UCLA Anderson School of Management
Daniel Andrei, Option Markets 232D, Fall 2013

MBA – Final Exam

December 2013

Date: ____________________

Your Name: ____________________

Your Equiz.me email address: ____________________

Your Signature:[1] ____________________

- This exam is open book, open notes. You can use a calculator or a computer, but be sure to show or explain your work.
- You are not allowed to communicate with anyone (verbally, in writing, or electronically), except for me, during the exam period.
- You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- If you are stuck on something, make an assumption, tell me what it is, and do the best you can. I give partial credit if you provide enough correct information.

TIME LIMIT: 1 hour and 30 minutes

TOTAL POINTS: 100

[1] As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them.
By signing the exam: (*i*) you certify your presence, and (*ii*) you state that you neither gave nor received help on the exam.

1 (20 points) When answering questions a thru e refer to the following table of commodity forward and spot prices. The **effective annual** risk free interest rate is 4.0%.

Expiration Time	**Corn Cents/Bushel**	**Soybeans Cents/Bushel**	**Pork Bellies Cents/Pound**
Today = spot	212	430	540
6 months	208	440	560
12 months	203	452	590
18 months	200	435	625
24 months	195	419	655

a. (4 points) What is the **effective annualized** lease rate on the 12-month corn forward contract?

Lease rate for corn:

b. (4 points) What is the **effective annualized** lease rate on the 18-month soybean forward contract?

Lease rate for soybeans:

c. (4 points) Which of the following terms most accurately describes the forward curve for corn over the next two years? Briefly justify why.

(i) Contango
(ii) Backwardation
(iii) Contango and backwardation
(iv) None of the above

d. (4 points) Which of the following terms most accurately describes the forward curve for soybeans over the next two years? Briefly justify why.

(i) Contango
(ii) Backwardation
(iii) Contango and backwardation
(iv) None of the above

e. (4 points) Which of the following terms most accurately describes the forward curve for pork bellies over the next two years? Briefly justify why.

(i) Contango
(ii) Backwardation
(iii) Contango and backwardation
(iv) None of the above

2 (8 points) A **naked option** is an option that is not combined with an offseting position in the underlying stock. The initial and maintenance margin required by the CBOE for a written naked put option is the greater of the following two calculations:

- A total of 100% of the proceeds of the sale, plus 20% of the underlying share prices, and less the amount (if any) by which the option is out of the money
- A total of 100% of the option proceeds plus 10% of the exercise price.

For a written naked call option, it is the greater of

- A total of 100% of the proceeds of the sale, plus 20% of the underlying share price, and less the amount (if any) by which the option is out of the money
- A total of 100% of the option proceeds plus 10% of the underlying share price.

a. (4 points) An investor writes five naked put option contracts on a stock **(each contract is on 100 shares of the stock)**. The **individual** option price is $15, the strike price is $80, and the stock price is $75. What is the initial margin requirement?

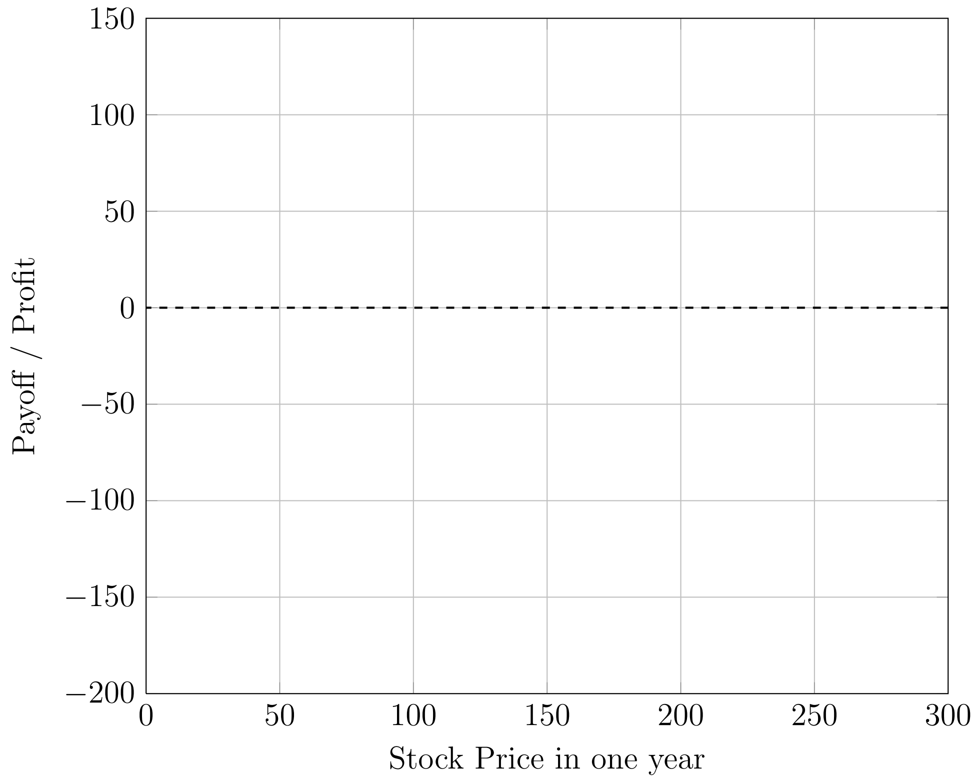
Margin requirement for put:

b. (4 points) An investor writes five naked call option contracts on a stock **(each contract is on 100 shares of the stock)**. The **individual** option price is $10, the strike price is $80, and the stock price is $75. What is the initial margin requirement?

Margin requirement for call:

3 (15 points) Suppose a stock pays no dividends and has a current price of \$150. The forward price for delivery in one year is \$157.50. Suppose the **annual effective** interest rate is 5%.

a. (5 points) Plot the payoff and profit diagrams for a long forward contract on the stock with a forward price of \$157.50. Are the payoff and profit graphs different? Why or why not? What is the height of the profit graph when the stock price is \$200? Clearly indicate this point on the diagram.



b. (5 points) You have a bullish view on the stock and therefore are considering two trading strategies: (*i*) buy the stock or (*ii*) buy the forward contract. Is there any advantage to buying the stock relative to buying the forward contract?

c. (5 points) Suppose the stock had a dividend yield of 2% **(yearly, continuously compounded)** and everything else stayed the same. Is there any advantage to buying the forward contract relative to buying the stock?

4 (25 points) Assume that **continuously compounded** zero-coupon bond yields with maturities 1, 2, and 3 years are 5.2%, 5.5%, and 5.8%, respectively.

a. (5 points) Calculate the zero-coupon bond prices with maturities 1, 2, and 3 years.

$P(0,1)$:

$P(0,2)$:

$P(0,3)$:

b. (5 points) What is the implied forward zero-coupon bond price from year 1 to year 2, quoted at time 0, $P_0(1,2)$? What is the implied forward zero-coupon bond price from year 2 to year 3, quoted at time 0, $P_0(2,3)$?

$P_0(1,2)$:

$P_0(2,3)$:

c. (5 points) What is the one-year implied forward rate from year 1 to year 2, prevailing at time 0, $r_0(1,2)$? What is the one-year implied forward rate from year 2 to year 3, prevailing at time 0, $r_0(2,3)$? **(both rates are continuously compunded)**

$r_0(1,2)$:

$r_0(2,3)$:

d. (5 points) Suppose that oil forward prices for 1 year and 3 years are \$20 and \$23. The 3-year swap price for oil is \$21.2723. What is the oil forward price with maturity 2 years, $F_{0,2}$?

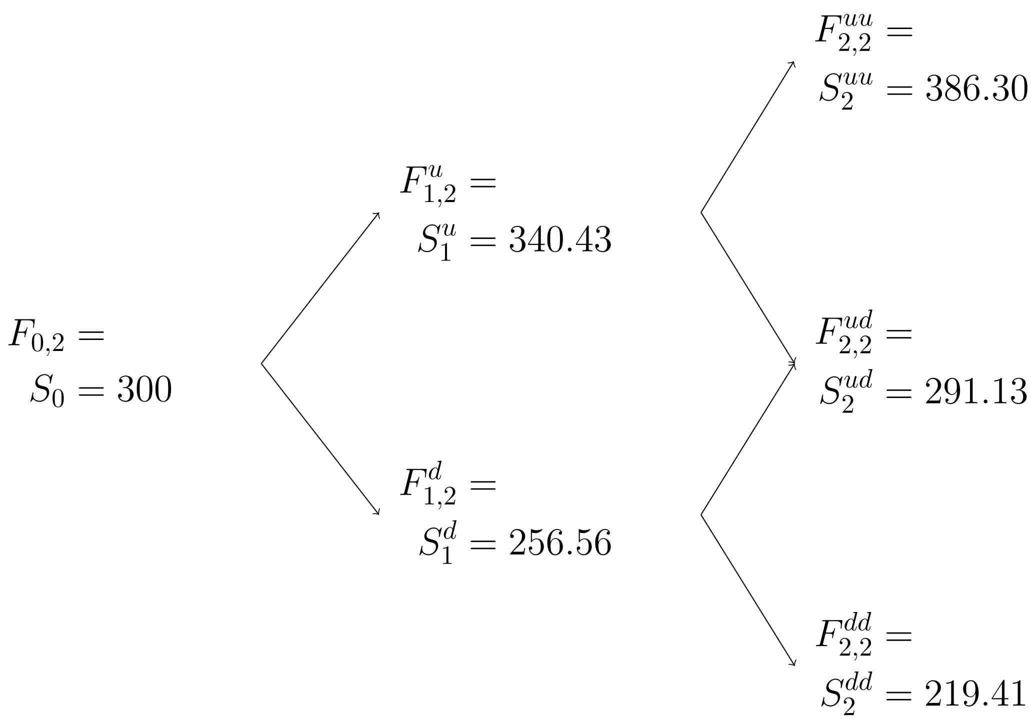
$F_{0,2}$:

e. (5 points) What is the 2-year swap price for oil?

2-year swap price:

5 (30 points) The price of a stock today (at time 0) is \$300. The volatility of the stock is $\sigma = 20\%$, the **annual continuously compounded interest rate** is $r = 5\%$, and the dividend yield on the stock is $\delta = 8\%$.

A two-period binomial tree, with 6 months between nodes, is depicted below. **The numbers on the tree represent prices of the stock at all nodes**, according to the parameters above.



a. (5 points) Consider a futures contract on the stock which expires in one year from today. Fill in the futures prices at each node in the tree above.

b. (5 points) What are the u and d parameters for the stock price? What are the u and d parameters for the futures price?

u for stock price:

d for stock price:

u for futures price:

d for futures price:

c. (4 points) The risk-neutral probability of an up move is $p^* = 0.4647$ (it can be verified that this probability is the same for the stock and for the futures). **Please use this probability for all the calculations below this point.**

Consider an European put option **on the stock** with maturity 6 months from today, with a strike price of \$300. What is the payoff of the put option in 6 months if the stock price goes up to \$340.43, $P^u_{S,1}$? What is the payoff of the put option in 6 months if the stock price goes down to \$256.56, $P^d_{S,1}$? What is the price today of this put option, $P_{S,0}$?

$P^u_{S,1}$:

$P^d_{S,1}$:

$P_{S,0}$:

d. (4 points) Consider an European put option **on the futures** with maturity 6 months from today, with a strike price of \$300. What is the payoff of the put option in 6 months if the stock price goes up to \$340.43, $P^u_{F,1}$? What is the payoff of the put option in 6 months if the stock price goes down to \$256.56, $P^d_{F,1}$? What is the price today of this put option, $P_{F,0}$?

$P^u_{F,1}$:

$P^d_{F,1}$:

$P_{F,0}$:

e. (4 points) Did you obtain $P_{S,0} = P_{F,0}$? Why or why not? If not, for which value of δ you would obtain $P_{S,0} = P_{F,0}$?

f. (5 points) Consider an European call option **on the futures** with maturity 6 months from today, with a strike price of \$300. Write down the put call parity relation between the futures call and futures put options, at time 0. What is the price today of of this call option, $C_{F,0}$?

Put call parity:

$C_{F,0}$:

g. (3 points) Prove, mathematically, that $F_{0,2} = F_{2,2}^{ud}$. That is, the forward price today is equal to the forward price in 2 periods if the stock goes up and down (or down and up).

6 (2 points) Assume the following Black-Scholes parameters for a stock and an European call option on it:

Price of the underlying	S_0
Strike price	K
Volatility of the underlying	σ
Interest rate (cont. coumpounded)	r
Maturity of the option	T_1
Dividend yield of the underlying	δ

a. (2 points) Consider a futures contract on the stock with maturity $T_2 > T_1$ and an European call option **on the futures**. The option has maturity T_1 and strike price K. Fill in the table below the Black-Scholes parameters used to compute this option's price. **All the values that you fill in must be functions of the parameters given above.**

Price of the underlying	
Strike price	
Volatility of the underlying	
Interest rate (cont. coumpounded)	
Maturity of the option	
Dividend yield of the underlying	

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Solutions to Final Exam (MBA)

A general comment: if you got the wrong answer and failed to show your work, there was no way to give partial credit.

1 a. The general formula in discrete time is:

$$\delta_l = \frac{1+r}{\left(\frac{F_{0,T}}{S_0}\right)^{\frac{1}{T}}} - 1 \tag{1}$$

The effective annualized lease rate on the 12-month corn forward contract is

$$\delta_l = 8.61\% \tag{2}$$

b. Using the same formula, the effective annualized lease rate on the 18-month soybean forward contract is

$$\delta_l = 3.2\% \tag{3}$$

c. Correct answer: (ii) Backwardation. Forward curve downward slopping.

d. Correct answer: (iii) Contango and backwardation. Forward curve upward slopping first then downward sloping, futures price becomes lower than spot price at long maturities.

e. Correct answer: (i) Contango. Forward curve upward slopping.

2 a. The put option is \$5 in the money. The first calculation gives

$$500 \times (15 + 0.2 \times 75) = \$15,000 \tag{4}$$

The second calculation gives

$$500 \times (15 + 0.1 \times 80) = \$11,500 \tag{5}$$

The initial margin requirement is therefore \$15,000.

b. Because the option is \$5 out of the money, the first calculation gives

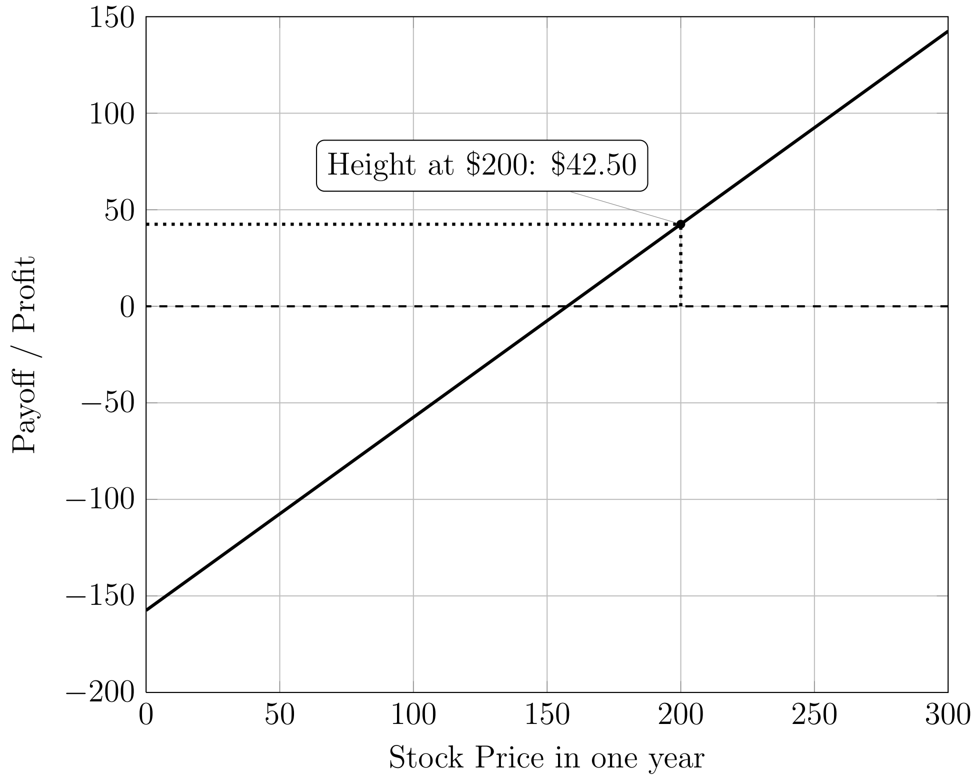
$$500 \times (10 + 0.2 \times 75 - 5) = \$10,000 \tag{6}$$

The second calculation gives

$$500 \times (10 + 0.1 \times 75) = \$8,750 \tag{7}$$

The initial margin requirement is therefore \$10,000.

3 a. It does not cost anything to enter into a forward contract. Therefore, the payoff diagram coincides with the profit diagram. The graphs have the following shape:



The profit of the long forward is $S_T - \$157.50$, where S_T is the value of one share at expiration. Thus, the height of the profit graph when the stock price is $200 equals $42.50 (as shown on the diagram above).

b. We can borrow $150 and buy the stock. We have to pay back $\$150 \times 1.05 = \$157.50$ in one year. Therefore, our total profit at expiration from the purchase of the stock is $S_T - \$157.50$, where S_T is the value of one share at expiration. As this profit is the same as a long forward contract, there is no advantage in investing in any instrument.

c. As the buyer of the stock, we receive the dividend. As the buyer of the forward contract, we do not receive the dividend, because we only have a claim to buy the stock in the future for a given price, but we do not own it yet. Therefore, it does matter now whether we buy the stock or buy the forward contract. Because everything else is the same as in part (a) and (b), it is now beneficial to buy the stock. **Note that you do not really need to use the dividend yield to answer this question.**

4 a. Zero-coupon bond prices are:

$$P(0,1) = e^{-0.052\times 1} = 0.9493 \tag{8}$$
$$P(0,2) = e^{-0.055\times 2} = 0.8958 \tag{9}$$
$$P(0,3) = e^{-0.058\times 3} = 0.8403 \tag{10}$$

b. Implied forward zero-coupon bond prices are:

$$P_0(1,2) = \frac{P(0,2)}{P(0,1)} = 0.9436 \tag{11}$$
$$P_0(2,3) = \frac{P(0,3)}{P(0,2)} = 0.9380 \tag{12}$$

c. Implied forward rates are computed from implied zero-coupon bond prices:

$$r_0(1,2) = -\ln\left[P_0(1,2)\right] = 5.8\% \tag{13}$$
$$r_0(2,3) = -\ln\left[P_0(2,3)\right] = 6.4\% \tag{14}$$

d. The 3-year swap price for oil is

$$\bar{F}_3 = \frac{P(0,1)\times F_{0,1} + P(0,2)\times F_{0,2} + P(0,3)\times F_{0,3}}{P(0,1)+P(0,2)+P(0,3)} \tag{15}$$
$$\$21.2723 = \frac{0.9493\times \$20 + 0.8958\times F_{0,2} + 0.8403\times \$23}{0.9403+0.8958+0.8403} \tag{16}$$

Solving for $F_{0,2}$ yields

$$F_{0,2} = \frac{\$21.2723\times(0.9493+0.8958+0.8403) - (0.9493\times \$20 + 0.8403\times \$23)}{0.8958} \tag{17}$$

Finally, we get $F_{0,2} = \$21$.

e. The 2-year swap price for oil is

$$\bar{F}_2 = \frac{P(0,1)\times F_{0,1} + P(0,2)\times F_{0,2}}{P(0,1)+P(0,2)} \tag{18}$$
$$\bar{F}_2 = \frac{0.9493\times \$20 + 0.8958\times \$21}{0.9493+0.8958} = \$20.49 \tag{19}$$

5 a. Use the formula $F_{t,T} = S_t e^{(r-\delta)(T-t)}$:

$$
\begin{tikzcd}[column sep=large]
 & & \begin{matrix} F^{uu}_{2,2} = 386.30 \\ S^{uu}_2 = 386.30 \end{matrix} \\
 & \begin{matrix} F^{u}_{1,2} = 335.36 \\ S^{u}_1 = 340.43 \end{matrix} \arrow[ur] \arrow[dr] & \\
\begin{matrix} F_{0,2} = 291.13 \\ S_0 = 300 \end{matrix} \arrow[ur] \arrow[dr] & & \begin{matrix} F^{ud}_{2,2} = 291.13 \\ S^{ud}_2 = 291.13 \end{matrix} \\
 & \begin{matrix} F^{d}_{1,2} = 252.74 \\ S^{d}_1 = 256.56 \end{matrix} \arrow[ur] \arrow[dr] & \\
 & & \begin{matrix} F^{dd}_{2,2} = 219.41 \\ S^{dd}_2 = 219.41 \end{matrix}
\end{tikzcd}
$$

b. The u and d parameters for the stock are:

$$u_{stock} = e^{(r-\delta)h+\sigma\sqrt{h}} = 1.1348 \tag{20}$$

$$d_{stock} = e^{(r-\delta)h-\sigma\sqrt{h}} = 0.8552 \tag{21}$$

The u and d parameters for the futures price are:

$$u_{futures} = e^{\sigma\sqrt{h}} = 1.1519 \tag{22}$$

$$d_{futures} = e^{-\sigma\sqrt{h}} = 0.8681 \tag{23}$$

Note that u and d can also be calculated directly using the values from the tree.

c. The payoffs in 6 months for the put on the stock are:

$$P^u_{S,1} = 0 \tag{24}$$

$$P^d_{S,1} = 43.44 \tag{25}$$

The price of the put option is

$$P_{S,0} = e^{-0.05\times 0.5}(0.4647 \times 0 + (1 - 0.4647) \times 43.44) = 22.68 \tag{26}$$

d. The payoffs in 6 months for the put on the futures are:

$$P_{F,1}^{u} = 0 \tag{27}$$
$$P_{F,1}^{d} = 47.26 \tag{28}$$

The price of the put option is

$$P_{F,0} = e^{-0.05\times 0.5}(0.4647 \times 0 + (1 - 0.4647) \times 47.26) = 24.67 \tag{29}$$

e. We do not obtain the same values. Because $r < \delta$, the market is in backwardation and thus $F_{0,2} < S_0$. For this reason, $P_{F,0} > P_{S,0}$.

If, instead, $r = \delta$, then $F_{t,2} = S_t$ at all nodes until maturity and thus we would obtain the same price for the two put options.

f. Remember that the futures price behaves like an asset which has a dividend yield equal to the risk free rate. Thus, the put call parity in this case is

$$C_{F,0} - P_{F,0} = F_{0,2}e^{-r\times 0.5} - Ke^{-r\times 0.5} \tag{30}$$
$$C_{F,0} - 24.67 = 291.13e^{-0.05\times 0.5} - 300e^{-0.05\times 0.5} \tag{31}$$

The price of the call option follows from the put call parity:

$$C_{F,0} = 24.67 + 291.13e^{-0.05\times 0.5} - 300e^{-0.05\times 0.5} \tag{32}$$
$$C_{F,0} = 16.03 \tag{33}$$

g. Since for the futures contract we have $u = e^{\sigma\sqrt{h}}$ and $d = e^{-\sigma\sqrt{h}}$, it follows that:

$$F_{2,2}^{ud} = F_{0,2}e^{\sigma\sqrt{h}}e^{-\sigma\sqrt{h}} = F_{0,2} \tag{34}$$

6 a. We price an European option on a futures contract by using the futures prices as the stock price and setting the dividend yield equal to the risk-free rate.

Price of the underlying	$S_0e^{(r-\delta)T_2}$
Strike price	K
Volatility of the underlying	σ
Interest rate (cont. coumpounded)	r
Maturity of the option	T_1
Dividend yield of the underlying	r