UCLA Anderson School of Management
Daniel Andrei, Option Markets 232D, Fall 2012

MBA — Final Exam

December 2012

Your Name: _____

Your Equiz.me email address: _____

Your Signature:[1] _____

- This exam is open book, open notes. You can use a calculator, but be sure to show or explain your work.
- You cannot use a computer. You are not allowed to communicate with anyone (verbally, in writing, electronically), except for me, during the exam period.
- You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- If you are stuck on something, make an assumption, tell me what it is, and do the best you can. I give partial credit if you provide enough correct information.
- If you got the wrong answer and failed to show your work, there is no way to give partial credit.

TOTAL POINTS: 100

TIME LIMIT: 1 hour and 30 minutes

[1] As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them.

By signing the exam: (*i*) you state that you adhere to the UCLA academic standards, (*ii*) you certify your presence, and (*iii*) you state that you neither gave nor received help on the exam.

1 (5 points) A symmetric butterfly spread consists in: Long call with strike K_1, long call with strike K_2, short 2 calls with strike $(K_1 + K_2)/2$. Can a symmetric butterfly spread have zero initial premium? Why or why not?

2 (5 points) A straddle consists in: Long call and long put with the same strike. True or False (no justification necessary): Straddles make money if the stock price moves away form the strike and ends far from it.

True: ☐ **False:** ☐

3 (18 points) A \$100 stock will pay a **discrete** \$15 dividend 180 days from today. This is the only dividend the stock will pay during the year. **The interest rate is zero.**

a. (8 points) What is the one-year forward price for the stock?

Forward price

b. (10 points) Suppose the forward price were \$1 greater than your answer in the previous part. Explain the transactions you would undertake so as to have **zero cash flow at time 0** and **positive cash flow in one year**. Be sure to explain what transactions you undertake at times 0 and 1, and what happens when the dividend is paid.

4 (10 points) Here are two forward curves and the corresponding swap price:

	Year 1 forward price	Year 2 forward price	Swap price
A	20	30	24.76
B	30	20	25.24

Explain why the swap price for curve A is below that for curve B.

5 (12 points) Suppose the current widget price is \$2 and the widget forward price for delivery in one month is \$2.50. The interest rate is zero. Widgets are stored.

a. (6 points) Assuming that widgets are stored, what is the one-month storage cost?

Storage cost

b. (6 points) If you short-sell a widget for one month, what will be the payment between you and the widget lender?

6 (10 points) You wish to buy corn at a fixed price for quarters **2** and **3**. Quarterly forward corn prices are

	Price at end of quarter:			
	1	2	3	4
Price	2.75	3.00	2.35	2.95

The effective quarterly interest rate is 3% (for all maturities above). What fixed price will you pay in quarters 2 and 3?

Fixed price

7 (22 points) Suppose the effective annual euro-denominated interest rate is 10% and the yen-denominated interest rate is zero. The euro price of a yen is 0.01.

a. (6 points) Let F denote the forward price and x_1 the exchange rate in one year. Write an expression for the payoff of a long forward contract.

Payoff

b. (6 points) What is the one-year forward price for a yen, expressed in euros?

One-year forward price

c. (10 points) Suppose the forward price is too high, i.e., it is **greater** than the theoretically correct price you have derived. What transactions would you undertake to arbitrage this mispricing? (Just say what you would do at time zero, there is no need to discuss future cash flows.)

8 (18 points) The time until expiration is 0.4 years and the underlying stock pays no dividends. **Do not assume that you know the interest rate.** Suppose you observe the following option prices:

Strike price	50	60
Call premium	5.7868	1.7679
Put premium	2.3091	7.9947

a. (8 points) What transaction using all four of these options will enable you to buy the stock forward at \$50 and sell the stock forward at \$60? (Use a "$+n$" to denote a purchase of n options and "$-n$" to denote the sale of n options.)

Option Quantities:

50-strike call: _____

50-strike put: _____

60-strike call: _____

60-strike put: _____

b. (10 points) What are the continuously compounded interest rate and the stock price? **(Hint: use your answer to part (a). What synthetic position did you construct?)** Be clear about what you have done; you will receive no credit for answering this by using the Black-Scholes formula to figure out the option price inputs.

Interest rate	Stock Price

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Solutions to Final Exam (MBA)

A general comment: if you got the wrong answer and failed to show your work, there was no way to give partial credit.

1 No. Since the symmetric butterfly spread has non-negative payoff, it must have positive value.

2 True.

3 a. The one-year prepaid forward price is $S_0 - PV(\text{Div}) = \$85$. The one-year forward price is the future value of the one-year prepaid forward price, or \$85.

b. If the forward price were \$1 too high ($F = \$86$), the arbitrage is to short the forward, which entails an obligation to sell the stock in one year. To offset this, borrow to buy the stock. At 180 days, when the dividend is paid, repay \$15 of the borrowing; this leaves \$85 of debt outstanding. At the end of one year, the payoff is

$$\underbrace{F - S_1}_{\text{Forward}} + S_1 - \$85 = F - \$85 = \$1$$

4 The swap price will be somewhere between \$20 and \$30.

Case A: Consider a party who is the fixed price payer and who is hedged with forward contracts. In year 1, the fixed price payer will make a net payment (year 1 forward price less swap price) and in year 2, the fixed price payer will receive a net payment (year 2 forward price less swap price). This is a loan. In order for it to carry a positive rate of interest, the swap price must be below the average price (\$25).

This logic in reverse explains why the swap price is above \$25 in case B.

The key idea in this case was the pattern of payments on the hedged swap transaction.

5 a. If the interest rate is zero, the minimum one-month storage cost is \$0.50. The reason is that you can buy a widget and sell it forward one month. This guarantees a return of \$0.50, gross of storage costs.

b. To short-sell a widget, you would locate an owner, borrow it, and sell, receiving \$2. Since you will return the widget in one month, it is as if you are storing the widget on behalf of the lender. Thus, the lender should be willing to pay you the \$0.50 storage cost. You then receive \$2.50 (\$2 for selling the widget and a \$0.50 payment from the lender). You could enter into a long forward contract

and guarantee a \$2.50 price to purchase the widget in the future, so you break even.

The key idea here is that by borrowing the widget to short-sell it, you save the lender storage costs; they should be willing to pass along this saving.

6 The swap price is

$$\frac{\frac{3.00}{1.03^2}+\frac{2.35}{1.03^3}}{\frac{1}{1.03^2}+\frac{1}{1.03^3}} = 2.6798$$

7 a. The payoff to a long forward (you are buying the currency) is $x_1 - F$.

b. The one-year forward price is $0.01 \times (1.1) = 0.011$.

c. If the yen forward price is too high, sell forward. This requires you to have a unit of foreign currency in one year, so lend in yen the present value of one yen, and borrow in Euros to finance this. The amount received from selling the yen at the forward price will more than repay the amount borrowed.

8 a. You create a synthetic forward purchase of the stock at \$50 (buy the 50 call and sell the 50 put) and a synthetic forward sale at \$60 (sell the 60 call and buy the 60 put). This will yield \$10 for certain. The cost of the strategy is

$$-C(50)+P(50)+C(60)-P(60) = -5.7868+2.3091+1.7679-7.9947 = -9.7045$$

This pays off \$10 in 0.4 years.

b. You have created a synthetic bond, the interest rate is

$$\ln(10/9.7045)/0.4 = 0.075$$

To determine the stock price, we can use put-call parity for either strike. This gives us

$$C(K) - P(K) + PV(K) = S$$

Thus, using the 50-strike options, we have

$$5.7868 - 2.3091 + 50e^{-0.075\times 0.4} = \$52$$

You can verify that the option prices above were computed using $S = 52$, $r = 0.075$, and $\sigma = 0.30$.