–221.
- Erickson, T., Whited, T., 2000. Measurement Error and the Relationship between Investment and  $q$ . *Journal of Political Economy* 108, 1027–1057.
- Erickson, T., Whited, T., 2012. Treating Measurement Error in Tobin’s  $q$ . *Review of Financial Studies* 25, 1286–1329.
- Farboodi, M., Matray, A., Veldkamp, L., 2017. Where Has All the Big Data Gone? *Princeton and NYU Working Paper*.
- Fazzari, S., Hubbard, R. G., Petersen, B., 1988. Financing Constraints and Corporate Investment. *Brookings Papers on Economic Activity* 1, 141–206.
- Gilchrist, S., Himmelberg, C. P., 1995. Evidence on the Role of Cash Flow for Investment. *Journal of Monetary Economics* 36, 541–572.
- Goldstein, I., Guembel, A., 2008. Manipulation and the Allocational Role of Prices. *Review of Economic Studies* 75, 133–164.
- Gomes, J., 2001. Financing Investment. *American Economic Review* 91, 1263–1285.
- Griliches, Z., 1979. Issues in Assessing the Contribution of Research and Development to Productivity Growth. *Bell Journal of Economics* 10, 92–116.
- Grossman, G. M., Helpman, E., 1991. Quality ladders in the theory of growth. *The Review of Economic Studies* 58, 43–61.
- Gutiérrez, G., Philippon, T., 2016. Investment-less Growth: An Empirical Investigation *NYU Working Paper*.
- Hall, B., 2002. The financing of research and development. *Oxford Review of Economic Policy* 18, 35–51.
- Hall, B. H., Mairesse, J., Mohnen, P., 2009. Measuring the Returns to R&D. NBER Working Papers 15622, National Bureau of Economic Research, Inc.
- Hall, R. E., 2001. The Stock Market and Capital Accumulation. *American Economic Review* 91, 1185–1202.
- Hayashi, F., 1982. Tobin’s Marginal  $q$  and Average  $q$ : A Neoclassical Interpretation. *Econometrica* 50, 213–224.
- Hennessy, C. A., Radnaev, B., 2018. Learning and Leverage Cycles in General Equilibrium: Theory and Evidence. *Review of Finance* 22, 311–335.
- Johannes, M., 2004. The Statistical and Economic Role of Jumps in Continuous-Time Interest Rate Models. *The Journal of Finance* 59, 227–260.
- Kaplan, S. N., Zingales, L., 1997. Do Investment-Cash Flow Sensitivities Provide Useful Measures of Financing Constraints? *Quarterly Journal of Economics* 112, 169–215.

- Keynes, J. M., 1936. *The General Theory of Employment, Interest and Money*. London: Macmillan.
- Klette, T., Kortum, S., 2004. Innovating Firms and Aggregate Innovation. *Journal of Political Economy* 112, 986–1018.
- Lindenberg, E. B., Ross, S., 1981. Tobin's  $q$  Ratio and Industrial Organization. *Journal of Business* 54, 1–32.
- Liptser, R. S., Shiryaev, A. N., 2001. *Statistics of Random Processes, vol. II. Applications*. Springer-Verlag, Second ed.
- Mairesse, J., Sassenou, M., 1991. R&D Productivity: A Survey of Econometric Studies at the Firm Level. *Science-Technology-Industry Review* 8, 9–43.
- Merton, R. C., 1971. Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory* 3, 373–413.
- Moyen, N., 2004. Investment-Cash Flow Sensitivities: Constrained versus Unconstrained Firms. *Journal of Finance* 59, 2061–2092.
- Moyen, N., Platikanov, S., 2012. Corporate Investments and Learning. *Review of Finance* 17, 1437–1488.
- Mussa, M., 1977. External and Internal Adjustment Costs and the Theory of Aggregate and Firm Investment. *Economica* 44, 163–178.
- Myers, S. C., Majluf, N. S., 1984. Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have. *Journal of Financial Economics* 13, 187–221.
- Peters, R. H., Taylor, L. A., 2017. Intangible capital and the investment- $q$  relation. *Journal of Financial Economics* 123, 251–272.
- Philippon, T., 2009. The Bond Market's  $q$ . *Quarterly Journal of Economics* 124, 1011–1056.
- Piazzesi, M., 2010. Affine term structure models. *Handbook of Financial Econometrics* 1, 691–766.
- Romer, P. M., 1990. Endogenous technological change. *Journal of political Economy* 98, S71–S102.
- Salinger, M. A., 1984. Tobin's  $q$ , Unionization, and the Concentration-Profits Relationship. *RAND Journal of Economics* 15, 159–170.
- Scheinkman, J. A., Xiong, W., 2003. Overconfidence and speculative bubbles. *Journal of political Economy* 111, 1183–1220.
- Subrahmanyam, A., Titman, S., 1999. The Going-Public Decision and the Development of Financial Markets. *Journal of Finance* 54, 1045–1082.
- Thompson, P., 2001. The Microeconomics of an R&D-Based Model of Endogenous Growth. *Journal of Economic Growth* 6, 263–83.
- Tobin, J., 1969. A General Equilibrium Approach to Monetary Theory. *Journal of Money, Credit and Banking* 1, 15–29.
- Warusawitharana, M., 2015. Research and development, profits, and firm value: A structural estimation. *Quantitative Economics* 6, 531–565.

## Appendix A.

### Appendix A.1. Aggregate data from NIPA tables and Flow of Funds

This section provides details behind the calculations of aggregate investment and Tobin's  $q$ .

*Investment.* The numerator of the investment rate is private nonresidential fixed investment, seasonally-adjusted at annual rates (row 2 of NIPA table 5.3.5). The denominator is the net capital stock, measured as the current-cost net stock of private nonresidential fixed assets (row 4 of table 1.1 of the BEA Fixed Asset Accounts Tables). Both investment and capital stock are deflated using the price index for private nonresidential fixed investment (row 2 of NIPA table 5.3.4). The capital stock is recorded only in the last quarter of each year, so we smooth these year-end values linearly to the other quarters of the year. The denominator of the investment rate is lagged by one quarter.

Note that private nonresidential fixed assets were redefined in 2013 to include intellectual property (IP) products. Investment in IP products is recorded in row 16 of table 5.3.5, and the current-cost net stock of IP products is recorded in row 7 of table 1.1. For comparison with older definitions, in Figures B.22 and B.23 in the online Internet Appendix we repeat the analysis of Figures 1 and 2 after subtracting out these series from the investment and capital stock.

*Tobin's  $q$ .* These calculations follow Hall (2001). The numerator of Tobin's  $q$  is the aggregate market value of corporate equity and corporate debt, minus corporate inventories. Aggregate market equity is series FL103164103 from the Fed's Flow of Funds website (note that this series was previously labeled FL103164003 until mid-2010). Aggregate corporate debt is measured as financial liabilities (series FL104190005Q), minus financial assets (series FL104090005Q), plus the market value of outstanding bonds, minus the book value of outstanding bonds. The book value of outstanding bonds is the sum of the outstanding amounts of taxable corporate bonds (series FL103163003Q) and tax-exempt corporate bonds (series FL103162000Q). Inventories are measured with private nonfarm inventories located in row 3 of NIPA Table 5.8.5.

The market value of bonds is calculated according to an algorithm employed in Hall (2001): Corporate bonds are assumed to be issued with ten-year maturities at a yield taken from a broad index (for taxable bonds, the BAA yield reported by Moody's; for tax-exempt bonds, the muni bond yields reported in the Federal Reserve's Table H.15). Market values are then recalculated for each vintage of bonds in each year by discounting their remaining scheduled payments at the then-prevailing yield, so that the market and book values of any vintage of bonds diverge after the issuance date.

The denominator of Tobin's  $q$  is the replacement cost of the corporate capital stock. This series is initialized in 1952 at the NIPA real net stock of private nonresidential fixed assets, then measured in later quarters by capitalizing real gross corporate fixed investment (series FU105013005) at an annual depreciation rate of 10%. Investment is deflated using the same NIPA deflator as above. As in Hall (2001), the replacement cost is then reinflated using the same deflator, to be consistent with the numerator of Tobin's  $q$  which is measured in nominal terms.

### Appendix A.2. Proof of Proposition 1 and of Corollary 1.1

The observable variables are the cash-flow process (16) and the signal (18). The unobservable variable is  $\mu_t$ . Write the dynamics of the observable variables  $\theta_t$  and  $s_t$ :

$$\begin{bmatrix} d\theta_t \\ ds_t \end{bmatrix} = \left( \underbrace{\begin{bmatrix} -\lambda\theta_t \\ 0 \end{bmatrix}}_{A_0} + \underbrace{\begin{bmatrix} \lambda \\ 0 \end{bmatrix}}_{A_1} \mu_t \right) dt + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B_1} dW_t^\mu + \underbrace{\begin{bmatrix} \sigma_\theta & 0 \\ 0 & \frac{1}{\sqrt{\Phi}} \end{bmatrix}}_{B_2} \begin{bmatrix} dW_t^\theta \\ dW_t^s \end{bmatrix}, \quad (\text{Appendix A.1})$$

and of the unobservable variable  $\mu_t$ :

$$d\mu_t = \left( \underbrace{\eta\bar{\mu}}_{a_0} + \underbrace{(-\eta)\mu_t}_{a_1} \right) dt + \underbrace{\sigma_\mu}_{b_1} dW_t^\mu + \underbrace{\begin{bmatrix} 0 & 0 \end{bmatrix}}_{b_2} \begin{bmatrix} dW_t^\theta \\ dW_t^s \end{bmatrix}. \quad (\text{Appendix A.2})$$

We will apply the following standard theorem.

**Theorem 1.** (Theorem 12.7, page 36 of [Liptser and Shiryaev, 2001](#)) Consider an unobservable process  $u_t$  and an observable process  $s_t$  with dynamics given by

$$du_t = [a_0(t, s_t) + a_1(t, s_t)u_t]dt + b_1(t, s_t)dZ_t^u + b_2(t, s_t)dZ_t^s \quad (\text{Appendix A.3})$$

$$ds_t = [A_0(t, s_t) + A_1(t, s_t)u_t]dt + B_1(t, s_t)dZ_t^u + B_2(t, s_t)dZ_t^s. \quad (\text{Appendix A.4})$$

All the parameters can be functions of time and of the observable process. The posterior mean (the filter) and the posterior variance (the Bayesian uncertainty) evolve according to (we drop the dependence of coefficients on  $t$  and  $s_t$  for notational convenience):

$$d\hat{u}_t = (a_0 + a_1\hat{u}_t)dt + [(b \circ B) + \zeta_t A_1^\top](B \circ B)^{-1}[ds_t - (A_0 + A_1\hat{u}_t)dt] \quad (\text{Appendix A.5})$$

$$\frac{d\zeta_t}{dt} = a_1\zeta_t + \zeta_t a_1^\top + (b \circ b) - [(b \circ B) + \zeta_t A_1^\top](B \circ B)^{-1}[(b \circ B) + \zeta_t A_1^\top]^\top, \quad (\text{Appendix A.6})$$

where

$$b \circ b = b_1 b_1^\top + b_2 b_2^\top \quad (\text{Appendix A.7})$$

$$B \circ B = B_1 B_1^\top + B_2 B_2^\top \quad (\text{Appendix A.8})$$

$$b \circ B = b_1 B_1^\top + b_2 B_2^\top. \quad (\text{Appendix A.9})$$

In our setup, we obtain

$$b \circ b = \sigma_\mu^2, \quad B \circ B = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \frac{\Phi+1}{\Phi} \end{bmatrix}, \quad b \circ B = \begin{bmatrix} 0 & \sigma_\mu \end{bmatrix}, \quad (\text{Appendix A.10})$$

and

$$[(b \circ B) + \zeta_t A_1^\top](B \circ B)^{-1} = \begin{bmatrix} \frac{\lambda \zeta_t}{\sigma_\theta^2} & \frac{\sigma_\mu \Phi}{1+\Phi} \end{bmatrix}. \quad (\text{Appendix A.11})$$

This yields

$$d\hat{\mu}_t = \eta(\bar{\mu} - \hat{\mu}_t)dt + \begin{bmatrix} \frac{\lambda}{\sigma_\theta^2} \zeta_t & \sigma_\mu \frac{\Phi}{1+\Phi} \end{bmatrix} \begin{bmatrix} d\theta_t - \lambda(\hat{\mu}_t - \theta_t)dt \\ ds_t \end{bmatrix}. \quad (\text{Appendix A.12})$$

Equation (21) of Proposition 1 results directly from (Appendix A.6):

$$\frac{d\zeta_t}{dt} = \frac{\sigma_\mu^2}{1+\Phi} - 2\eta\zeta_t - \frac{\lambda^2 \zeta_t^2}{\sigma_\theta^2}. \quad (\text{Appendix A.13})$$

This deterministic process has the following steady-state solution

$$\bar{\zeta} = \frac{\sigma_\theta^2}{\lambda^2} \left( \sqrt{\eta^2 + \frac{1}{1+\Phi} \frac{\lambda^2 \sigma_\mu^2}{\sigma_\theta^2}} - \eta \right). \quad (\text{Appendix A.14})$$

The (observable) process  $\theta$  can be written in two ways:

$$d\theta_t = \lambda(\mu_t - \theta_t)dt + \sigma_\theta dW_t^\theta \quad (\text{Appendix A.15})$$

$$d\theta_t = \lambda(\hat{\mu}_t - \theta_t)dt + \sigma_\theta d\widehat{W}_t^\theta. \quad (\text{Appendix A.16})$$

The first equation is written under the physical (true) probability measure. The second equation is written under the filtration of the firm, and  $\widehat{W}_t^\theta$  is a standard Brownian motion under this filtration. Intuitively, the second equation shows how the firm interprets the dynamics of the observable process  $\theta$ .

From (Appendix A.16), we obtain:

$$d\theta_t - \lambda(\hat{\mu}_t - \theta_t)dt = \sigma_\theta d\widehat{W}_t^\theta. \quad (\text{Appendix A.17})$$

Furthermore, we can write the signal as

$$ds_t = dW_t^\mu + \frac{1}{\sqrt{\Phi}} dW_t^s = \sqrt{\frac{\Phi+1}{\Phi}} d\widehat{W}_t^s, \quad (\text{Appendix A.18})$$

where  $\widehat{W}_t^s$  is a standard Brownian motion independent of  $\widehat{W}_t^\theta$ . This yields equation (20) in Proposition 1:

$$d\hat{\mu}_t = \eta(\bar{\mu} - \hat{\mu}_t)dt + \begin{bmatrix} \frac{\lambda}{\sigma_\theta} \zeta_t & \sigma_\mu \sqrt{\frac{\Phi}{1+\Phi}} \end{bmatrix} \begin{bmatrix} d\widehat{W}_t^\theta \\ d\widehat{W}_t^s \end{bmatrix}. \quad (\text{Appendix A.19})$$

The cash-flow shocks  $d\widehat{W}_t^\theta$  result from (Appendix A.15)-(Appendix A.16):

$$d\widehat{W}_t^\theta = dW_t^\theta + \frac{\lambda}{\sigma_\theta} (\mu_t - \hat{\mu}_t)dt, \quad (\text{Appendix A.20})$$

and the information shocks  $d\widehat{W}_t^s$  result from (Appendix A.18):

$$d\widehat{W}_t^s = \sqrt{\frac{\Phi}{1+\Phi}} ds_t. \quad (\text{Appendix A.21})$$

This completes Proposition 1. After replacement of (Appendix A.14) in (Appendix A.19), we obtain

$$d\hat{\mu}_t = \eta(\bar{\mu} - \hat{\mu}_t)dt + \frac{\sigma_\theta}{\lambda} \left( \sqrt{\eta^2 + \frac{1}{1+\Phi} \frac{\lambda^2 \sigma_\mu^2}{\sigma_\theta^2}} - \eta \right) d\widehat{W}_t^\theta + \sigma_\mu \sqrt{\frac{\Phi}{1+\Phi}} d\widehat{W}_t^s, \quad (\text{Appendix A.22})$$

and thus the instantaneous variance of the filter  $\hat{\mu}_t$  is

$$\text{Var}_t[d\hat{\mu}_t] = \sigma_\mu^2 - \frac{2\eta\sigma_\theta^2}{\lambda^2} \left( \sqrt{\eta^2 + \frac{1}{1+\Phi} \frac{\lambda^2 \sigma_\mu^2}{\sigma_\theta^2}} - \eta \right) \quad (\text{Appendix A.23})$$

$$= \sigma_\mu^2 - 2\eta\bar{\zeta}. \quad (\text{Appendix A.24})$$

We can then compute

$$\frac{\partial \text{Var}_t[d\hat{\mu}_t]}{\partial \sigma_\mu} = 2\sigma_\mu \left( 1 - \frac{\eta\sigma_\theta}{(1+\Phi)\sqrt{\eta^2\sigma_\theta^2 + \frac{\lambda^2\sigma_\mu^2}{1+\Phi}}} \right) > 0 \quad (\text{Appendix A.25})$$

and

$$\frac{\partial \text{Var}_t[d\hat{\mu}_t]}{\partial \Phi} = \frac{\eta\sigma_\theta\sigma_\mu^2}{(1+\Phi)^2\sqrt{\eta^2\sigma_\theta^2 + \frac{\lambda^2\sigma_\mu^2}{1+\Phi}}} > 0, \quad (\text{Appendix A.26})$$

which yields Corollary 1.1.  $\square$

### Appendix A.3. Discretization used for simulations

#### Appendix A.3.1. Model with innovation jumps

For the model of Section 3.2, the following two processes have to be simulated:

$$d\theta_t = \lambda(\bar{\mu} - \theta_t)dt + \sigma_\theta dW_t^\theta + JdN_t \quad (\text{Appendix A.27})$$

$$d\nu_t = -\kappa\nu_t dt + \sigma_\nu dW_t^\nu. \quad (\text{Appendix A.28})$$

We implement the following discretization scheme:

$$\theta_{t+\Delta t} = \theta_t e^{-\lambda\Delta t} + \bar{\mu}(1 - e^{-\lambda\Delta t}) + \sigma_\theta \sqrt{\frac{1 - e^{-2\lambda\Delta t}}{2\lambda}} \varepsilon_t^\theta + JN_{\Delta t} \quad (\text{Appendix A.29})$$

$$\nu_{t+\Delta t} = \nu_t e^{-\kappa\Delta t} + \sigma_\nu \sqrt{\frac{1 - e^{-2\kappa\Delta t}}{2\kappa}} \varepsilon_t^\nu, \quad (\text{Appendix A.30})$$

where  $\varepsilon_t^\theta$  and  $\varepsilon_t^\nu$  are i.i.d. standard normal variables,  $\varepsilon_t^\theta, \varepsilon_t^\nu \sim \mathcal{N}(0, 1)$ , and  $N_{\Delta t}$  is a Poisson random variable with parameter  $h(\Phi)\Delta t$ . Due to the presence of jumps, we simulate the two processes at a high frequency (daily), then we sample data points at yearly frequency in order to build Figure 5.

#### Appendix A.3.2. Model with learning

The process for the shock to the purchase price of capital, which is the same in both models, is given in (Appendix A.28), and its discretization is provided in (Appendix A.30). Two remaining processes have to be simulated under the filtration of the firm:

$$d\theta_t = \lambda(\widehat{\mu}_t - \theta_t)dt + \sigma_\theta d\widehat{W}_t^\theta \quad (\text{Appendix A.31})$$

$$d\widehat{\mu}_t = \eta(\bar{\mu} - \widehat{\mu}_t)dt + \Omega d\widehat{W}_t^\theta + \sigma_\mu \sqrt{\frac{\Phi}{1 + \Phi}} d\widehat{W}_t^s, \quad (\text{Appendix A.32})$$

where we define

$$\Omega \equiv \frac{\sigma_\theta}{\lambda} \left( \sqrt{\eta^2 + \frac{1}{1 + \Phi} \frac{\lambda^2 \sigma_\mu^2}{\sigma_\theta^2}} - \eta \right). \quad (\text{Appendix A.33})$$

We implement the following discretization of (Appendix A.31)-(Appendix A.32):

$$\theta_{t+\Delta t} = \theta_t e^{-\lambda\Delta t} + \widehat{\mu}_t(1 - e^{-\lambda\Delta t}) + \sigma_\theta \sqrt{\frac{1 - e^{-2\lambda\Delta t}}{2\lambda}} \varepsilon_t^\theta \quad (\text{Appendix A.34})$$

$$\widehat{\mu}_{t+\Delta t} = \widehat{\mu}_t e^{-\eta\Delta t} + \bar{\mu}(1 - e^{-\eta\Delta t}) + \sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}} \left( \Omega \varepsilon_t^\theta + \sigma_\mu \sqrt{\frac{\Phi}{1 + \Phi}} \varepsilon_t^s \right), \quad (\text{Appendix A.35})$$

where  $\varepsilon_t^s$  is an i.i.d. standard normal variable,  $\varepsilon_t^s \sim \mathcal{N}(0, 1)$ , independent of  $\varepsilon_t^\theta$  and  $\varepsilon_t^\nu$ . We simulate these two processes at yearly frequency.

#### Appendix A.3.3. Simulations

Once we have simulated the above time series, we compute  $q(x_t)$  for each point in the state space, where  $x_t = \{\theta_t, \nu_t\}$  in the model with innovations and  $x_t = \{\theta_t, \widehat{\mu}_t, \nu_t\}$  in the model with learning. Then we use the first-order condition for investment, which is given in equation (14) or (25), to compute the investment-capital ratio for each simulated point,

$$\frac{I_t}{K_t} = -\frac{1}{a} + \frac{1}{a} q(x_t) - \frac{1}{a} \nu_t. \quad (\text{Appendix A.36})$$

This provides all the data necessary for the plots.