Asset Pricing with Investor Learning About the Fed's Ability to Control Inflation

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Abstract

This paper builds an asset pricing model to investigate the stock market implications of uncertainty surrounding the Federal Reserve's (Fed) inflation-fighting ability. In the model, investors learn about the Fed's ability to control inflation. This learning process amplifies the market risk premium and return volatility, particularly during highly accommodative or restrictive monetary policy cycles. The effect is stronger during restrictive cycles, as learning magnifies stock responses to inflation surprises. Moreover, a decline in the Fed's perceived ability further boosts risk premia and volatility by fueling expectations of more persistent inflation. Empirical tests support the model's predictions, demonstrating the central role of beliefs about the Fed's ability in driving asset prices.

Keywords: Asset Pricing, Learning, Inflation, Federal Reserve
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1 Introduction

This paper develops a novel asset pricing model to investigate how investor learning about the Federal Reserve's (henceforth, "Fed") ability to control inflation affects financial markets. Inflation, a critical macroeconomic variable with significant implications for economic stability and societal well-being, has returned to the forefront of economic concerns following the COVID-19 pandemic and the unprecedented policy responses it triggered. This resurgence has created a period of exceptional uncertainty surrounding inflation's future path, leading to a fundamental question: How do investors' perceptions of the Fed's ability to control inflation affect asset prices? To answer this question and quantify its implications, our asset pricing model incorporates a mechanism through which investors learn about the Fed's inflation-fighting ability, and demonstrates how this learning process shapes asset risk premia and volatility.

Our model departs from existing work by incorporating a time-varying and unobservable parameter that captures the Fed's ability to control inflation into an otherwise standard equilibrium asset-pricing framework. This parameter, which we interpret as the Fed's inflation-fighting *ability*, directly impacts long-term inflation expectations, and therefore, long-run inflation risk. Investors do not directly observe the Fed's ability. Instead, they learn about it by observing realized inflation, which is influenced by monetary policy through a standard Taylor rule. This dynamic establishes a feedback loop: monetary policy influences inflation, observed inflation influences investor beliefs about the Fed's ability, and these beliefs, in turn, drive macroeconomic variables and asset prices.

The model yields two novel insights. First, uncertainty about the Fed's ability to control inflation generates an "inflation-control uncertainty" premium in the stock market. This premium arises because investors are unsure about the long-run effectiveness of monetary policy, adding a new layer of risk to asset prices. The uncertainty premium is particularly pronounced when monetary policy deviates from a neutral stance, whether restrictive or accommodative. Intuitively, when the Fed is actively using its tools, the stakes of correctly gauging its ability are higher. The model demonstrates that this uncertainty amplifies the market risk premium and market return volatility during these periods.

Second, the model uncovers an asymmetric response of asset prices to monetary policy, dependent on the policy stance. When the Fed adopts a restrictive stance (i.e., raising interest rates to combat high inflation), positive inflation surprises are perceived as "doubly bad" news. They not only signal higher-than-expected inflation but also erode investor confidence in the Fed's ability to control it. This leads to a larger increase in the risk premium and a sharper decline in asset prices than under an accommodative stance, where positive inflation surprises can be perceived as a sign of the Fed successfully moving inflation toward a higher target. The asymmetry implies that the market risk premium and market volatility are higher during restrictive policy cycles than during accommodative ones.

These two novel effects, which arise from the interaction between investor learning and the monetary policy stance, are amplified by the agent's perception of the Fed's ability to control inflation. When the perceived ability is low, the agent sees inflation as more persistent and long-run risk as more severe. This in turn magnifies the sensitivity of asset prices to monetary policy and inflation surprises, leading to a higher market risk premium and market return volatility.

To quantify the model's predictions, we estimate its parameters using maximum likelihood on U.S. macroeconomic data (real GDP, Federal funds rate, inflation, and the output gap) from 1954 to 2023. Importantly, the estimation does not rely on asset price data. The model nevertheless generates asset pricing moments that closely replicate those observed in the data, including a realistic average real interest rate (1%), nominal interest rate (4.5%), market risk premium (6%), and market return volatility (12%).

We test the model's predictions using the S&P 500 as a proxy for the market. We construct empirical time series for the market risk premium, market return volatility, market price-dividend ratio, real interest rate, and expected output growth rate, and compare them to their model-implied counterparts. The results confirm the model's predictions. As predicted by the model, we find that the expected output growth rate, the real interest rate, and the market price-dividend ratio are positively associated with the output gap and negatively associated with inflation. We document that the market risk premium and market return volatility are negatively related to the Fed's perceived ability to control inflation and exhibit an asymmetric U-shaped relationship with the monetary policy stance. Specifically, both the risk premium and volatility are higher when the Fed deviates from a neutral stance, and this effect is more pronounced during restrictive policy cycles. These relationships are statistically significant and robust across different specifications, both in the model and in the data. Finally, we conduct predictive regressions of future market returns on the price-dividend ratio, the real interest rate, and expected output growth. The results show that these variables are significant predictors of future returns, in the model and the data, further validating the model's ability to capture real-world asset pricing dynamics.

Literature Our work contributes to several strands of the literature. Primarily, it contributes to the literature on asset pricing under uncertainty, particularly how investor learning shapes risk premia and volatility. We extend this line of research by developing a novel theoretical framework that incorporates learning about a key macroeconomic parameter—the Fed's ability to control inflation—within a standard long-run risk model (Bansal and Yaron, 2004). While existing studies have examined the role of investor perceptions and beliefs about monetary policy (e.g., Bauer, Pflueger, and Sunderam, 2024; Caballero and Simsek, 2022; Bianchi, Ludvigson, and Ma, 2022), as well as how these beliefs are formed (e.g., Coibion and Gorodnichenko, 2012, 2015), the primary contribution of this paper is an analysis of the asset pricing implications of learning about the effectiveness of monetary policy in controlling inflation. A contemporaneous empirical study by Ghaderi, Seo, and Shaliastovich (2024) examines investor preferences for different inflation ranges using survey data. Their finding that investors dislike both very high and very low inflation outcomes, preferring a moderate range, aligns with the asset pricing implications of our model, in which uncertainty about the Fed's ability to control inflation generates a U-shaped relationship between the market risk premium (and volatility) and the monetary policy stance.

Our paper builds upon the literature analyzing the interplay between incomplete information, monetary policy, and asset prices. Specifically, our work connects to Bansal and Shaliastovich (2013), who demonstrate that long-run risks can generate asset price predictability in bond and currency markets. We extend their line of research by examining how incomplete information about the Fed's ability to control inflation generates predictability in equity markets (see also Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch, 2018; Xiong and Yan, 2010; Veronesi, 1999, 2000; Ai, 2010; Buraschi and Jiltsov, 2005; Wachter, 2006; Detemple, 1986). By incorporating uncertainty about the Fed's inflation-fighting ability, we provide a novel perspective on the sources of risk premia and return volatility. Our work contributes to the broader literature on asset pricing in monetary economies (Bhamra, Dorion, Jeanneret, and Weber, 2023; Gallmeyer, Hollifield, Palomino, and Zin, 2007; Gil de Rubio Cruz, Osambela, Palazzo, Palomino, and Suarez, 2022; Palazzo and Yamarthy, 2022; Bonelli, Palazzo, and Yamarthy, 2024; Danthine and Donaldson, 1986; Pflueger and Rinaldi, 2022; Bakshi and Chen, 1996). We identify a novel channel through which monetary policy drives asset prices: by shaping investor beliefs about the central bank's ability to achieve its inflation target, affecting long-run inflation risk.

The paper proceeds as follows: Sections 2 and 3 outline the model and parameter estimation. Section 4 presents the implications and empirical tests. Section 5 concludes.

2 The Economic Environment

Consider a continuous-time infinite-horizon economy with a single representative agent. The agent derives utility from consumption C, which equals aggregate output δ in equilibrium. The agent has Kreps-Porteus preferences (Epstein and Zin, 1989; Weil, 1990) with subjective discount rate ρ, relative risk aversion γ, and elasticity of intertemporal substitution ψ. We denote the agent's indirect utility by J_t. Formally, the indirect utility satisfies

$$J_t = \mathbb{E}_t \left[\int_t^\infty h(C_s, J_s) ds \right], \tag{1}$$

where, following Duffie and Epstein (1992), the aggregator $h(C, J)$ is

$$h(C,J) = \frac{\rho}{1-1/\psi} \left(\frac{C^{1-1/\psi}}{[(1-\gamma)J]^{1/\theta-1}} - (1-\gamma)J \right), \quad \text{with } \theta \equiv \frac{1-\gamma}{1-1/\psi}. \tag{2}$$

We assume that real aggregate output, δ, follows the dynamic process

$$\frac{d\delta_t}{\delta_t} = \mu_{\delta,t} dt + \sigma_\delta dB_{\delta,t}, \tag{3}$$

where $\mu_{\delta,t}$ is the expected growth rate of output, $\sigma_\delta > 0$ is a constant, and B_δ is a one-dimensional Brownian motion. The expected growth rate $\mu_{\delta,t}$ will be determined endogenously from the agent's optimality conditions.

In this economy, the consumption good price level, p_t, evolves according to

$$\frac{dp_t}{p_t} = \pi_t dt, \tag{4}$$

where π_t is the inflation rate, i.e., the instantaneous growth rate of the price level.

Inflation Dynamics and Monetary Policy Having defined π_t as the inflation rate, we now turn to its dynamics and the role of monetary policy in shaping it. To justify the affine inflation dynamics assumed in this paper, we draw on insights from Cochrane (2024) and the New-Keynesian literature (Rotemberg, 1982; Calvo, 1983; Galí, 2015), which provide a theoretical foundation for modeling inflation as a process influenced by the output gap and monetary policy. Specifically, the inflation rate π_t follows a *forward-looking Phillips curve*, which, in deviation-from-mean form, can be expressed as

$$\pi_t^d = \beta \mathbb{E}_t[\pi_{t+1}^d] + \kappa y_t \tag{5}$$

where y_t is the (zero-mean) output gap, and $\pi_t^d \equiv \pi_t - \overline{\pi}$ is inflation's deviation from its mean $\overline{\pi}$. Here, y_t is a reduced-form variable that captures short-term fluctuations relevant for inflation dynamics.

Rewriting this in continuous time (see Cochrane, 2024) and rearranging yields

$$\mathbb{E}_t\left[d\pi_t^d\right] = -\rho_\pi \pi_t^d dt - \rho_y y_t dt, \tag{6}$$

where $\rho_\pi \equiv \left(1 - \frac{1}{\beta}\right)$ and $\rho_y \equiv \frac{\kappa}{\beta}$. Reintroducing the mean level $\overline{\pi}$ leads to an affine drift for the inflation process:

$$E_t[d\pi_t] = \rho_\pi(\overline{\pi} - \pi_t)dt - \rho_y y_t dt. \tag{7}$$

Monetary policy adheres to the well-known Taylor rule (Taylor, 1993), which links the nominal interest rate $r_{N,t}$ to deviations of inflation from target and the output gap:

$$r_{N,t} = \overline{r}_N + \beta_\pi(\pi_t - \overline{\pi}) + \beta_y y_t, \tag{8}$$

where $\overline{r}_N$ denotes the long-run nominal interest rate, and β_π and β_y capture the monetary authority's responsiveness to inflation and the output gap, respectively.

By substituting the Taylor rule (8) into the Phillips-curve-based inflation equation (7), the drift of the inflation process becomes

$$\mathbb{E}_t[d\pi_t] = \underbrace{\left(\rho_\pi - \tfrac{\rho_y \beta_\pi}{\beta_y}\right)}_{\equiv \lambda_\pi}\left(\overline{\pi} - \pi_t\right)dt - \frac{\rho_y}{\beta_y}\left(r_{N,t} - \overline{r}_N\right)dt. \tag{9}$$

Rewriting, we obtain the following affine process for π_t:

$$d\pi_t = \lambda_\pi \Big(\overline{\pi}_t - \pi_t\Big) dt + \sigma_\pi dB_{\pi,t}, \quad \text{where} \quad \overline{\pi}_t \equiv \overline{\pi} - a\Big(r_{N,t} - \overline{r}_N\Big) \text{ and } a \equiv \frac{\rho_y}{\beta_y \lambda_\pi}. \tag{10}$$

This affine specification ensures analytical tractability and allows the application of filtering theory to model investor learning.

In equation (10), $\overline{\pi}_t$ denotes the long-term inflation expectation, which evolves based on the Fed's policy stance $(r_{N,t} - \overline{r}_N)$ and directly impacts the inflation dynamics. The parameter λ_π measures the speed at which inflation reverts toward $\overline{\pi}_t$, while $\sigma_\pi > 0$ captures the instantaneous volatility of inflation. The Brownian B_π accounts for exogenous shocks to inflation unrelated to policy. Finally, the parameter a quantifies how strongly the Fed's policy stance $(r_{N,t} - \overline{r}_N)$ influences changes in the long-term inflation expectation.

Separately, the output gap follows a mean-reverting process

$$dy_t = -\lambda_y y_t dt + \sigma_y dB_{y,t}, \tag{11}$$

with $\lambda_y > 0$, $\sigma_y > 0$, and B_y uncorrelated with B_δ and B_π.

2.1 Unobserved Ability and Investor Learning

The novel feature of our framework is that the parameter a, which governs how strongly monetary policy shifts long-term inflation expectations, is unobserved by the agent. One can interpret a as the Fed's *ability* to control inflation: higher a implies that tighter (looser) policy drives inflation more quickly down (up) toward its long-term target. Importantly, we allow a to be time-varying to reflect the evolving nature of the economy and the new challenges faced by monetary policymakers. As economies evolve, the effectiveness of tools like the Taylor rule may change, prompting the agent to question their current efficacy in managing inflation. To capture these dynamics, we model a as an unobserved mean-reverting process:

$$da_t = -\lambda_a a_t dt + \sigma_a dB_{a,t}, \tag{12}$$

where $\lambda_a, \sigma_a > 0$ and B_a is independent of the other Brownian motions.

The representative agent observes realized inflation $\{\pi_s\}_{s \leq t}$ and nominal interest rates $\{r_{N,s}\}_{s \leq t}$. From these observables, the agent filters out $\widehat{a}_t \equiv \mathbb{E}[a_t \mid \mathcal{F}_t^{\pi, r_N}]$. Standard filtering theory (Liptser and Shiryaev, 2001) implies the following stochastic differential equations for the posterior mean $\widehat{a}_t$ and the posterior variance $\nu_{a,t} \equiv \mathbb{E}[(a_t - \widehat{a}_t)^2 \mid \mathcal{F}_t^{\pi, r_N}]$:

$$d\widehat{a}_t = -\lambda_a \widehat{a}_t dt - \frac{(r_{N,t} - \overline{r}_N)\lambda_\pi \nu_{a,t}}{\sigma_\pi} d\widehat{B}_{\pi,t}, \tag{13}$$

$$d\nu_{a,t} = \left[\sigma_a^2 - 2\lambda_a \nu_{a,t} - \left(\frac{(r_{N,t} - \overline{r}_N)\lambda_\pi \nu_{a,t}}{\sigma_\pi}\right)^2\right] dt. \tag{14}$$

Here, $\widehat{B}_{\pi,t}$ is the $\mathcal{F}_t^{\pi, r_N}$-Brownian motion capturing *surprise* changes in inflation relative to the agent's beliefs.

Post-filtering, the agent's perceived inflation dynamics are

$$d\pi_t = \lambda_\pi \left[\overline{\pi} - \widehat{a}_t (r_{N,t} - \overline{r}_N) - \pi_t\right] dt + \sigma_\pi d\widehat{B}_{\pi,t}, \tag{15}$$

where we define the agent's long-term inflation expectation as $\overline{\pi}_t^\ell \equiv \overline{\pi} - \widehat{a}_t(r_{N,t} - \overline{r}_N)$.

Under a restrictive policy ($r_{N,t} > \overline{r}_N$), equation (13) shows that a positive inflation surprise lowers $\widehat{a}_t$, thereby raising long-term inflation expectations and signaling a reduced perception of the Fed's ability to control inflation. Conversely, a negative inflation surprise under this regime increases $\widehat{a}_t$, reflecting an improved perception of the Fed's ability.

This dynamic creates an asymmetric response of $\widehat{a}_t$ to inflation surprises. Under a restrictive policy, a positive surprise indicates a potential loss of Fed control, while under an accommodative policy ($r_{N,t} < \overline{r}_N$), the same surprise suggests the Fed is successfully steering inflation toward a higher target. This asymmetry, with differing implications of inflation surprises based on policy stance, is central to our asset pricing results.

Recent monetary policy episodes illustrate this asymmetry. During the COVID-19 pandemic, the Fed's accommodative stance and resulting positive inflation surprises were interpreted as evidence of effective policy, bolstering its perceived ability to manage inflation. In contrast, during the 2022-23 tightening cycle, positive inflation surprises under a restrictive regime, coupled with stubbornly high inflation, signaled policy ineffectiveness, reducing the Fed's perceived ability. These episodes highlight how identical economic news—a positive inflation surprise—can have markedly different implications for perceived Fed ability, and consequently for asset prices, depending on the prevailing monetary policy stance.

Monetary Policy Stance The *monetary policy stance*, defined as

$$\phi_t \equiv r_{N,t} - \overline{r}_N = \beta_\pi(\pi_t - \overline{\pi}) + \beta_y y_t,$$

captures the Fed's deviation from a neutral policy. The monetary policy stance not only shapes inflation dynamics but also directly drives the agent's learning about the Fed's ability to control inflation, as $d\widehat{a}_t$ depends on ϕ_t through the observed inflation surprises. A positive ϕ_t indicates "restrictive" policy, $\phi_t = 0$ is "neutral," and $\phi_t < 0$ is "accommodative."

2.2 Asset Pricing

Our primary objective is to understand how investor learning about the Fed's ability, combined with the monetary policy stance, shape equilibrium asset prices.

Solving for the equilibrium in this economy involves writing the HJB equation:

$$\max_C \{h(C, J) + \mathcal{L}J\} = 0, \tag{16}$$

with the differential operator $\mathcal{L}J$ following from Itô's lemma. Consistent with existing work (e.g., Benzoni, Collin-Dufresne, and Goldstein, 2011), we guess the following value function:

$$J(C, \pi, \widehat{a}, y, \nu_a) = \frac{C^{1-\gamma}}{1-\gamma}\left[\rho e^{I(x_t)}\right]^{\theta}, \tag{17}$$

where $I(x_t)$ is the log wealth-consumption ratio and $x_t \equiv [\pi_t \ \widehat{a}_t \ \phi_t \ \nu_{a,t}]^\top$ denotes the state vector. Note that the state vector does not include $\mu_{\delta,t}$, which in our model is endogenously determined in equilibrium as a function of the other state variables, as shown below.

We substitute the guess (17) into the HJB equation (16) and impose the market-clearing condition $C_t = \delta_t$. This yields a partial differential equation for the log wealth-consumption ratio, $I(x_t)$, which we solve numerically using Chebyshev polynomials (Judd, 1998). Appendix A provides a detailed description of the solution method and numerical procedure.

State Price Density and Market Prices of Risk Following Duffie and Epstein (1992), the state price density in this economy is given by

$$\xi_t = \exp\left[\int_0^t h_J(C_s, J_s)ds\right] h_C(C_t, J_t), \tag{18}$$

where $h_J(\cdot)$ and $h_C(\cdot)$ are, respectively, the partial derivatives of the aggregator $h(\cdot)$ with respect to J and C. In our setting, this becomes

$$\xi_t = \exp\left[\int_0^t \left(\tfrac{\theta-1}{e^{I(x_s)}} - \rho\theta\right) ds\right] \rho^{\theta} \delta_t^{-\gamma}\left(e^{I(x_t)}\right)^{\theta-1}. \tag{19}$$

The economy is driven by a three-dimensional Brownian vector $\widehat{B}_t = [B_{\delta,t} \ \widehat{B}_{\pi,t} \ B_{y,t}]^\top$. Applying Itô's lemma to (18) and matching terms with

$$\frac{d\xi_t}{\xi_t} = -r_{R,t}dt - m_t^\top d\widehat{B}_t, \tag{20}$$

yields the equilibrium real risk-free rate $r_{R,t}$ and the (three-dimensional) market price of risk

$m_t = [m_{\delta,t}\ m_{\pi,t}\ m_{y,t}]^\top$. The components of m_t are given by:

$$m_{\delta,t} = \gamma\sigma_\delta, \tag{21}$$

$$m_{\pi,t} = (1-\theta)\Big(\sigma_\pi I_\pi - \tfrac{\lambda_\pi \nu_{a,t}}{\sigma_\pi} I_{\widehat{a}}\phi_t\Big), \tag{22}$$

$$m_{y,t} = (1-\theta)\sigma_y I_y, \tag{23}$$

where I_z denotes the partial derivative of the log wealth-consumption ratio $I(x_t)$ with respect to a state variable $z \in \{\pi, \widehat{a}, y\}$, and ϕ_t is the monetary policy stance defined above.

The market price of risk $m_{\delta,t}$ is positive, reflecting the agent's aversion to consumption risk. For $m_{\pi,t}$, we expect $I_\pi < 0$ (as higher inflation reduces expected consumption growth and the wealth-consumption ratio) and $I_{\widehat{a}} > 0$ (as greater trust in the Fed's ability to control inflation raises the wealth-consumption ratio). Under a restrictive policy ($\phi_t > 0$) and preference for early resolution of uncertainty ($1-\theta > 0$), $m_{\pi,t}$ is negative, indicating a willingness to accept lower returns for inflation-hedging assets. Its magnitude increases with stronger preferences for early resolution $(1-\theta)$, higher inflation volatility (σ_π), and greater uncertainty about the Fed's ability $(\nu_{a,t})$. For $m_{y,t}$, $I_y > 0$ implies that the agent requires a positive return premium for exposure to the output gap. Its magnitude also grows with higher output gap volatility (σ_y) and stronger preferences for early resolution $(1-\theta)$.

Real Risk-Free Rate and Fisher Equation We derive the real risk-free rate $r_{R,t}$ as follows. From (19) and (20), applying Itô's lemma implies

$$r_{R,t} = \rho + \frac{\mu_{\delta,t}}{\psi} - \frac{\gamma(1+\psi)}{2\psi}\sigma_\delta^2 - \frac{1-\theta}{2}\Big(\sigma_{W,t}^2 - \sigma_\delta^2\Big), \tag{24}$$

where

$$\sigma_{W,t}^2 \equiv \sigma_\delta^2 + \sigma_y^2 I_y^2 + \Big(\sigma_\pi I_\pi - \tfrac{\lambda_\pi \nu_{a,t}\phi_t}{\sigma_\pi} I_{\widehat{a}}\Big)^2 \tag{25}$$

is the instantaneous variance of wealth. The first two terms in (24) are the standard drivers of the real rate (time preference and expected consumption growth), while the last two capture precautionary-savings effects due to consumption and wealth risk.

The nominal interest rate $r_{N,t}$ follows from the Fisher equation,

$$r_{N,t} = r_{R,t} + \pi_t, \tag{26}$$

so that taking unconditional expectations in (26) yields a closed-form expression for the neutral level of nominal rates,

$$\overline{r}_N = \rho + \overline{\pi} + \frac{\overline{\mu}_\delta}{\psi} - \frac{\gamma(1+\psi)}{2\psi}\sigma_\delta^2 - \frac{1-\theta}{2}\Big(\sigma_\pi^2 \overline{I}_\pi^2 + \sigma_y^2 \overline{I}_y^2\Big), \tag{27}$$

where $\overline{\mu}_\delta$ is the long-run mean of consumption growth, and $\overline{I}_\pi, \overline{I}_y$ are the partial derivatives of $I(x_t)$ evaluated at $\pi_t = \overline{\pi}, \widehat{a}_t = 0, \phi_t = 0$, and $\nu_{a,t} = \overline{\nu}_a$.

Endogenous Consumption Growth and Non-Neutrality In equilibrium, since $C_t = \delta_t$, the drift of consumption must satisfy:

$$\mu_{\delta,t} = \psi\Big(r_{N,t} - \pi_t - \rho\Big) + \frac{\gamma(1+\psi)}{2}\sigma_\delta^2 + \frac{\psi(1-\theta)}{2}\Big(\sigma_{W,t}^2 - \sigma_\delta^2\Big). \tag{28}$$

Combined with the inflation process (15) and the Taylor rule (8), this equation shows that monetary policy influences the real interest rate $r_{R,t}$ and, consequently, $\mu_{\delta,t}$. Thus, monetary policy is non-neutral, as it directly affects consumption growth dynamics.[1]

Further insight can be gained by considering the agent's intertemporal trade-off. Let

[1] One can interpret this channel as a reduced-form representation of real-world frictions allowing changes in the nominal interest rate to influence real allocations. While standard New Keynesian models derive non-neutrality from frictions like price stickiness or nominal rigidities that endogenize inflation, our model begins with an inflation process and requires the expected growth rate to adjust endogenously in equilibrium. This reduced-form approach allows us to focus on the role of investor learning and monetary policy stance without explicitly modeling underlying frictions.

$c_t = \log \delta_t$. By discretizing (3), we obtain

$$\mathbb{E}_t[c_{t+1} - c_t] = \mu_{\delta,t} - \tfrac{\sigma_\delta^2}{2}.$$

Using (28) in this expression and rearranging yields

$$c_t - \mathbb{E}_t[c_{t+1}] = \psi\Big(\rho + \pi_t - r_{N,t}\Big) - \frac{\gamma(1+\psi)-1}{2}\sigma_\delta^2 - \frac{\psi(1-\theta)}{2}\Big(\sigma_{W,t}^2 - \sigma_\delta^2\Big). \tag{29}$$

This relationship, derived from the representative agent's first-order condition for consumption today versus consumption tomorrow, links current versus expected future consumption to the nominal rate, the subjective discount rate, and inflation. The agent consumes more today relative to tomorrow when either the subjective discount rate ρ or the inflation rate π_t is high, and consumes less today relative to tomorrow when the nominal interest rate $r_{N,t}$ is high. It also includes an "excess variance of wealth" adjustment, $\sigma_{W,t}^2 - \sigma_\delta^2$, which acts as a precautionary-savings motive when the agent prefers early resolution of uncertainty.[2]

Equilibrium Stock Market As in Bansal and Yaron (2004), the representative firm in this economy pays a real dividend D_t satisfying

$$\frac{dD_t}{D_t} = \Big[(1-\alpha)\overline{\mu}_\delta + \alpha\mu_{\delta,t}\Big]dt + \sigma_D dB_{D,t}, \tag{30}$$

where $B_{D,t}$ is an independent Brownian motion, α governs the exposure of dividends to consumption growth, and σ_D controls dividend volatility.

Denote the market log price-dividend ratio by $\Pi(x_t)$, which solves a partial differential equation described in Appendix A. By Itô's lemma, the diffusion components of market

[2] The final term in (29) acts as an "exogenous preference shifter" in monetary economies. A change in this term can be interpreted as a discount rate shock (Galí, 2015, Chapter 3). In our model this shock is endogenous and driven by the excess variance of wealth, $\sigma_{W,t}^2 - \sigma_\delta^2$. An increase in the excess variance of wealth results in lower consumption today relative to tomorrow because the representative agent prefers early resolution of uncertainty. Thus, a higher excess variance of wealth boosts precautionary saving and discourages current consumption.

returns are:

$$s_{\delta,t} = 0, \tag{31}$$

$$s_{\pi,t} = \sigma_\pi \Pi_\pi - \tfrac{\lambda_\pi \nu_{a,t}}{\sigma_\pi} \Pi_{\widehat{a}} \phi_t, \tag{32}$$

$$s_{y,t} = \sigma_y \Pi_y, \tag{33}$$

$$s_{D,t} = \sigma_D. \tag{34}$$

The Brownian $B_{D,t}$ has zero price of risk in our calibration (its correlation with other shocks is set to zero), so multiplying the prices of risk (21)–(23) by the corresponding diffusions (31)–(33) gives the equilibrium market risk premium:

$$\begin{aligned} RP_t = {} & (1-\theta)\Big(\sigma_y^2 \Pi_y I_y + \sigma_\pi^2 \Pi_\pi I_\pi\Big) \\ & - (1-\theta)\nu_{a,t}\lambda_\pi \phi_t \Big(\Pi_\pi I_{\widehat{a}} + \Pi_{\widehat{a}} I_\pi\Big) + (1-\theta)\tfrac{\lambda_\pi^2 \nu_{a,t}^2}{\sigma_\pi^2} \Pi_{\widehat{a}} I_{\widehat{a}} \phi_t^2. \end{aligned} \tag{35}$$

We expect $\Pi_\pi < 0$, $\Pi_{\widehat{a}} > 0$, and $\Pi_y > 0$, mirroring the same signs in the wealth-consumption ratio $I(\cdot)$.

Two primary forces influence the risk premium. First, for a representative agent with a preference for early resolution of uncertainty $(1-\theta > 0)$, the term $(1-\theta)\left(\sigma_y^2 \Pi_y I_y + \sigma_\pi^2 \Pi_\pi I_\pi\right)$ in (35) is positive. Its magnitude declines with the agent's perception of the Fed's ability to control inflation, $\widehat{a}_t$. When $\widehat{a}_t$ is large, the Fed's strong ability to control inflation ensures faster mean reversion to lower levels, reducing inflation persistence. This decreases long-run inflation risk and lowers the risk premium.

The uncertainty channel $\nu_{a,t}$ is the second force influencing the market risk premium. It appears in the second-row terms of equation (35), forming a quadratic expression in the monetary policy stance ϕ_t. Since $\Pi_{\widehat{a}} I_{\widehat{a}} > 0$, the quadratic term generates a U-shape for agents who prefer early resolution of uncertainty $(1-\theta > 0)$, meaning uncertainty about the Fed's ability to control inflation increases the risk premium when the Fed deviates from a neutral policy stance. The linear term in ϕ_t introduces asymmetry: because $\Pi_\pi I_{\widehat{a}} + \Pi_{\widehat{a}} I_\pi < 0$, the risk premium is higher under a restrictive policy stance than under an accommodative one. This asymmetry stems from equation (13), which shows how learning amplifies the effects of inflation surprises when policy is restrictive but dampens them when policy is accommodative. These effects are amplified by higher uncertainty, more extreme policy stances, and stronger preferences for early resolution of uncertainty.

Finally, the market return variance is

$$\sigma_t^2 = \sigma_D^2 + \sigma_y^2 \Pi_y^2 + \sigma_\pi^2 \Pi_\pi^2 - 2\lambda_\pi \nu_{a,t} \Pi_{\widehat{a}} \Pi_\pi \phi_t + \tfrac{\lambda_\pi^2 \nu_{a,t}^2}{\sigma_\pi^2} \Pi_{\widehat{a}}^2 \phi_t^2. \tag{36}$$

Similarly to the risk premium, uncertainty about the Fed's ability increases market return variance whenever the Fed deviates from a neutral policy, with a larger effect under restrictive monetary stances due to the asymmetric learning response in (13).

Overall, two forces shape the market's response to monetary policy, reflecting the paper's main contributions. First, the Fed's perceived ability to control inflation, $\widehat{a}_t$, reduces long-run inflation risk, leading to lower risk premia and dampened return variance. This reflects expectations that monetary policy will effectively return inflation to its target, with a stronger Fed reducing both the persistence of inflation shocks and their impact on asset prices. Second, uncertainty about the Fed's ability to control inflation, captured by $\nu_{a,t}$, amplifies both the risk premium and return variance when the Fed deviates from a neutral policy stance. This effect is particularly pronounced under a restrictive policy, as concerns grow that the Fed may struggle to rein in inflation.

3 Parameter Estimation and Model Fit

We estimate our model's parameters using maximum likelihood and monthly U.S. data on real Gross Domestic Product (GDP), the Federal funds rate, the Consumer Price Index (CPI), and the output gap, spanning July 1954 to December 2023.[3] Real GDP is sourced from the NIPA tables, while the Federal funds rate, CPI, and output gap are from the Federal Reserve Bank of St. Louis (FRED).

Figure 1 shows the time series of annualized real GDP growth, the output gap, CPI inflation, and the Federal funds rate. In the mid-1970s and early 1980s, the Federal funds rate rose above 10% to curb high inflation, as seen in the third panel. These high rates contributed to the economic downturns evident in the first two panels.

Table 1 reports the maximum likelihood estimates. We use the log real GDP growth rate, log CPI growth rate, continuously compounded output gap, and continuously compounded Federal funds rate as proxies for $\log(\delta_{t+\Delta}/\delta_t)$, π_t, y_t, and $r_{N,t}$, respectively. The estimates for the nominal interest rate's sensitivities to inflation (β_π) and the output gap (β_y) confirm that rates respond more strongly to inflation than to output (Clarida, Gali, and Gertler, 2000; Ang, Boivin, Dong, and Loo-Kung, 2011). Inflation and the output gap exhibit low mean-reversion speeds, indicating persistence, while the Fed's *ability to control inflation*, a_t, reverts more quickly to its mean. The sample's average inflation is 3.38%, compared to a mean nominal interest rate of 4.44%, implying a mean real rate of about 1%. Historically, average inflation has exceeded the Fed's current 2% target.

Investor Learning About the Fed's Ability Figure 2 shows the agent's estimate of the Fed's ability to control inflation, $\widehat{a}_t$ (solid line, left axis), alongside the historical inflation rate (dashed line, right axis). Shaded vertical bands denote the tenures of different Fed Chairs. The estimate $\widehat{a}_t$ fluctuates significantly over the sample, reflecting changes in investor beliefs about the Fed's ability. When inflation is above its historical mean, changes in inflation and $\widehat{a}_t$ are negatively correlated (-0.52). In contrast, the correlation is positive (0.61) when inflation is below its mean. This pattern aligns with the paper's learning mechanism: during high-inflation periods, rising inflation suggests the Fed may be losing control, while in deflationary periods, falling inflation signals the same.

[3] The maximum likelihood procedure is detailed in Appendix B.

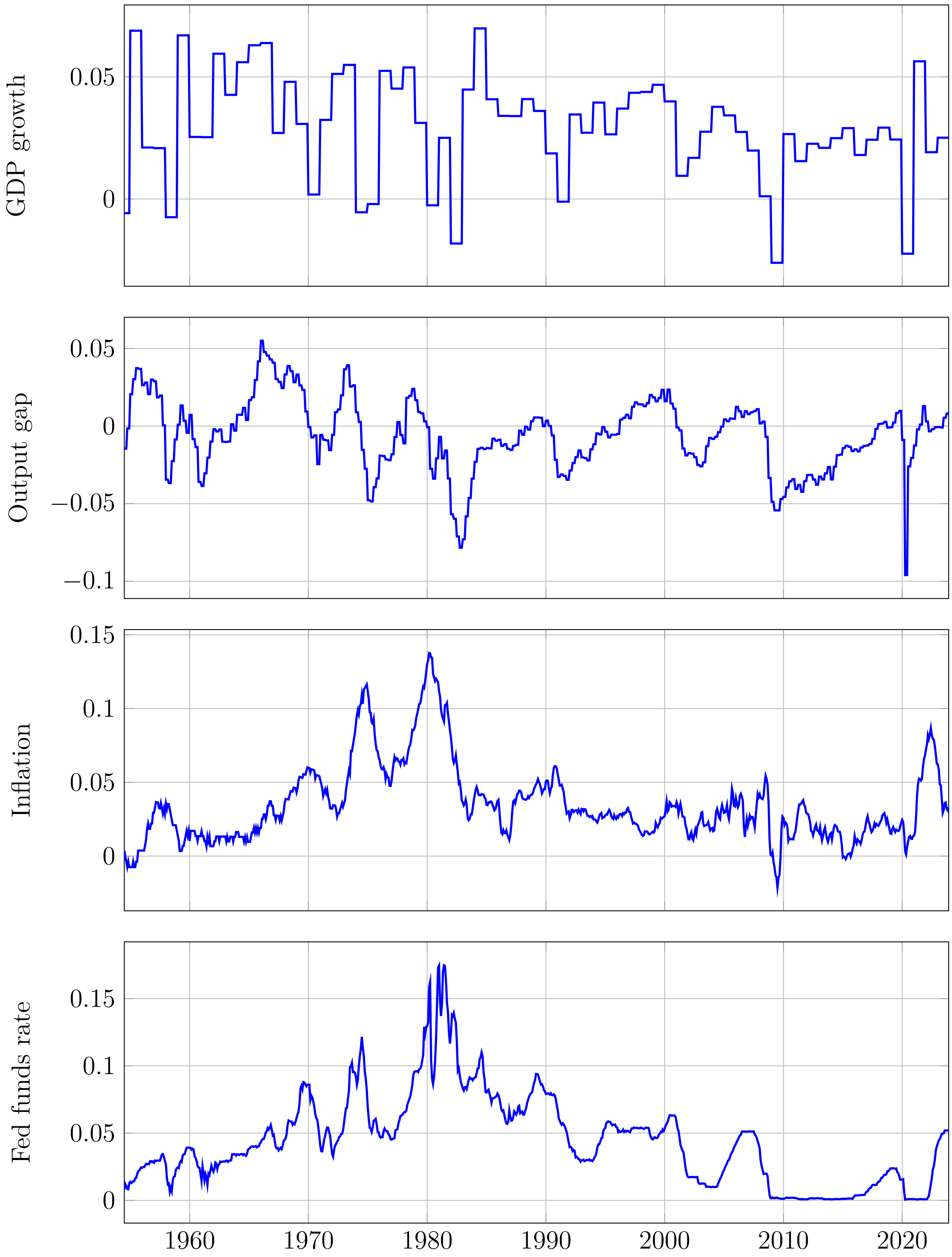

Figure 1: GDP Growth, Output Gap, Inflation, and Federal Funds Rate. This figure plots the observed annualized U.S. real GDP growth rate (first panel), output gap (second panel), CPI inflation rate (third panel), and Federal funds rate (fourth panel).

Output growth volatility	σ_δ	0.0212*** (0.0005)
Output gap volatility	σ_y	0.0215*** (0.0007)
Output gap mean-reversion speed	λ_y	0.4219*** (0.0507)
Mean nominal interest rate	$\overline{r}_N$	0.0444*** (0.0011)
Interest rate sensitivity to inflation	β_π	0.4666*** (0.0277)
Interest rate sensitivity to output gap	β_y	0.1332*** (0.0216)
Inflation volatility	σ_π	0.0131*** (0.0011)
Mean inflation	$\overline{\pi}$	0.0338*** (0.0032)
Inflation mean-reversion speed	λ_π	0.2436*** (0.0262)
Volatility of the Fed's ability to control inflation	σ_a	0.7434*** (0.1709)
Mean-reversion speed of the Fed's ability to control inflation	λ_a	0.9365*** (0.2000)

Table 1: Parameter Values Estimated by Maximum Likelihood.
This table reports the parameter values estimated by maximum likelihood. The estimation procedure is detailed in Appendix B. Output data is in real terms. Standard errors are reported in brackets, and statistical significance at the 10%, 5%, and 1% levels is denoted by *, **, and ***, respectively. The data is at the monthly frequency from July 1954 to December 2023.

The time series of $\widehat{a}_t$ tracks key policy episodes. For example, $\widehat{a}_t$ fell sharply during the inflationary peaks of the mid-1970s and early 1980s, reflecting investors' belief that the Fed was losing control of inflation. In contrast, $\widehat{a}_t$ was positive during Paul Volcker's tenure (1979–87), consistent with his success in reducing inflation, and during Ben Bernanke's tenure (2006–14) amid the global financial crisis. During the COVID-19 pandemic, the Fed's accommodative policies and positive inflation surprises boosted $\widehat{a}_t$. However, during the 2022-23 tightening cycle, the Fed's battle against stubbornly high inflation, evidenced by continuing positive inflation surprises even under a restrictive regime, reduced $\widehat{a}_t$. This asymmetry reflects the learning process in (13), where $\widehat{a}_t$ adjusts based on the sign of ϕ_t.

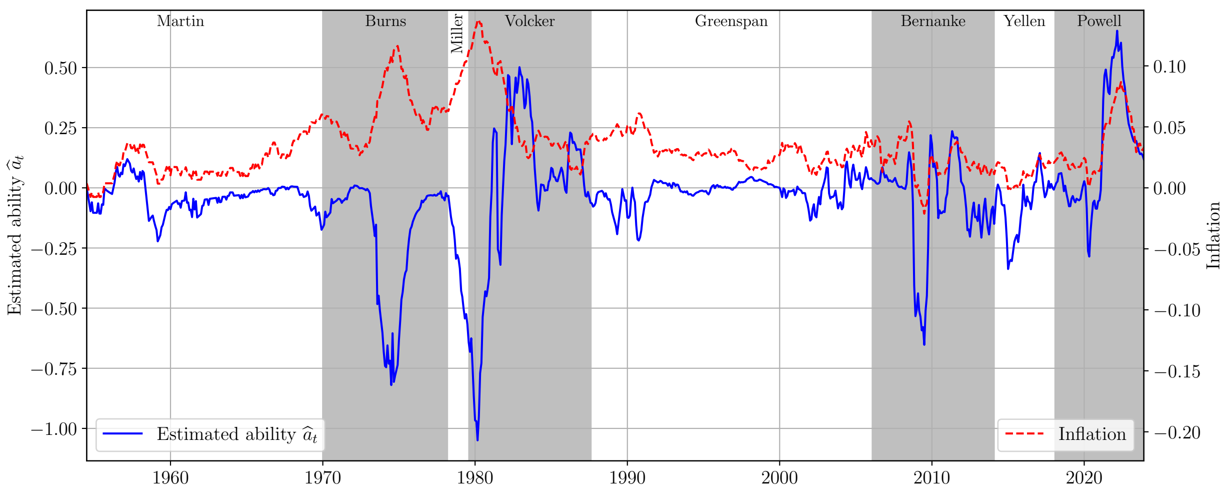

Figure 2: Agent's Estimate of the Fed's Ability to Control Inflation.
The figure juxtaposes the agent's estimate of the Fed's ability to control inflation, $\widehat{a}_t$ (solid line, left axis), with the historical inflation rate (dashed line, right axis). The alternating shaded bands mark the tenures of different Fed Chairmans. The time series of $\widehat{a}_t$ is extracted from the maximum likelihood estimation. The data is at the monthly frequency from July 1954 to December 2023.

Monetary Policy Stance and Inflation Expectations Figure 3 displays the Fed's monetary policy stance, $\phi_t = r_{N,t} - \overline{r}_N$ (top panel), and the agent's long-term inflation expectation, $\overline{\pi}_t^\ell \equiv \overline{\pi} - \widehat{a}_t\phi_t$ (bottom panel). A positive ϕ_t indicates restrictive policy, while a negative ϕ_t reflects an accommodative stance. Over the sample, the Fed maintained restrictive policy from late 1965 to early 1992 and again near the end of the sample, while policy was mostly accommodative from mid-1992 to late 2021. The monetary policy stance is highly persistent, with a volatility of 1.3% and an autocorrelation of 0.987.

The agent's long-term inflation expectation, influenced by updates to $\widehat{a}_t$, also shows persistent fluctuations (volatility of 1.25%, autocorrelation of 0.93) and correlates positively with survey-based measures of inflation expectations.[4] Notable lows in $\overline{\pi}_t^\ell$ occurred in early 1982 and during the Great Recession (2008–2009). Highs were observed during the 1973 oil crisis, the late 1970s, and mid-2021, reflecting periods of significant inflationary pressure.

[4]The agent's long-term inflation expectation, $\overline{\pi}_t^\ell$, has a 0.42 correlation with the median 5-year inflation forecast obtained from the Federal Reserve Bank of Philadelphia's Survey of Professional Forecasters (SPF). The SPF 5-year inflation forecast is available since 2005Q3. Regressing the SPF 5-year inflation forecast on $\overline{\pi}_t^\ell$ yields a coefficient of 0.16, significant at the 1% level (t-stat = 3.96).

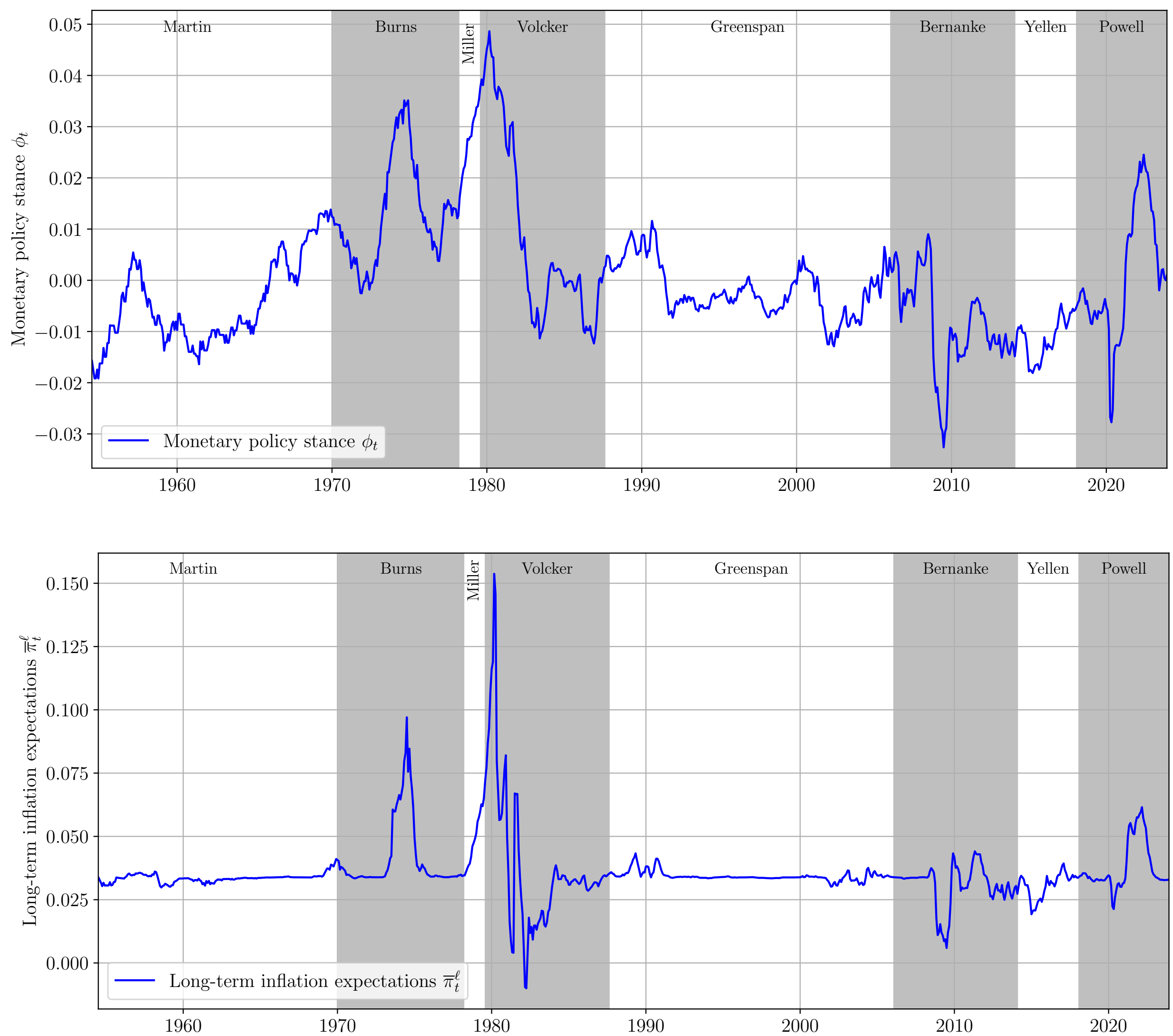

Figure 3: Monetary Policy Stance and Inflation Expectations.
This figure plots the Fed monetary policy stance $\phi_t = r_{N,t} - \overline{r}_N$ (top panel) and the agent's long-term inflation expectation (bottom panel), denoted as $\overline{\pi}_t^\ell \equiv \overline{\pi} - \widehat{a}_t(r_{N,t} - \overline{r}_N)$. These time series are extracted from the maximum likelihood estimation. The data is at the monthly frequency from July 1954 to December 2023.

Asset Pricing Calibration and Moments Following Bansal and Yaron (2004), we set the agent's relative risk aversion to $\gamma = 10$, the elasticity of intertemporal substitution to $\psi = 2$, the subjective discount rate to $\rho = 0.0095$, and the dividend exposure to output growth to $\alpha = 2.5$. These are standard in the long-run risk literature. Dividend growth volatility is set to $\sigma_D = 0.0575$, matching historical S&P 500 dividend volatility.

Table 2 compares asset-pricing moments from the data (first column) with those implied

Moment	**Data**	**Model**
Real interest rate	0.0095	0.0106
Nominal interest rate	0.0444	0.0444
Market risk premium	0.0596	0.0569
Market return volatility	0.1490	0.1178

Table 2: Asset-pricing Moments.
This table presents asset-pricing moments, with the first column displaying empirical moments and the second column showing model-implied counterparts. Empirical moments are obtained using the Fed funds rate as the nominal interest rate, the difference between the Fed funds rate and the CPI inflation rate as the real interest rate, and the S&P 500 as the market. Model-implied moments are obtained by inputting the state variable time series from the maximum likelihood estimation into the model. The data is at the monthly frequency from July 1954 to December 2023.

by the model (second column). Empirical moments are based on the Fed funds rate as the nominal interest rate, the difference between the Fed funds rate and CPI inflation as the real rate, and the S&P 500 as the market. Model-implied moments are derived by feeding the state variable time series from the maximum likelihood estimation into the model.

The model produces a real interest rate of about 1% and a nominal rate of 4.4%, both aligning with the data. The model-implied risk premium is 5.7%, close to the empirical value of 6.0%, while market return volatility is 11.8%, slightly below the observed value of 14.9%. These findings indicate that our framework, despite being estimated solely with macro and policy data, captures realistic asset-pricing moments, including plausible real and nominal interest rates, a market risk premium, and return volatility.

4 Asset Pricing Implications and Empirical Evidence

This section examines the model's asset pricing implications. We present its predictions and provide supporting empirical evidence. The results are based on numerical solutions using parameters estimated in Section 3. Details on the numerical solution are in Appendix A.

4.1 Model Predictions for Asset Prices

Figure 4 plots the expected consumption growth $\mu_{\delta,t}$, the real interest rate $r_{R,t}$, and the log price-dividend ratio Π_t as functions of inflation (π_t) and the output gap (y_t). As we will show in Table 3, these two variables (π_t and y_t) are the primary drivers of $\mu_{\delta,t}$, $r_{R,t}$, and Π_t, explaining over 99.9% of their variation in the model.

The figure shows that both expected consumption growth ($\mu_{\delta,t}$) and the real interest rate ($r_{R,t}$) decline as inflation rises. According to equation (28), higher inflation leads to lower expected consumption growth because it encourages the agent to consume more today. Furthermore, when the nominal interest rate's sensitivity to inflation (β_π) is less than one, the Fisher equation (26) and the Taylor rule (8) imply that higher inflation lowers the real interest rate. Finally, a rise in the output gap (y_t) increases the nominal interest rate via the Taylor rule, which boosts expected consumption growth as shown in equation (28), and raises the real interest rate through the Fisher equation. This outcome reflects the agent's reduced demand for risk-free savings when anticipating higher future consumption, resulting in a higher real interest rate.

The bottom left panel of Figure 4 shows that the price-dividend ratio decreases with inflation. This occurs because, as per equation (28), higher inflation reduces expected growth. This mechanism introduces long-run risk into the economy: if inflation is persistent, an agent who prefers early resolution of uncertainty dislikes its fluctuations. The bottom right panel of Figure 4 shows that the price-dividend ratio increases with the output gap, as expected growth rises with the output gap, consistent with equation (28).

Figure 5 plots the market risk premium and market return volatility as functions of their primary drivers: the Fed's ability to control inflation ($\widehat{a}_t$) and the Fed's monetary policy stance (ϕ_t). As will be shown in Table 4, these two factors explain over 99.7% of the variation in both measures.

The top left panel of the figure shows that the risk premium declines as the Fed's ability to control inflation ($\widehat{a}_t$) increases. As discussed in relation to equation (35), inflation is a

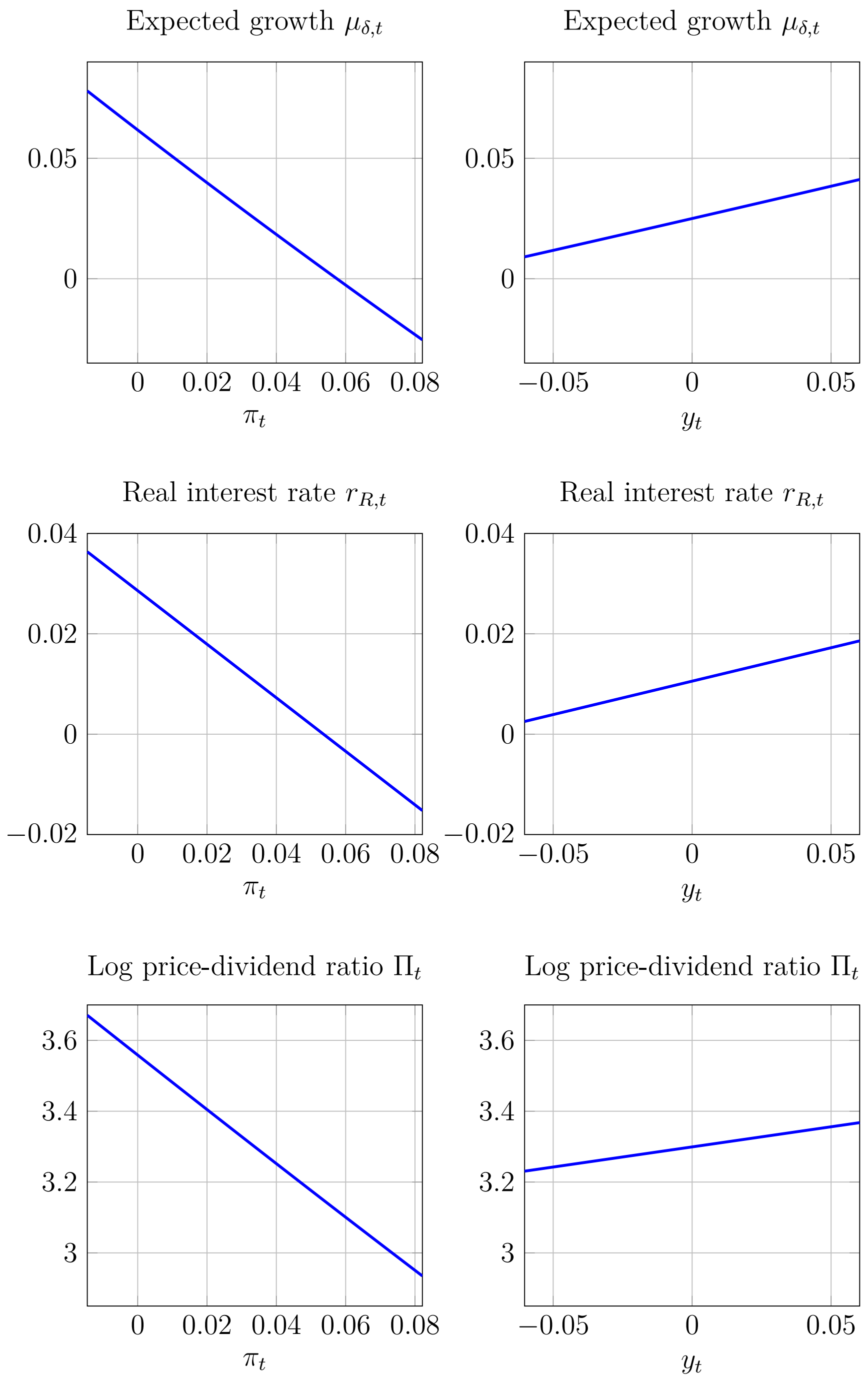

Figure 4: Model Predictions for Expected Consumption Growth, Real Interest Rate, and Market Price-Dividend Ratio.
The figure displays the model-implied expected consumption growth ($\mu_{\delta,t}$), real interest rate ($r_{R,t}$), and log price-dividend ratio (Π_t) as functions of inflation (π_t, left panels) and the output gap (y_t, right panels). The model is solved numerically using parameters estimated in Section 3; see Appendix A for details.

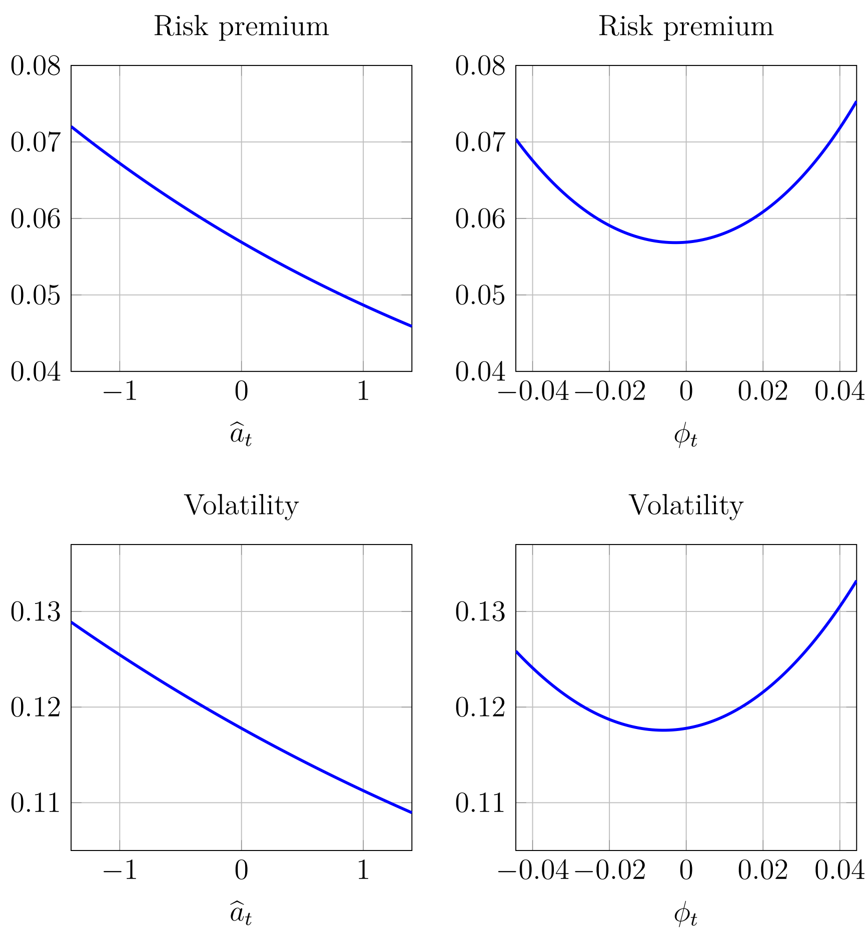

Figure 5: Asset Pricing Implications: Model Predictions for the Market Risk Premium and Return Volatility.
The figure plots the model's market risk premium and return volatility against the Fed's perceived ability to control inflation ($\widehat{a}_t$, left) and the monetary policy stance (ϕ_t, right). See Section 3 for parameters and Appendix A for the numerical method.

source of long-run risk. Therefore, a stronger ability to control it is valuable, reducing the risk premium.

Figure 5's top right panel shows that the risk premium exhibits a U-shaped relationship with the monetary policy stance (ϕ_t). This pattern arises from uncertainty about the Fed's ability to control inflation. In equation (35), the second row is quadratic in ϕ_t, which implies that deviations from neutral policy increase the risk premium. Equation (35) also reveals an asymmetry: the risk premium is higher under restrictive policy, though this effect is modest given our estimates. This asymmetry occurs because, with very restrictive policy, higher inflation is doubly harmful (raising prices and lowering perceived Fed ability), while lower inflation is doubly beneficial. This amplifies market sensitivity to inflation shocks, increasing the risk premium.

The bottom panels of Figure 5 illustrate how market return volatility varies with the Fed's ability to control inflation and the monetary policy stance. The patterns are similar to those for the risk premium: volatility decreases as the Fed's ability improves and follows a U-shaped relationship with the monetary policy stance. The underlying reasons for these patterns mirror those discussed for the risk premium.

4.2 Empirical Evidence

We now assess the empirical validity of the model's predictions. We start by regressing both empirical and model-implied quantities—including the expected output growth rate, real interest rate, market price-dividend ratio, market risk premium, and market return volatility—on the state variables. This analysis tests whether the data supports the relationships shown in Figures 4 and 5.

We obtain *empirical* measures of expected output growth, the real interest rate, the market risk premium, and market return volatility as follows. Expected output growth is estimated using an ARMA(1,1) model applied to realized GDP growth. The real interest rate is simply the difference between the Fed funds rate and CPI inflation. To estimate the market risk premium, we regress the one-year-ahead S&P 500 excess return on the current log dividend yield and realized variance, following Fama and French (1989); Cochrane (2008) and French, Schwert, and Stambaugh (1987); Guo (2006). Finally, we model market return volatility using an Exponential GARCH(1,1,1) model (Nelson, 1991) applied to the S&P 500 excess return residual (i.e., the excess return minus the estimated risk premium).[5]

Model-implied measures for the expected output growth rate, real interest rate, market

[5] S&P 500 returns, dividend yields, and realized variance data are from Amit Goyal's website. In the ARMA(1,1) model for expected output growth, the AR(1) coefficient is positive and significant at 1%, the MA(1) is insignificant, and results hold with AR(1) or ARMA(2,2). For the risk premium, log dividend yield and realized variance load positively (significant at 1% and 5%). The Exponential GARCH(1,1,1) captures asymmetric volatility. The ARCH(1) and GARCH(1) terms are positive and statistically significant at the 1% level, while the LEVERAGE(1) term is negative and significant at the 1% level.

	Expected output growth		Real interest rate		Log price-dividend ratio	
	Model	Data	Model	Data	Model	Data
π_t	-1.0395^{***}	-0.0935^{***}	-0.5334^{***}	-0.1035^{***}	-7.7655^{***}	-6.4404^{***}
	(0.0032)	(0.0215)	(0.0000)	(0.0379)	(0.0188)	(0.4121)
y_t	0.2618^{***}	0.4706^{***}	0.1332^{***}	0.1619^{***}	1.0987^{***}	1.7354^{***}
	(0.0022)	(0.0255)	(0.0000)	(0.0402)	(0.0083)	(0.5385)
R^2	0.999	0.331	1.000	0.038	0.999	0.196
Obs.	834	834	834	834	834	834

Table 3: Expected Output Growth, Real Interest Rate, and Log Price-Dividend Ratio: Model vs. Data.
The table presents relationships between expected output growth, the real interest rate, the log price-dividend ratio, and inflation (π_t) and the output gap (y_t), for both the model and the data. Standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels. Monthly data: July 1954 – December 2023.

price-dividend ratio, market risk premium, and market return volatility are obtained by using the time series of state variables estimated via maximum likelihood in Section 3 as inputs to the model.

We use regression analysis to test the relationships shown in Figure 4. Table 3 reports the results, which confirm the figure's patterns. Both the model and the data show that the expected output growth rate ($\mu_{\delta,t}$), real interest rate ($r_{R,t}$), and market log price-dividend ratio (Π_t) decline with inflation and rise with the output gap. These effects are statistically significant and, in the model, explain nearly all the variation in these quantities.

These relationships arise because higher inflation or a higher output gap leads to more restrictive monetary policy, raising the nominal interest rate through the Taylor rule (8). Combined with the Fisher equation (26), this implies that the real interest rate falls with inflation and rises with the output gap. Equation (24) shows that expected output growth depends linearly on the real interest rate, explaining its similar behavior. Finally, higher expected growth increases the price-dividend ratio, reflecting higher valuations.

Table 4 presents the empirical and model-implied relationships between the market risk premium (Panel A), market return volatility (Panel B), and their primary drivers: the Fed's ability to control inflation ($\widehat{a}_t$), the monetary policy stance (ϕ_t), and its square (ϕ_t^2). The

Panel A: Market risk premium vs. state variables

	Risk premium		Risk premium		Risk premium		Risk premium		Risk premium	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
$\widehat{a}_t$	-0.0194^{***}	-0.0355^{***}					-0.0096^{***}	-0.0153^{**}	-0.0097^{***}	-0.0200^{***}
	(0.0010)	(0.0041)					(0.0001)	(0.0067)	(0.0001)	(0.0070)
ϕ_t			0.1885^{***}	0.5968^{***}					0.0052^{***}	0.3792^{***}
			(0.0245)	(0.0586)					(0.0010)	(0.0788)
ϕ_t^2					13.8913^{***}	26.5550^{***}	9.5004^{***}	19.5721^{***}	9.3448^{***}	8.2385^{*}
					(0.2309)	(1.8314)	(0.0927)	(3.4931)	(0.1013)	(4.2598)
R^2	0.776	0.078	0.276	0.084	0.897	0.099	0.998	0.107	0.998	0.128
Obs.	834	834	834	834	834	834	834	834	834	834

Panel B: Market return volatility vs. state variables

	Volatility		Volatility		Volatility		Volatility		Volatility	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
$\widehat{a}_t$	-0.0143^{***}	-0.0374^{***}					-0.0074^{***}	-0.0167	-0.0075^{***}	-0.0231^{**}
	(0.0007)	(0.0090)					(0.0001)	(0.0119)	(0.0001)	(0.0118)
ϕ_t			0.1389^{***}	0.7010^{***}					0.0083^{***}	0.5228^{***}
			(0.0175)	(0.1347)					(0.0009)	(0.1846)
ϕ_t^2					10.0796^{***}	27.6997^{***}	6.7222^{***}	20.1037^{***}	6.4752^{***}	4.4785
					(0.1579)	(5.8885)	(0.0407)	(7.7414)	(0.0518)	(9.7348)
R^2	0.788	0.031	0.281	0.041	0.886	0.039	0.996	0.042	0.997	0.056
Obs.	834	834	834	834	834	834	834	834	834	834

Table 4: Asset Pricing Implications: Market Risk Premium and Return Volatility vs. the Fed's Ability to Control Inflation and the Monetary Policy Stance. The table presents relationships between the market risk premium (Panel A), return volatility (Panel B), and the Fed's perceived ability to control inflation ($\widehat{a}_t$), the monetary policy stance (ϕ_t), and its square (ϕ_t^2), for both the model and the data. Standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels. Monthly data: July 1954 – December 2023.

"Model" columns confirm the results in Figure 5, showing these factors explain over 99% of the variation in both quantities. In both the model and the data, an increase in the Fed's ability to control inflation significantly reduces the market risk premium and return volatility. As the Fed's inflation control improves, the likelihood of extreme future inflation during restrictive cycles (or very low inflation during accommodative cycles) diminishes—see equation (15). This reduces inflation persistence and the associated long-run risk, leading to a lower market risk premium and decreased market return volatility in equilibrium.

Both the model and the data show that the market risk premium and return volatility increase significantly with the monetary policy stance (ϕ_t). As discussed earlier, inflation surprises are amplified during restrictive cycles due to investor learning. A positive inflation surprise under restrictive policy is doubly bad news, as it reduces confidence in the Fed's ability. In contrast, the same surprise under accommodative policy can be partially good news, signaling that the Fed is successfully moving inflation towards a higher target. This asymmetry contributes to the higher risk premium and volatility observed when monetary policy is restrictive.

Finally, Table 4 confirms that the market risk premium and return volatility increase with the square of the monetary policy stance (ϕ_t^2). This quadratic relationship, predicted by the model and highlighted in the last terms of equations (35) and (36), arises from uncertainty about the Fed's ability to control inflation. While the data supports this positive relationship in some specifications, its statistical significance weakens when the linear term (ϕ_t) is included in the regression.

Predictive Regressions Having established the model's ability to capture unconditional relationships among key macroeconomic and financial variables, we now examine its asset pricing implications for return predictability. We use predictive regressions to test the forecasting power of the market log price-dividend ratio, real interest rate, and expected output growth, controlling for inflation and the Fed's perceived ability.

Table 5 presents predictive regressions of future market excess returns on the market log price-dividend ratio (Panel A), real interest rate (Panel B), and expected output growth rate (Panel C), controlling for inflation (π_t) and the Fed's ability to manage it ($\widehat{a}_t$). In both the model and the data, Panel A shows that the price-dividend ratio negatively predicts future returns, consistent with Boudoukh, Michaely, Richardson, and Roberts (2007); Cochrane (2008); van Binsbergen and Koijen (2010). Inflation and the Fed's ability to control inflation load negatively on future market returns. Empirically (in the data), these variables explain about 10% of the variation in one- and two-year returns and 7% in ten-year returns.

Panels B and C show that, in both the model and the data, the real interest rate and expected output growth rate negatively and significantly predict future market returns (controlling for inflation and the Fed's ability). In the data, the R^2 values for the real interest

Panel A: Future market return vs. market price-dividend ratio

	1-year future return		**1-year future return**		**2-year future return**		**2-year future return**		**10-year future return**		**10-year future return**	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
pd_t	-0.6347^{***}	-0.1037^{***}	-0.6132^{***}	-0.1008^{***}	-0.3848^{***}	-0.0722^{***}	-0.3521^{***}	-0.0708^{***}	-0.3154^{***}	-0.0304^{***}	-0.2987^{***}	-0.0292^{***}
	(0.1335)	(0.0142)	(0.1326)	(0.0151)	(0.0676)	(0.0101)	(0.0641)	(0.0102)	(0.0436)	(0.0035)	(0.0445)	(0.0036)
π_t	-5.1131^{***}	-1.9102^{***}	-5.0062^{***}	-1.9556^{***}	-2.9895^{***}	-1.2497^{***}	-2.8266^{***}	-1.2726^{***}	-2.4605^{***}	-0.3364^{***}	-2.3771^{***}	-0.3555^{***}
	(1.0459)	(0.2306)	(1.0333)	(0.2230)	(0.5493)	(0.1240)	(0.5224)	(0.1247)	(0.3533)	(0.0463)	(0.3555)	(0.0483)
$\widehat{a}_t$			-0.0250	-0.0267			-0.0381^{***}	-0.0134			-0.0195^{***}	-0.0112^{**}
			(0.0200)	(0.0307)			(0.0106)	(0.0146)			(0.0049)	(0.0057)
R^2	0.023	0.101	0.025	0.102	0.018	0.100	0.028	0.100	0.066	0.067	0.080	0.069
Obs.	834	834	834	834	834	834	834	834	834	834	834	834

Panel B: Future market return vs. real interest rate

	1-year future return		**1-year future return**		**2-year future return**		**2-year future return**		**10-year future return**		**10-year future return**	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
$r_{R,t}$	-6.2834^{***}	-0.5041^{**}	-6.3009^{***}	-0.5100^{**}	-3.4601^{***}	-0.4315^{***}	-3.4819^{***}	-0.4351^{***}	-2.5674^{***}	-0.1631^{**}	-2.5795^{***}	-0.1651^{**}
	(1.0798)	(0.2365)	(1.0629)	(0.2346)	(0.5675)	(0.1310)	(0.5479)	(0.1335)	(0.3661)	(0.0669)	(0.3770)	(0.0663)
π_t	-3.5385^{***}	-1.2925^{***}	-3.6326^{***}	-1.4474^{***}	-1.8474^{***}	-0.8283^{***}	-1.9650^{***}	-0.9245^{***}	-1.3804^{***}	-0.1567^{***}	-1.4457^{***}	-0.2101^{***}
	(0.5996)	(0.2174)	(0.5848)	(0.2134)	(0.3266)	(0.1099)	(0.3231)	(0.1103)	(0.2090)	(0.0469)	(0.2137)	(0.0530)
$\widehat{a}_t$			-0.0352^{*}	-0.0641^{**}			-0.0440^{***}	-0.0398^{***}			-0.0244^{***}	-0.0221^{***}
			(0.0193)	(0.0305)			(0.0105)	(0.0142)			(0.0049)	(0.0072)
R^2	0.030	0.050	0.034	0.057	0.020	0.048	0.033	0.054	0.061	0.014	0.083	0.024
Obs.	834	834	834	834	834	834	834	834	834	834	834	834

Panel C: Future market return vs. expected output growth

	1-year future return		**1-year future return**		**2-year future return**		**2-year future return**		**10-year future return**		**10-year future return**	
	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data
$\mu_{\delta,t}$	-3.0944^{***}	-2.0433^{***}	-3.3709^{***}	-1.9921^{***}	-1.5367^{***}	-1.1317^{***}	-1.8183^{***}	-1.0973^{***}	-1.1161^{***}	-0.8106^{***}	-1.2796^{***}	-0.7952^{***}
	(0.5440)	(0.2756)	(0.5335)	(0.2879)	(0.2823)	(0.1685)	(0.2780)	(0.1686)	(0.1730)	(0.0725)	(0.1893)	(0.0730)
π_t	-3.4031^{***}	-1.4463^{***}	-3.8173^{***}	-1.5252^{***}	-1.5983^{***}	-0.8974^{***}	-2.0200^{***}	-0.9505^{***}	-1.1704^{***}	-0.2217^{***}	-1.4152^{***}	-0.2454^{***}
	(0.5957)	(0.2053)	(0.5836)	(0.1974)	(0.3152)	(0.1061)	(0.3238)	(0.1018)	(0.1926)	(0.0438)	(0.2114)	(0.0453)
$\widehat{a}_t$			-0.0522^{***}	-0.0349			-0.0531^{***}	-0.0235^{*}			-0.0308^{***}	-0.0105^{*}
			(0.0189)	(0.0276)			(0.0106)	(0.0122)			(0.0050)	(0.0055)
R^2	0.029	0.108	0.037	0.110	0.016	0.082	0.034	0.084	0.046	0.130	0.080	0.132
Obs.	834	834	834	834	834	834	834	834	834	834	834	834

Table 5: Asset Pricing Implications for Return Predictability: Using the Price-Dividend Ratio, Real Interest Rate, and Expected Output Growth to Forecast Market Returns.
The table compares model and data predictions for future market excess returns based on the log price-dividend ratio (pd_t, Panel A), real interest rate ($r_{R,t}$, Panel B), and expected output growth ($\mu_{\delta,t}$, Panel C), controlling for inflation (π_t) and the Fed's perceived ability to control it ($\widehat{a}_t$). Standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels. Monthly data: July 1954 – December 2023.

rate are roughly 5% for one- and two-year horizons and 2% for ten years; for expected output growth, they are about 11%, 8%, and 13% for one-, two-, and ten-year horizons, respectively.[6]

Overall, this section shows that the model's predictions are supported by the data. Expected output growth, the real interest rate, and the market price-dividend ratio decline with inflation and rise with the output gap. The market risk premium and return volatility decrease with the Fed's perceived ability to control inflation and increase quadratically with more restrictive monetary policy, consistent with the model's predictions. Finally, predictive regressions of future market returns on the price-dividend ratio, real interest rate, and expected output growth yield consistent results in both the model and the data, underscoring the model's relevance for understanding asset pricing dynamics.

5 Conclusion

This paper develops an asset pricing model to explore the impact of investor learning about the Fed's ability to control inflation on financial markets. The model demonstrates that the Fed's perceived inflation-fighting ability is a crucial determinant of equilibrium risk premia and return volatility. When investors perceive the Fed as capable, risk premia and volatility decrease, reflecting lower long-run inflation risk. Conversely, doubts about the Fed's ability amplify these measures, particularly during restrictive monetary policy periods. Amplification occurs because inflation surprises are perceived asymmetrically: positive surprises under restrictive policy erode confidence in the Fed, exacerbating market risk. Empirical tests using U.S. data from 1954 to 2023 support these predictions, highlighting the role of investor learning about the Fed in shaping asset price dynamics.

The findings underscore the importance of central bank credibility for asset markets.

[6]The relationship among stock returns, interest rates, and inflation has been widely debated in the literature. Fama and Schwert (1977) and Campbell (1987) document a negative relationship between market returns and nominal interest rates, while Campbell and Ammer (1993) find a weak positive relationship with real interest rates. The link between market returns and inflation is examined in Fama and Schwert (1977), Fama (1981), Geske and Roll (1983), and Kaul (1987). The interplay between expected returns and output growth is explored in Fama (1981, 1990) and Ritter (2005).

While the Fed has more effective tools to combat inflation than in the past, our model suggests that credibility—specifically, investors' confidence in its ability to manage inflation—may be the most valuable asset in the current economic environment.

Future research could explore the asset pricing implications of time-varying investor attention, potentially amplifying the effects documented here. Investigating heterogeneous beliefs and the impact of rapid information flow would also provide valuable insights, as would extending the analysis to an international context. Exploring these areas through the lens of asset pricing will be key to predicting market reactions to policy changes.

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Appendix

A Details on Model Resolution in Section 2

Learning: To obtain the agent's posterior mean $\widehat{a}_t \equiv \mathbb{E}[a_t|\mathcal{F}_t^{\pi,r_N}]$ and the posterior variance $\nu_{a,t} \equiv \mathbb{E}[(a_t - \widehat{a}_t)^2|\mathcal{F}_t^{\pi,r_N}]$ as in (13)-(14), apply Theorem 12.7 in Liptser and Shiryaev (2001) with:

$$A_0 = \lambda_\pi(\overline{\pi} - \pi_t), \quad A_1 = -\lambda_\pi(r_{N,t} - \overline{r}_N), \quad B_1 = 0, \quad B_2 = [0 \ \sigma_\pi], \tag{A37}$$

$$a_0 = \lambda_a, \quad a_1 = -\lambda_a, \quad b_1 = \sigma_a, \quad b_2 = [0 \ 0]. \tag{A38}$$

The surprise change in inflation according to the agent's information set $\mathcal{F}^{\pi,r_N}$ is

$$d\widehat{B}_{\pi,t} = dB_{\pi,t} + \frac{\lambda_\pi}{\sigma_\pi}(\widehat{a}_t - a_t)(r_{N,t} - \overline{r}_N)dt. \tag{A39}$$

HJB equation: The partial differential equation (PDE) that results from (16)-(17) is:

$$0 = e^{-I} - \rho + \frac{\gamma - 1}{\theta}\left(\frac{\gamma\sigma_\delta^2}{2} - \mu_{\delta,t}\right) + \lambda_\pi\left[\widehat{a}_t(\overline{r}_N - r_{N,t}) + \overline{\pi} - \pi_t\right]I_\pi - \lambda_a\widehat{a}_t I_{\widehat{a}} - \lambda_y y_t I_y \tag{A40}$$

$$+ \frac{\sigma_\pi^2}{2}I_{\pi\pi} + \frac{(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}_a^2}{2\sigma_\pi^2}I_{\widehat{a}\widehat{a}} + \frac{\sigma_y^2}{2}I_{yy} + (\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}_a I_{\pi\widehat{a}} \tag{A41}$$

$$+ \frac{\theta\sigma_\pi^2}{2}I_\pi^2 + \frac{\theta(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\breve{\nu}_a^2}{2\sigma_\pi^2}I_{\widehat{a}}^2 + \frac{\theta\sigma_y^2}{2}I_y^2 + \theta(\overline{r}_N - r_{N,t})\lambda_\pi\breve{\nu}_a I_\pi I_{\widehat{a}}. \tag{A42}$$

To derive this PDE, we set $\nu_{a,t} = \breve{\nu}_a$, where $\breve{\nu}_a$ represents the empirical average of $\nu_{a,t}$. This reduces the number of state variables, simplifying the numerical solution process. The theoretical results in Section 2 are not affected by this assumption. Our numerical analysis of the model with time-varying $\nu_{a,t}$ shows that the price-dividend ratio barely changes in response to $\nu_{a,t}$, though the solution process becomes much slower. Additionally, the empirical dynamics of $\nu_{a,t}$ indicate very little variation in $\nu_{a,t}$. For these reasons, we fix $\nu_{a,t}$ to its empirical average $\breve{\nu}_a$.

The PDE for $I(\pi_t, \widehat{a}, y)$ is solved numerically using the Chebyshev collocation method (Judd, 1998). That is, we approximate the function $I(\pi_t, \widehat{a}, y)$ as follows:

$$I(\pi_t, \widehat{a}, y) \approx \mathcal{P}(\pi_t, \widehat{a}, y) = \sum_{i=0}^{I}\sum_{j=0}^{J}\sum_{k=0}^{K} a_{i,j,k}T_i[\pi] \times T_j[\widehat{a}] \times T_k[y], \tag{A43}$$

where $T_m[\cdot]$ is the Chebyshev polynomial of order m. The interpolation nodes are obtained by meshing the scaled roots of the Chebyshev polynomials of order $I+1$, $J+1$, and $K+1$. We scale the roots of the Chebyshev polynomials such that they cover approximately 99% of the unconditional distributions of the three state variables (which are all mean-reverting).

The polynomial $\mathcal{P}(\pi_t, \widehat{a}, y)$ and its partial derivatives are then substituted into the PDE, and the resulting expression is evaluated at the interpolation nodes. This yields a system of $(I+1) \times (J+1) \times (K+1)$ equations with $(I+1) \times (J+1) \times (K+1)$ unknowns (the coefficients $a_{i,j,k}$). This system of equations is solved numerically.

To verify the solution method's accuracy and address potential concerns about anomalous numerical outcomes, we employed two distinct platforms (Mathematica and Python) and multiple grid dimensions for solving the PDE. In all cases, the results were consistently similar.

Finally, the PDE for the market log price dividend ratio Π_t of the asset that is a claim to the dividend process (30) is given by:

$$0 = e^{-\Pi} - r_{R,t} + \overline{\mu}_\delta + \alpha \left(\mu_{\delta,t} - \overline{\mu}_\delta\right) \tag{A44}$$

$$+ \left(\lambda_\pi[\widehat{a}_t(\overline{r}_N - r_{N,t}) + \overline{\pi} - \pi_t] - m_{\pi,t}\sigma_t\right)\Pi_\pi - \left(\lambda_a \widehat{a}_t + \frac{m_{\pi,t}(\overline{r}_N - r_{N,t})\lambda_\pi \check{\nu}_a}{\sigma_\pi}\right)\Pi_{\widehat{a}} \tag{A45}$$

$$- \left(\lambda_y y_t + m_{y,t}\sigma_y\right)\Pi_y + \frac{\sigma_\pi^2}{2}\Pi_{\pi\pi} + \frac{(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\check{\nu}_a^2}{2\sigma_\pi^2}\Pi_{\widehat{a}\widehat{a}} + \frac{\sigma_y^2}{2}\Pi_{yy} + (\overline{r}_N - r_{N,t})\lambda_\pi\check{\nu}_a\Pi_{\pi\widehat{a}} \tag{A46}$$

$$+ \frac{\sigma_\pi^2}{2}\Pi_\pi^2 + \frac{(\overline{r}_N - r_{N,t})^2\lambda_\pi^2\check{\nu}_a^2}{2\sigma_\pi^2}\Pi_{\widehat{a}}^2 + \frac{\sigma_y^2}{2}\Pi_y^2 + (\overline{r}_N - r_{N,t})\lambda_\pi\check{\nu}_a\Pi_\pi\Pi_{\widehat{a}}. \tag{A47}$$

We first replace the solution for the log-wealth consumption ratio I in the real interest rate $r_{R,t}$ and in the market prices of risk $m_{\delta,t}$, $m_{\pi,t}$, and $m_{y,t}$, which are then replaced in the above PDE. We then solve for the market log price-dividend ratio Π using the same numerical procedure as above.

B Maximum Likelihood Estimation in Section 3

U.S. GDP is from NIPA tables. Real values are used as proxies for the output δ_t. The Fed funds rate and output gap are from FRED, and their continuously compounded values are used as proxy for the nominal interest rate $r_{N,t}$ and the output gap y_t, respectively. The year-over-year log growth rate of the Consumer Price Index (CPI) is the proxy for π_t. Time series are at the monthly frequency from July 1954 to December 2023.

The GDP growth rate volatility is obtained by maximizing the following log-likelihood function

$$l_\delta(\Theta_\delta; u_{\delta,\Delta}, \ldots, u_{\delta,J\Delta}) = \sum_{j=1}^{J} \log\left(\frac{1}{(2\pi)^{1/2}\sqrt{\sigma_\delta^2\Delta}}\right) - \frac{1}{2}\left(\sigma_\delta^2\Delta\right)^{-1} u_{\delta,j\Delta}^2, \tag{B48}$$

where $\Delta = 1/12$, $\Theta_\delta \equiv (\sigma_\delta)^\top$, J is the number of observations, $\top$ is the transpose operator, and

$$u_{\delta,t+\Delta} = \log\left(\delta_{t+\Delta}/\delta_t\right) - \left(\text{avg(GDP growth)} - \frac{1}{2}\sigma_\delta^2\right)\Delta. \tag{B49}$$

avg(GDP growth) stands for the annualized empirical average of the GDP growth rate.

The parameters driving the Taylor rule are obtained by maximizing the following log-likelihood function

$$l_r(\Theta_r; u_{r,\Delta}, \ldots, u_{r,J\Delta}) = \sum_{j=1}^{J} \log\left(\frac{1}{(2\pi)^{1/2}\sqrt{\sigma_r^2\Delta}}\right) - \frac{1}{2}\left(\sigma_r^2\Delta\right)^{-1} u_{r,j\Delta}^2, \tag{B50}$$

where $\Theta_r \equiv (\overline{r}_N, \beta_\pi, \beta_y, \sigma_r)^\top$ and

$$u_{r,t} = r_{Nt} - \left[\overline{r}_N + \beta_\pi\left(\pi_t - \text{avg(Inflation)}\right) + \beta_y y_t\right]. \tag{B51}$$

The annualized empirical average of the inflation rate is denoted by avg(Inflation).

To obtain the parameters driving inflation, we discretize the solutions of the stochastic differential equations in (13) and (15) as follows

$$\pi_{t+\Delta} = \pi_t e^{-\lambda_\pi\Delta} + \widehat{\overline{\pi}}_t\left(1 - e^{-\lambda_\pi\Delta}\right) + \sqrt{var_\pi}\,\epsilon_{\pi,t+\Delta}, \tag{B52}$$

$$\widehat{\overline{\pi}}_t = \overline{\pi} - \widehat{a}_t(r_{Nt} - \overline{r}_N) \tag{B53}$$

$$\widehat{a}_{t+\Delta} = \widehat{a}_t e^{-\lambda_a \Delta} - \frac{(r_{Nt} - \overline{r}_N)\lambda_\pi \nu_{a,t}}{\sigma_\pi}\sqrt{\frac{1 - e^{-2\lambda_a \Delta}}{2\lambda_a}}\epsilon_{\pi,t+\Delta}, \tag{B54}$$

$$\nu_{a,t+\Delta} = \nu_{a,t} + \left[\sigma_a^2 - 2\lambda_a \nu_{a,t} - \left(\frac{(r_{Nt} - \overline{r}_N)\lambda_\pi \nu_{a,t}}{\sigma_\pi}\right)^2\right]\Delta, \tag{B55}$$

where $var_\pi = \frac{\sigma_\pi^2}{2\lambda_\pi}\left(1 - e^{-2\lambda_\pi \Delta}\right)$ and $\epsilon_{\pi,t+\Delta}$ is a normally distributed random variable with mean zero and variance one. The parameters driving inflation are obtained by maximizing the following log-likelihood function

$$l_\pi(\Theta_\pi; u_{\pi,\Delta}, \ldots, u_{\pi,J\Delta}) = \sum_{j=1}^{J} \log\left(\frac{1}{(2\pi)^{1/2}\sqrt{var_\pi}}\right) - \frac{1}{2}\left(var_\pi\right)^{-1} u_{\pi,j\Delta}^2, \tag{B56}$$

where $\Theta_\pi \equiv (\sigma_\pi, \overline{\pi}, \lambda_\pi, \sigma_a, \lambda_a)^\top$ and

$$u_{\pi,t+\Delta} = \pi_{t+\Delta} - \left[\pi_t e^{-\lambda_\pi \Delta} + \widehat{\overline{\pi}}_t\left(1 - e^{-\lambda_\pi \Delta}\right)\right]. \tag{B57}$$

The updating rule for $\widehat{a}_t$ and $\nu_{a,t}$ are provided in (B54) and (B55), respectively.