UCLA Anderson School of Management
Daniel Andrei, Option Markets 232D, Fall 2012

FEMBA — Take Home Exam

Saturday, November 10, 2012

Your Name: _____

Your Equiz.me email address: _____

Your Signature:[1] _____

- Be sure to show or explain your work. You may present calculations in non-reduced form (e.g., as "$e^{0.095} - 1$").
- If you are stuck on something, make an assumption, tell me what it is, and do the best you can. I give partial credit if you provide enough correct information.
- If you got the wrong answer and failed to show your work, there is no way to give partial credit.

TOTAL POINTS: 100

DEADLINE: Sunday, November 18, 2012, 5:00 PM (Pacific Time)

- Email a scanned copy of the take home exam to daniel.andrei@anderson.ucla.edu **PRIOR TO THE DEADLINE. PLEASE EMAIL ONLY ONCE.**
- **IMPORTANT:** Please mail original copy of the exam to:

  Daniel Andrei
  UCLA Anderson School of Management
  110 Westwood Plaza, Suite C420
  Los Angeles, CA 90095

  I should receive your mail before Monday, November 26, 2012.

[1] As a member of the UCLA Anderson academic community, the highest standards of academic behavior are expected of you. It is your responsibility to make yourself aware of these standards (specifically regarding plagiarism, individual work, and team work) and adhere to them.
By signing the exam you state that you adhere to the UCLA academic standards.

1 (24 points) Answer the following questions.

a. (4 points) The effective annual interest rate is 9.5%. What is the equivalent continuously compounded interest rate?

Interest rate

b. (4 points) An S&P 500 forward contract with 9 months to expiration has a price of 916.3467. The S&P index is currently at 900 and the continuously compounded annual risk free rate is 4%. What is the continuous dividend yield on the index?

Dividend yield

c. (4 points) ABC has a price of \$80 and DEF has a price of \$70. Both are non-dividend paying stocks. An 18-month option costing \$22 has the payoff

$$\max(0, S_{\text{ABC},1.5} - S_{\text{DEF},1.5})$$

In words, at expiration (in 1.5 years) the option permits ABC to be received by giving up DEF. What is the price of an option that has the payoff

$$\max(0, S_{\text{DEF},1.5} - S_{\text{ABC},1.5})$$

(Note that this option permits DEF to be received by giving up ABC.)

Option price

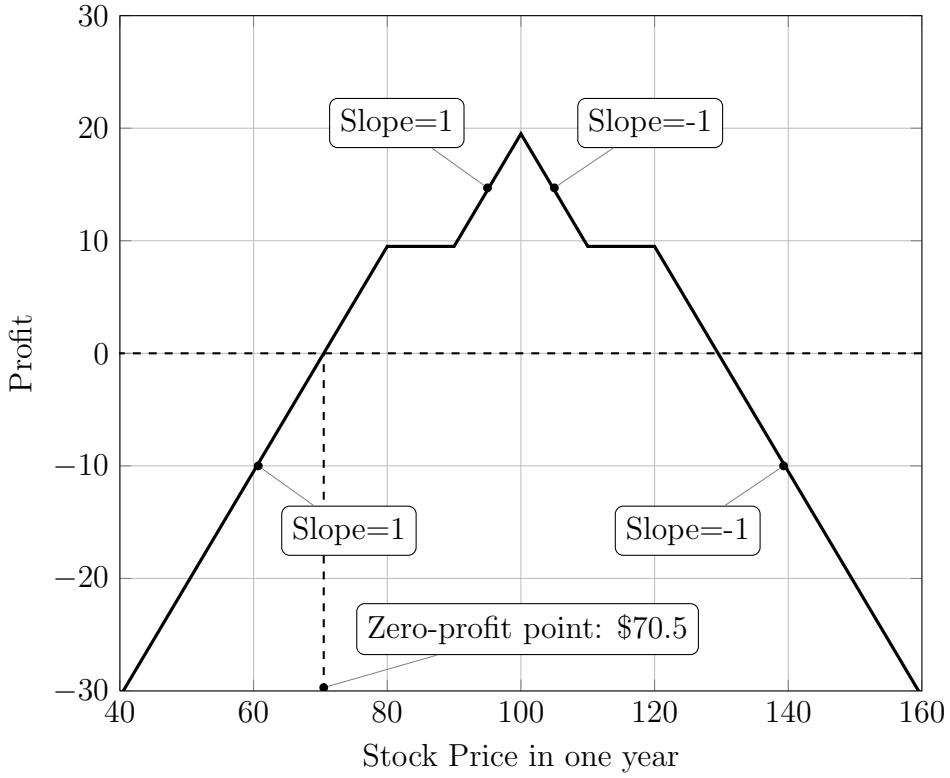
d. (4 points) Suppose you buy a 80-strike put and a 110-strike call and you delta-hedge the position. Suppose the current stock price is \$100. If the stock price does not move over one day, do you make or lose money? Why?

e. (4 points) Suppose you buy an 100-strike call with maturity 1 year and sell a 100-strike call with maturity 3 months. Suppose the current stock price is \$100. If the stock price does not move over one day, do you make or lose money? Why?

f. (4 points) The one-year forward price for copper is \$3 and the two-year forward price is \$2.50. The two-year swap price (where one unit is delivered in year one and one in year two) is 2.7556. What is the one-year implied **effective** forward interest rate one year from today?

Implied forward interest rate

2 (20 points) Below is a **profit** diagram for a position. All options have 1 year to maturity and the stock price today is \$100. The yearly interest rate (continuously compounded) is 8%. The underlying asset (the stock) is not paying any dividends.



a. (5 points) In the table below, record option quantities which construct the diagram. Use only **puts** and a share position. Slopes are marked on the diagram. Denote a purchased position with "+" and a written position with "−".

Strike	80	90	100	110	120	Shares
Puts						

b. (5 points) In the table below, record option quantities which construct the diagram. Use only **calls** and a share position. Slopes are marked on the diagram. Denote a purchased position with "+" and a written position with "−".

Strike	80	90	100	110	120	Shares
Calls						

c. (5 points) What is the cash flow required to establish the position at the outset (i.e., what is the premium of this position)? **Hint:** *Use the zero-profit point from the diagram.*

d. (5 points) Suppose that all options had 6 months to maturity instead of one year, while the stock price is still \$100. Would the profit diagram with calls shift up or down compared to the diagram above? Why? What about the diagram with puts?

3 (13 points) An option pays the **square root** of the stock price less the strike price of \$2.50 if the owner of the option chooses to exercise it. For example, if the stock price is \$100, the claim when exercised pays $\$10 - \$2.50 = \$7.50$. Assume that the annual continuously compounded interest rate is 12%, the annual dividend yield on the stock is 0, and there are 5 months between nodes. Use the tree on the following page to answer the following questions. **Pay attention to the extra information provided in the tree.**

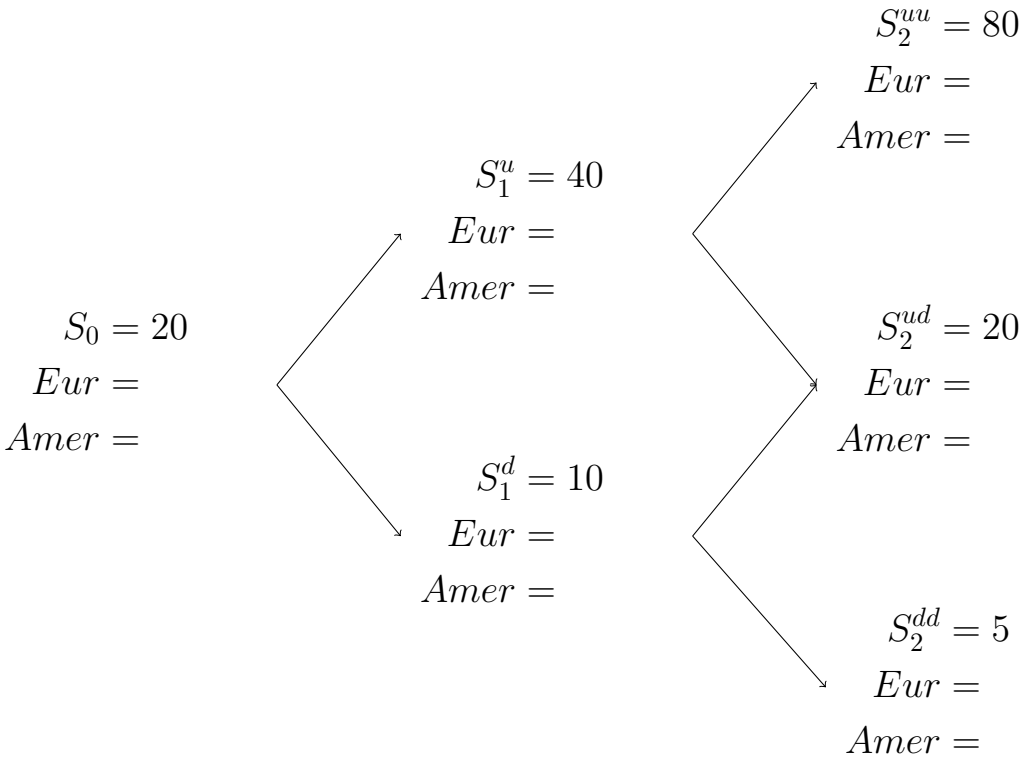
a. (4 points) What is the annualized volatility used to construct the tree? **Hint:** *Start first by finding u and d.*

Volatility:

b. (4 points) What is the risk-neutral probability of an up move?

Risk-neutral Probability:

c. (5 points) At each node in the tree, fill in the prices for the American and European versions of this option. (When you exercise the American version at time t, you receive $\sqrt{S_t} - \$2.50$, as at expiration.) Put an asterisk at each node where the American option is exercised.



4 (23 points) Below is some information about two options. Suppose you buy one 45-strike call, one 45-strike put, and 0.2 shares. You finance the position by borrowing at the continuously compounded risk-free rate of 6%. The stock price is $40 and the volatility is 30%. **You are to answer this question using *only* the information provided here. You *must* show your calculations to receive credit.**

Strike = 45	Call	Put
Price	2.7099	7.4981
Delta	0.3926	-0.5967
Gamma	0.0336	0.0336
Vega	0.1441	0.1441
Theta	-0.0067	-0.0060
Rho	0.1159	-0.2798
Elasticity	5.7957	-3.1832

a. (4 points) If the share price rises $1 right after you enter the position, what is the approximate change in the value of the position? Why?

Change in value of position

b. (4 points) How large a change in the stock price is a one standard deviation change over one day? (1 year = 365 days)

Change in stock price

c. (5 points) Suppose that over the course of one day, the stock price does not move. Approximately what will be the profit on your position?

Change in value

d. (5 points) Suppose that over the course of one day, the stock price moves significantly more than one standard deviation. Will you show a profit or loss on the position? Why?

Gain or Loss?

e. (5 points) Suppose that the annual continuously compounded expected return on the stock is 14% and recall that the continuously compounded risk-free rate is 6%. What is the annualized expected return on the option and stock position? **Hint:** *Compute first the elasticity of the position, as a weighted average of individual component elasticities. Also, keep in mind that the elasticity of the stock is 1.*

Expected Return

5 (20 points) IttyGroup issues stock with a current share price of \$100. This stock (known as "Dividends-In-Reverse Equity", or DIRE) will require shareholders to **pay** \$20 to the firm 6 months from today. (This is a new kind of stock issued by impoverished financial firms.) This is the only cash flow associated with the stock during the year. The continuously-compounded annual interest rate is 8%.

a. (5 points) Assume a shareholder receives one share of IttyGroup today **but pays in one year**. What is the price that the shareholder will pay in one year?

Price in one year

b. (5 points) Assume a shareholder pays today **but receives one share of IttyGroup in one year**. What is the price that the shareholder pays today (i.e., the one-year **prepaid** forward price)?

Price today

c. (5 points) What is the one-year forward price for the stock?

Forward price

d. (5 points) Suppose that you short the IttyGroup stock and plan to hold the short position for one year. What happens at the time of the reverse dividend? (In the answer, consider both the lender and the short-seller.)

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Solutions to Take Home Exam (FEMBA)

A general comment: if you got the wrong answer and failed to show your work, there was no way to give partial credit.

1 a. $\ln(1.095) = 0.0908$. You can check this by verifying that $e^{0.0908} = 1.095$.

b. We know that $916.3467 = 900e^{(0.04-\delta)\times 0.75}$. Solving for δ, we have

$$\ln(916.3467/900) = (0.04 - \delta) \times 0.75$$

which implies that

$$\delta = 0.04 - \ln(916.3467/900)/0.75 = 0.016$$

c. You can view the first option as an ABC call where the strike is DEF, and the second as an ABC put where the strike is DEF. Thus, by put-call parity, we have

$$\$22 = \text{Put} + 80 - 70 \implies \text{Put} = \$12$$

d. You have a strangle and you are in between the strikes (both options are out-of-the money). Your profit comes from a big move. If the stock price doesn't move, you would expect to lose money. (Since both options are out-of-the money, their value will decrease as we approach maturity.)

(You would get credit for any plausible story that you have. For example, you could possibly show a case where the position loses money.

e. You have a calendar spread, so you take advantage of time decay and make money.

f. Comparing the swap price to the forward prices, there is an implied loan of $3 - 2.7556 = 0.2444$ in year one and $2.50 - 2.7556 = 0.2556$ in year two. The implied interest rate is $0.2556/0.2444 - 1 = 0.046$.

2 a. The position with puts is as follows:

Strike	80	90	100	110	120	stock
Puts	-1	+1	-2	+1	-1	-1

b. The position with calls is as follows:

Strike	80	90	100	110	120	stock
Calls	-1	+1	-2	+1	-1	+1

c. We use the zero-profit point at stock price of \$70.5 to back-up the premium of the position. At \$70.5 none of the calls have value at maturity. Thus, the payoff of the position at \$70.5 is \$70.5. The profit is then

$$\begin{aligned} \$70.5 - \text{Future Value(Premium)} &= 0 \\ \$70.5 - \text{Premium} \times e^{0.08} &= 0 \end{aligned}$$

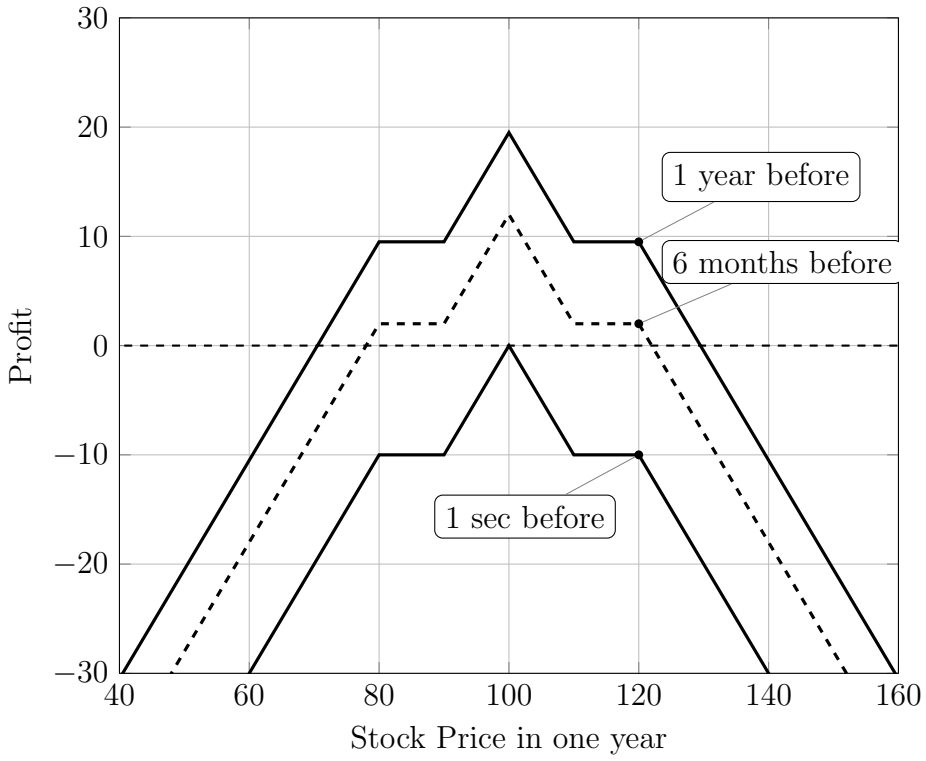
It follows that the premium of the position is $e^{-0.08} \times \$70.5 = \$65.08$.

d. Say we are one second before maturity and the stock price is \$100. The payoff of the strategy in one second is \$90. Thus, the premium of the position one second before maturity should be very close to \$90, whereas the future value of the premium should be \$90.

Say now we are in between, at 6 months before maturity, and the stock price is \$100. The future value of the premium of the position should be between \$70.5 (future value of the premium one year before maturity, see previous point) and \$90 (future value of the premium one second before maturity). This is illustrated in the diagram below.

Thus, the profit diagram would shift down compared with the diagram above.

The positions created with puts must behave identically to those created with calls, because both have the same profit diagram.



3 a. The per-period volatility is $0.5 \times \ln(u/d) = 0.5 \times \ln(4) = 0.6931$. Annualized volatility is $0.6931 \times \sqrt{12/5} = 1.0738$, or 107.38%.

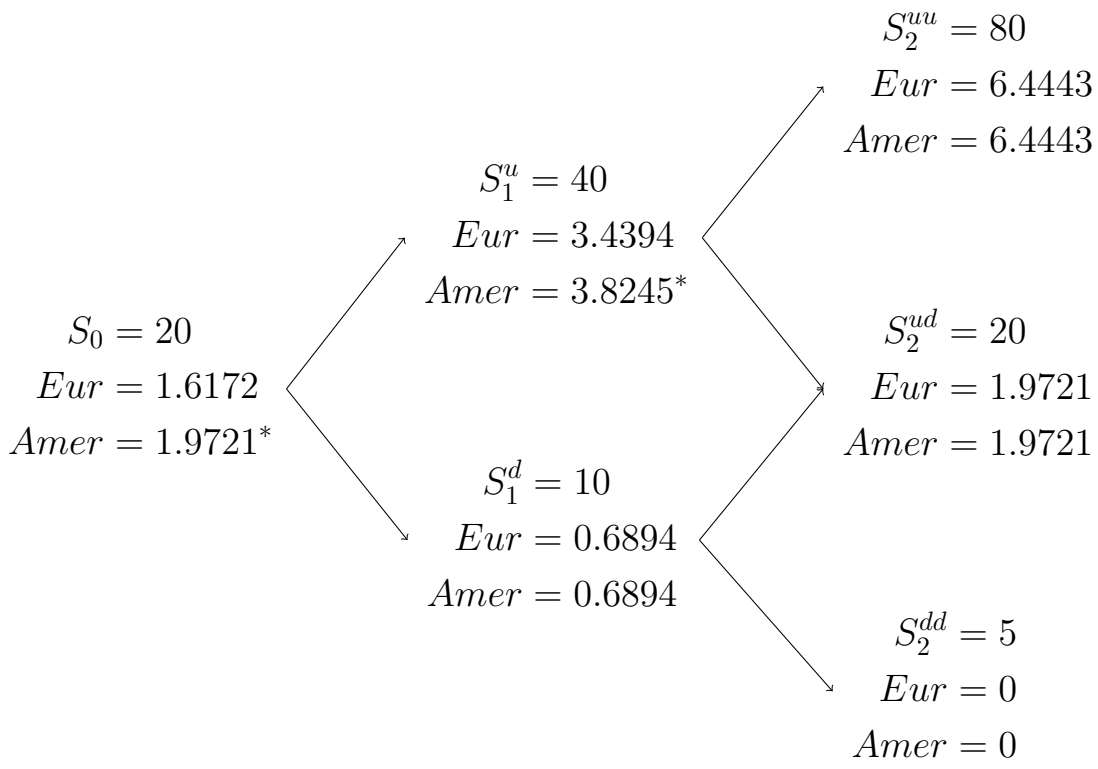
b. The risk-neutral probability of an up move is

$$p = \frac{e^{0.12 \times 0.4167} - 0.5}{2.0 - 0.5} = 0.3675$$

c. This claim is valued exactly like any other option. The expiration payoff is $\max(0, \sqrt{S_T} - \$2.50)$. Then you work backward. u and d are 2 and 0.5, respectively. For example, when the stock price is 40, the price of the European claim is

$$e^{-.12 \times 0.4167} \left[0.3675 \times 6.4442 + (1 - 0.3675) \times 1.9721\right] = 3.4394$$

The value of the American claim is \$3.8245, since $\sqrt{40} - 2.50 = 3.8245 > 3.4394$.



4 a. The delta of the position is

$$0.3926 - 0.5967 + 0.20 = -0.0041$$

The change in value is approximately zero (you are almost delta-hedged).

b. Since $\sigma = 30\%$, the one-day change on a \$40 stock is approximately $\$40 \times 0.30 \times \sqrt{1/365} = \$0.6281$

c. If the share price does not move, there is time decay on the two options and interest on the cost of the position. The cost of the position is

$$\text{Cost} = 2.7099 + 7.4981 + 0.20 \times 40 = \$18.2081$$

One day's interest expense is

$$\$18.2081 \times (e^{0.06/365} - 1) = \$0.00299$$

Thus, the loss on the position when the stock price does not move is the sum of the thetas plus the interest expense:

$$-\$0.00669 - \$0.00605 - \$0.00299 = -\$0.01573$$

If you ignore here the interest expense and only compute (correctly) the sum of the thetas, you have full credit.

d. The gamma on the position is positive (both the purchased call and put have positive gammas), hence, a move in excess of the standard deviation will generate a profit.

e. The elasticity of the position is a weighted average of the individual component elasticities. Thus,

$$\text{Elasticity} = \frac{2.7099}{18.2081} \times 5.7957 + \frac{7.4981}{18.2081} \times (-3.1832) + \frac{0.20 \times 40}{18.2081} \times 1 = -0.0089$$

Thus, the expected return on the position is

$$r + \Omega(r_M - r) = .06 + (-0.0089) \times (0.14 - 0.06) = 0.0593$$

If you don't ignore the borrowing, the position has zero investment and the notion of a rate of return is not well-defined, although in that case the expected *dollar* return will be the risk premium.

5 a. The current share price is \$100. In one year the shareholder will have to pay a higher price, due to interest rate cost. Price in one year is $\$100 \times e^{0.08} = \$108.3287$.

b. The \$20 payment is a negative dividend; by holding the prepaid forward rather than the stock you benefit by virtue of not having to pay the \$20. The one-year prepaid forward price is $S_0 - PV_{0,0.5}(\text{Div}) = \$100 - (-\$20) \times e^{-0.08 \times 0.5} = \$119.2158$.

c. The one-year forward price is the future value of the one-year prepaid forward price, $e^{0.08} \times 119.258 = 129.1449$.

d. If you are short the stock at the time of the reverse dividend, the share *lender* will have to pay \$20 to the share borrower. This leaves the share lender in the same positoin as if they still had the share, and compensates the borrower for the price increase due to the reverse dividend.