UCLA Anderson School of Management

Daniel Andrei, Derivative Markets 237D, Winter 2014

MFE – Midterm

February 2014

Date: ______________________

Your Name: ______________________

Your Equiz.me email address: ______________________

Your Signature:[1] ______________________

- This exam is open book, open notes. You can use a calculator or a computer, but be sure to show or explain your work.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period.
- You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- If you are stuck on something, make an assumption, tell me what it is, and do the best you can. I give partial credit if you provide enough correct information.

TIME LIMIT: 1 hour and 30 minutes

TOTAL POINTS: 100

[1] As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them.

By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help on the exam.

1 (10 points) In a securities market there are two non dividend-paying stocks, A and B. The current prices for A and B are both \$100. One year from now, there are three possible scenarios:

Scenario	Payoff A	Payoff B
1	\$200	\$0
2	\$50	\$0
3	\$0	\$300

Let C_A be the price of a European call option on stock A and P_B be the price of a European put option on stock B. Both options have a strike price of \$95 and expire in one year. The continuously compounded risk-free interest rate is 10%.

Calculate $P_B - C_A$.

$P_B - C_A$:

2 (50 points) Consider a binomial model with $T = 2$, $h = 1$, $r = 0.07$ (continuously compounded), $u = 1.25$, $d = 0.8$, and $S_0 = 300$.

a. (10 points) Determine the initial price C_0 of an European call with maturity $T = 2$ and strike price $K = \$300$. What is the replicating portfolio at time $t = 0$? Determine the price of the European call at time $t = 1$ when the asset went up, that is, C_1^u. Determine the price of the European call at time $t = 1$ when the asset went down, that is, C_1^d.

European call price C_0:

Replicating portfolio, Δ_0^C and B_0^C:

European call price C_1^u:

European call price C_1^d:

b. (10 points) Consider an European put with maturity $T = 2$ and strike price $K = \$300$. Determine the price of the European put at time $t = 1$ when the asset went up, that is, P_1^u. Determine the price of the European put at time $t = 1$ when the asset went down, that is, P_1^d.

European put price P_1^u:

European put price P_1^d:

c. (5 points) A **chooser option** (also known as an **as-you-like-it option**) becomes a put or a call at the discretion of the owner. In our particular example here, the owner has the right to choose at time $t = 1$ the call option from point (a) or the put option from point (b). Determine the price of the chooser option at time $t = 1$ when the asset went up, that is, X_1^u. Determine the price of the chooser option at time $t = 1$ when the asset went down, that is, X_1^d.

Chooser option price X_1^u:

Chooser option price X_1^d:

d. (10 points) Compute the risk-neutral probability of an increase in the stock price, p^*. Using risk-neutral valuation, determine the price of the chooser option at time 0, X_0.

Risk-neutral probability p^*:

Price of chooser option X_0:

e. (5 points) Consider an European put with maturity $T = 1$ and strike price $K = \$300e^{-0.07} = \$279.718$. Determine the price of this European put at time $t = 0$, P_0^{new}.

Price of new put option P_0^{new}:

f. (10 points) Add the price of the European call obtained at point (a) with the price of the European put obtained at point (e) and compare the result with the value of the chooser option at time $t = 0$, obtained at point (d). You should obtain the same value. Justify why this is the case.

3 (25 points) Suppose that $S_0 = \$100$, $K = \$100$, $r = 0.08$ (continuously compounded), $\sigma = 0.30$, $\delta = 0$, and $T = 1$. Consider a 2-period binomial tree (that is, $h = 0.5$) and a Geometric Average Strike Asian call option whose payoff at maturity is given by

$$\max\left[0, S_{2h} - \sqrt{S_h \times S_{2h}}\right] \tag{1}$$

where S_h and S_{2h} represent the price of the risky asset after one and two binomial periods, respectively.

a. (10 points) What is the price of the Geometric Average Strike Asian call option at time $t = 0$, C_0^G?

Price of the Geometric Average Strike Asian call C_0^G:

b. (5 points) Do you expect to obtain a higher or a lower price than an equivalent ordinary European call option on the same underlying and with the same strike and maturity? Justify your answer.

c. (5 points) Do you expect to obtain a higher or a lower price than an equivalent **Arithmetic** Average Strike Asian call option on the same underlying and with the same strike and maturity? Justify your answer.

d. (5 points) When would an average strike option make sense?

4 (15 points) Consider a 3-month American call option with strike \$41.5 on a non dividend-paying stock. The current price of the stock is \$40 and its annualized volatility is 30%. Assume that the Black-Scholes framework holds and that the delta of the call option today is 0.5.

Determine the current price of the American call option, C_0.

Price of call option C_0:

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Solutions to Midterm (MFE)

A general comment: if you got the wrong answer and failed to show your work, there was no way to give partial credit. Please keep this in mind for the final exam.

1 The problem is to find the price of a security that pays:

Scenario	Payoff $P_B - C_A$
1	-\$10
2	\$95
3	\$0

Replicate this payoff with a portfolio of stocks A and B and a position X in the risk-free asset:

$$\Delta_A 200 + \Delta_B 0 + Xe^{0.1} = -10 \tag{2}$$

$$\Delta_A 50 + \Delta_B 0 + Xe^{0.1} = 95 \tag{3}$$

$$\Delta_A 0 + \Delta_B 300 + Xe^{0.1} = 0 \tag{4}$$

The solution is $\Delta_A = -0.7$, $\Delta_B = -0.4333$, $X = 117.629$, and thus the value of $P_B - C_A$ is \$4.2955.

2

a. The call option price is $C_0 = \$53.80$. The replicating portfolio is $\Delta_0^C = 0.71$ and $B_0^C = -\$157.94$. If the asset goes up in the first period, $C_1^u = \$95.28$. If the asset goes down in the first period, $C_1^d = \$0$.

b. We can compute the put prices with the parity relationship: $P_1^u = \$0$, $P_1^d = \$39.72$.

c. The holder chooses the call option if the asset goes up and thus $X_1^u = \$95.28$. The holder chooses the put option if the asset goes down and thus $X_1^d = \$39.72$.

d. The risk-neutral probability is $p^* = 0.60557$. The price of the chooser option at time 0 is $X_0 = \$68.41$.

e. The new put option price is $P_0^{new} = \$14.61$.

f. We have

$$\$53.80 + \$14.61 = \$68.41 \tag{5}$$

This can be justified by using the parity relationship:

$$\max[C_1, P_1] = \max\left[C_1, C_1 - S_1 + \$300e^{-0.07}\right] \quad (6)$$
$$= C_1 + \max\left[\$300e^{-0.07} - S_1, 0\right] \quad (7)$$
$$= C_1 + P_1^{new} \quad (8)$$

The chooser option is a portfolio composed of the call from point (a) and the put from point (e).

3 a. You can use any technique to price the option (replicating portfolio or risk-neutral pricing). The tree is non-recombining. At maturity, there are 4 possible strikes: \$145.97, \$118.07, \$95.50, and \$77.24.

The risk neutral probability p^* is 0.447165. Hence, the price of the option at $t = 0$ is

$$C_0^G = \$6.55 \quad (9)$$

b. The average call should cost less because of the averaging factor required to calculate the strike, and thus a lower payoff volatility.

c. A geometric average is always less than or equal to an arithmetic average. This makes the strike of the Geometric Average option lower than the strike of the Arithmetic Average option. Thus

$$\text{Geometric Average Strike Asian call} \geq \text{Arithmetic Average Strike Asian call} \quad (10)$$

d. Such an option pays off when there is a difference between the average asset price over the life of the option and the asset price at expiration. It could be used for insurance in a situation where we accumulated an asset over a period of time and then sold the entire accumulated position at once. Similarly, a holder of a currency payable could purchase an average strike call option to hedge against the appreciation of the currency and, in the same time, benefit from the currency's depreciation. See more in McDonald, Chapter 14.2.

4 It is never optimal to exercise an American call before maturity if the stock pays no dividends. Thus, we can price the call option using the Black-Scholes formula

$$C_0 = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad (11)$$

We know that the delta of the option is $N(d_1)$ and in our case equals 0.5. Thus, $d_1 = 0$. It follows that $d_2 = d_1 - \sigma\sqrt{T} = -0.15$ and $N(d_2) = 0.44038$. We have

$$C_0 = 40 \times 0.5 - 41.5 \times e^{-r \times 0.25} \times 0.44038 \quad (12)$$

The only unknown here is r. Use the equation:

$$d_1 = \frac{\ln(40/41.5) + (r + 0.3^2/2)0.25}{0.3\sqrt{0.25}} \tag{13}$$

which gives $r = 0.1023$. Thus, $C_0 = \$2.19$.