Investments - Midterm

Daniel Andrei

Tuesday, April 13 2010, 13h00

Solution

Duration: 2 hours. Documentation and calculators authorized

Problem 1 (Multiple Choice, 30 points)

One correct answer per question. You do not need to justify your answer.

Exercise 1. Which of the following is NOT a characteristic of a money market instrument?

a. liquidity
b. marketability
c. long maturity
d. liquidity premium
*e. c and d

Money market instruments are short term instruments with high liquidity and marketability; they do not have long maturities nor pay liquidity premiums.

Exercise 2. Commercial paper is a short-term security issued by ___ to raise funds.

a. the Federal Reserve Bank
b. commercial banks
*c. large established firms
d. the New York Stock Exchange
e. state and local governments

Commercial paper is unsecured financing issued directly by large, presumably safe corporations.

Exercise 3. The stocks on the Dow Jones Industrial Average

a. have remained unchanged since the creation of the index.
b. are very representative of the entire market.
*c. are changed occasionally as circumstances dictate.
d. consist of stocks on which the investor cannot lose money.
e. b and c.

The stocks on the DJIA are not representative of the entire market, have been changed occasionally since the creation of the index, and one can lose money on any stock.

Exercise 4. In the mean-standard deviation graph, which one of the following statements is true regarding the indifference curve of a risk averse investor?

a. it is the locus of portfolios that have the same expected rates of return and different standard deviations.

b. it is the locus of portfolios that have the same standard deviations and different rates of return.

*c. it is the locus of portfolios that offer the same utility according to returns and standard deviations.

d. it connects portfolios that offer increasing utilities according to returns and standard deviations.

e. none of the above.

Indifference curves plot trade-off alternatives that provide equal utility to the individual (in this case, the trade-offs are the risk-return characteristics of the portfolios).

Exercise 5. A portfolio has an expected rate of return of 15% and a standard deviation of 15%. The risk-free rate is 6%. An investor has the following utility function utility function: $U = E(r) - (A/2)\sigma^2$. Which value of A makes this investor indifferent between the risky portfolio and the risk-free asset?

a. 5
b. 6
c. 7
*d. 8
e. none of the above

$$\begin{aligned} 0.06 &= 0.15 - \frac{A}{2}0.15^2 \\ A &= 8 \end{aligned}$$

Exercise 6. Covariance

a. measures the extent to which the returns of two risky assets move together.
b. may be interpreted as to sign only.
c. may be interpreted as to both sign and magnitude.
*d. a and b.
e. a and c.

Covariance may not be interpreted as to magnitude; the correlation coefficient my be interpreted both as to sign and magnitude.

Exercise 7. Use the following information to answer questions 7 to 10. You invest $100 in a risky asset with an expected rate of return of 12% and a standard deviation of 15% and a T-bill with a rate of return of 5%. What percentage of your money must be invested in the risky asset and the risk-free asset, respectively, to form a portfolio with an expected return of 9%?

a. 85% and 15%
b. 75% and 25%
c. 67% and 33%
*d. 57% and 43%
e. cannot be determined

$$\begin{aligned} 0.09 &= \omega 0.12 + (1-\omega)\, 0.05 \\ \omega &= 0.57 \end{aligned}$$

Exercise 8. Use the information from Exercise 7. What percentages of your money must be invested in the risk-free asset and the risky asset, respectively, to form a portfolio with a standard deviation of 9%?

a. 30% and 70%
b. 50% and 50%
c. 60% and 40%
*d. 40% and 60%
e. cannot be determined

$$\begin{aligned} 0.09 &= 0.15\omega \\ \omega &= 0.6 \end{aligned}$$

Exercise 9. Use the information from Exercise 7. A portfolio that has an expected outcome of $115 is formed by

a. investing $100 in the risky asset.
b. investing $80 in the risky asset and $20 in the risk-free asset.
*c. borrowing $43 at the risk-free rate and investing the total amount ($143) in the risky asset.
d. borrowing $33 at the risk-free rate and investing the total amount ($133) in the risky asset.
e. cannot be formed

$$\begin{aligned} 1.15 &= \omega 1.12 + (1-\omega)\,1.05 \\ \omega &= 1.43 \end{aligned}$$

Exercise 10. Use the information from Exercise 7. The slope of the Capital Allocation Line formed with the risky asset and the risk-free asset is equal to

*a. 0.4667
b. 0.8000
c. 0.5667
d. 0.4000
e. cannot be determined

$$(0.12 - 0.05)/0.15 = 0.4667$$

Exercise 11. Use the following information to answer questions 11 to 12. Consider two perfectly negatively correlated risky securities A and B. A has an expected rate of return of 15% and a standard deviation of 18%. B has an expected rate of return of 8% and a standard deviation of 12%. The weights of A and B in the global minimum variance portfolio are ___ and ___, respectively.

a. 0.60; 0.60
b. 0.40; 0.40
c. 0.60; 0.40
*d. 0.40; 0.60
e. none of the above

$$\begin{aligned} [\omega(18\% + 12\%) - 12\%]^2 &= 0 \\ \omega &= 0.4 \end{aligned}$$

Exercise 12. Use the information from Exercise 11. The risk-free portfolio that can be formed with the two securities will earn ___ rate of return.

a. 13.8%
b. 12.2%
*c. 10.8%
d. 9.9%
e. none of the above

$$E(r_p) = 0.4(15\%) + 0.6(8\%) = 10.8\%$$

Exercise 13. According to the Capital Asset Pricing Model (CAPM), the expected rate of return on any security is equal to

a. $r_f + \beta E(r_M)$
b. $r_f + \sigma[E(r_M) - r_f]$
*c. $r_f + \beta[E(r_M) - r_f]$
d. $E(r_M) + r_f$
e. none of the above

The expected rate of return on any security is equal to the risk free rate plus the systematic risk of the security (beta) times the market risk premium, $E(r_M - r_f)$.

Exercise 14. You invest $600 in security A with a beta of 1.2 and $400 in security B with a beta of 0.90. The beta of the resulting portfolio is

a. 1.05
b. 1.02
c. 1.10
*d. 1.08
e. none of the above

$$0.6(1.2) + 0.4(0.90) = 1.08$$

Exercise 15. The exploitation of security mispricing in such a way that risk-free economic profits may be earned is called ___

*a. arbitrage
b. capital asset pricing
c. factoring
d. fundamental analysis
e. none of the above

Arbitrage is earning of positive profits with a zero (risk-free) investment.

Exercise 16. Consider the multifactor APT. The risk premiums on the factor 1 and factor 2 portfolios are 5% and 3%, respectively. The risk-free rate of return is 10%. Stock A has an expected return of 19% and a beta on factor 1 of 0.8. Stock A has a beta on factor 2 of ___.

a. 1.33
b. 1.50
*c. 1.67
d. 2.00

e. none of the above

$$
\begin{aligned}
0.19 &= 0.1 + 0.05\,(0.8) + 0.03\,(x) \\
x &= 1.67
\end{aligned}
$$

Exercise 17. If the currency of your country is depreciating, the result should be to ___ exports and to ___ imports

a. stimulate, stimulate
*b. stimulate, discourage
c. discourage, stimulate
d. discourage, discourage
e. not affect, not affect

Depreciating currency means that country's goods and services are cheaper and thus that country's exports are stimulated. Likewise, goods and services of other countries are now more expensive; and thus imports are discouraged.

Exercise 18. ___ are analysts who use information concerning current and prospective profitability of a firm to assess the firm's fair market value

a. credit analysts
*b. fundamental analysts
c. systems analysts
d. technical analysts
e. specialists

Fundamentalists use all public information in an attempt to value a stock (while hoping to identify undervalued securities).

Exercise 19. Active portfolio managers try to construct a risky portfolio with ___.

*a. a higher Sharpe measure than a passive strategy
b. a lower Sharpe measure than a passive strategy
c. the same Sharpe measure as a passive strategy
d. very few securities
e. none of the above

A higher Sharpe measure than a passive strategy is indicative of the benefits of active management

Exercise 20. Shares of several foreign firms are traded in the U.S. markets in the form of ___.

*a. ADRs
b. ECUs
c. single-country funds
d. all of the above
e. none of the above

American Depository Rights (ADRs) allow U.S. investors to invest in foreign stocks via transactions on the U.S. stock exchanges.

Problem 2 (Short Questions, 25 points)

Question 1

(1) Fill in the table below the coefficients of relative risk aversion and absolute risk aversion for the different utility functions, as I did for the logarithmic utility case.

(2) Explain intuitively the difference between decreasing and increasing absolute risk aversion. Experimental and empirical evidence is mostly consistent with decreasing or increasing absolute risk aversion?

Name	Function	Relative Risk Aversion	Absolute Risk Aversion
Exponential	$-e^{-aR_p}$	aR_p	a
Logarithmic	$\log(R_p)$	1	$1/R_p$
CRRA	$\frac{R_p^{1-\rho}}{1-\rho}$	ρ	ρ/R_p
Quadratic	$R_p - \frac{b}{2}R_p^2$	$\frac{bR_p}{1-bR_p}$	$\frac{b}{1-bR_p}$

Solution

(2) Decreasing absolute risk aversion means that absolute risk aversion decreases as wealth increases, or greater acceptance of risky situations with greater wealth. Increasing absolute risk aversion is the opposite. Experimental and empirical evidence is mostly consistent with decreasing absolute risk aversion.

Question 2

(1) The table below outlines the steps involved when using a constant mix strategy. The T-bills rate is 0%. The first two lines are already completed. Fill in the next lines.

(2) What shape has the payoff of the constant mix strategy? Is this the correct strategy in this case and why? If not, what would be your strategy?

Case	Stock Market	Stock	T-bills	Assets	% in Stocks
Initial	100	60.00	40.00	100	60.0%
After Change	90	54.00	40.00	94.00	57.4%
After Rebalancing	90	56.40	37.60	94.00	60.0%
After change	100	62.67	37.60	100.27	62.5%
After Rebalancing	100	60.16	40.11	100.27	60.0%

Solution

(2) Constant mix requires the sale of stocks as they rise and the purchase of

stocks as they fall. It has a concave payoff. This is the correct strategy when the market ends up near its starting point (reversal), as in this case.

Question 3

Consider an equally-weighted portfolio with N assets. The weight of each of the N assets is $\omega_n = 1/N$. From the class notes, we know that the variance of this portfolio can be written as

$$\sigma_p^2 = \frac{1}{N}\bar{\sigma^2} + \frac{N-1}{N}\bar{\sigma} \tag{1}$$

where $\bar{\sigma^2}$ is the average of the N asset return variances, and $\bar{\sigma}$ is the average of the $N(N-1)$ asset return covariances. Intuitively, the average correlation in asset returns will be approximately $\bar{\rho} \approx \bar{\sigma}/\bar{\sigma^2}$. Thus, the risk of a well-diversified portfolio depends on the average variances and correlations of the assets included in the portfolio. Consider 2 cases: $(i)\bar{\sigma^2} = 0.2$ and $\bar{\rho} = 0$ and (ii) $\bar{\sigma^2} = 0.2$ and $\bar{\rho} = 0.4$. Compute, for each of the two cases, the minimum number of assets needed to have a portfolio variance equal to half of the average variance of the assets, i.e. $\sigma_p^2 = \bar{\sigma^2}/2$. Comment and interpret your results.

Solution

In both cases N solves the equation

$$\begin{aligned}\frac{1}{N}\bar{\sigma^2} + \frac{N-1}{N}\bar{\rho}\bar{\sigma^2} &= \frac{1}{2}\bar{\sigma^2} \\ \frac{1-\bar{\rho}+\bar{\rho}N}{N} &= \frac{1}{2} \\ N &= 1 + \frac{1}{1-2\rho}\end{aligned}$$

We find N=2 in the first case and N=6 in the second case. Intuition: if the average correlation is larger, more assets are needed to diversify to the same level of variance, $\sigma_p^2 = \bar{\sigma^2}/2$.

Question 4

The Treasury bill rate is 4%, and the expected return on the market portfolio is 12%. On the basis of the Capital Asset pricing Model:

1. What is the risk premium on the market?
2. What is the required return on an investment with a beta of 1.5?
3. If the market expects a return of 11.2% from stock XYZ, what is its beta?

Solution

(1) $r_M - r_f = 0.12 - 0.04 = 0.08$,

(2) using the Security Market Line we find $r = 0.16$,

(3) using the same relationship we find $\beta = 0.9$.

Question 5

The following question illustrates the APT. Imagine that there are only two pervasive macroeconomic factors. Investments X, Y, and Z have the following sensitivities to these factors:

Investment	b_1	b_2
X	1.75	0.25
Y	-1.00	2.00
Z	2.00	1.00

We assume that the expected risk premium is 4% on factor 1 and 8 percent on factor 2. Treasury bills obviously offer zero risk premium.

1. According to APT, what is the risk premium on each of the three stocks?
2. Suppose you buy $200 of X and $50 of Y and sell $150 of Z. What is the sensitivity of your portfolio to each of the two factors? What is the expected risk premium?

Solution
(1) $r_X = 0.09$*,* $r_Y = 0.12$*,* $r_Z = 0.16$*,*

(2) We invest 200% in stock X, 50% in stock Y and -150% in stock Z. The sensitivities of this portfolio to the factors are 0 to factor 1 and 0 to factor 2. Because the sensitivities are both zero, the expected risk premium is zero.

Problem 3 (CAPM, 20 points)

You are given the following information about possible investments:

Asset	Mean Return	Std. Dev.	Correlation with Market
Value Stocks	18%	30%	1.0
Growth Stocks	???%	20%	0.5
Gold	???%	20%	-0.5
T-bills	6%	0%	0.0

1. If the market standard deviation is 20%, what are the CAPM betas of each of these assets?
   Solution
   Use the fact that:
   $$\beta = \frac{Cov\left(r_j, r_m\right)}{\sigma_m^2} = \frac{\rho_{jm}\sigma_j}{\sigma_m}$$
   The value stock beta is 1.5, growth stock beta is 0.5, gold beta is -0.5 and T-bill beta is 0.

2. Assume all assets are priced correctly according to the CAPM. What are the expected returns of the market, growth stocks and gold?
   Solution
   For value stock the SML implies:
   $$0.18 = 0.06 + 1.5\left[\mathbb{E}\left[r_m\right] - 0.06\right]$$

which yields an expected return on the market equal to 14%. For Growth stocks the SML implies:

$$0.06 + 1.5\,[0.14 - 0.06] = 0.10$$

For Gold the SML implies:

$$0.06 - 0.5\,[0.14 - 0.06] = 0.02$$

3. In addition to the assets described above you are told that the expected return on ABC, Inc. equity shares is -14% and its beta is -2. Does the market correctly price this firm? If your answer is yes the proceed to Problem 4, otherwise explain whether it is over or undervalued and how you would take advantage of this mispricing.
   Solution
   ABC's expected return -14%, while the CAPM predicts that the expected return should be
   $$0.06 - 2.0\,[0.14 - 0.06] = -0.10$$
   Thus, the expected return is too low implying that ABC is overpriced.
   To take advantage of this mispricing, we should sell ABC's shares and buy a portfolio with the same beta as ABC. There are many ways to create such a portfolio. For example, we can use a portfolio of the market and T-bills by matching the portfolio beta to that of ABC.
   $$\begin{aligned} -2 &= (1-\omega)\,\beta_{riskless} + \omega\beta_{Market} \\ -2 &= (1-\omega)\,0 + \omega 1 \\ -2 &= \omega \end{aligned}$$
   Thus our portfolio should contain -200 percent of our wealth in the market and 300% in T-bills. The expected gain on this strategy is 4%.

4. Is the portfolio that you constructed in part (3) riskless? That is, do you expect to gain a profit for sure next period?
   Solution
   No. We eliminated systematic risk but there is still residual risk. On average we expect to make a positive expected return but we are still exposed to the idiosyncratic risk of the firm's return.

Problem 4 (APT, 25 points)

Suppose that the following two-index model describes returns:

$$R_i = a_i + b_{i1}I_1 + b_{i2}I_2 + e_i \tag{2}$$

Furthermore, assume that $\mathbb{E}\,[e_i e_j] = 0$. As shown in class, if an investor holds a well-diversified portfolio, residual risk will tend to go to zero and only systematic risk will matter. The only terms in the preceding equation that affect the systematic risk in a portfolio are b_{i1} and b_{i2}. Let us hypothesize the existence of three widely diversified portfolios shown in the following table.

Portfolio	Expected Return $\bar{R}_i$	b_{i1}	b_{i2}
A	15%	1.0	0.6
B	14%	0.5	1.0
C	10%	0.3	0.2

1. We know from the concepts of geometry that three points determine a plane just as two points determine a line. Recall that the equation of a plane can be written as

$$R_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} \tag{3}$$

   Find the equation of the plane in $\bar{R}_i$, b_{i1}, and b_{i2} space defined by these three portfolios (Hint: by substituting in the values of $\bar{R}_i$, b_{i1}, and b_{i2} for portfolios A, B, and C, obtain three equations with three unknowns: λ_0, λ_1, and λ_2).
   Solution
   The equation of the plane is

$$\bar{R}_i = 0.0775 + 0.05 b_{i1} + 0.0375 b_{i2}$$

2. Since a weighted combination of points on a plane *(where the weights sum to one)* also lies on the plane, all portfolios constructed from portfolios A, B, and C lie on the same plane. Assume a portfolio E exists with an expected return of 15%, a b_{i1} of 0.6, and a b_{i2} of 0.6. Find a combination of portfolios A, B, and C (call it portfolio D) that has the same systematic risk as portfolio E.
   Solution
   You can construct portfolio D by placing 1/3 of the funds in portfolio A, 1/3 in portfolio B, and 1/3 in portfolio C. The b_{pj}s of the portfolio are

$$\begin{aligned} b_{p1} &= \frac{1}{3}(1.0) + \frac{1}{3}(0.5) + \frac{1}{3}(0.3) = 0.6 \\ b_{p2} &= \frac{1}{3}(0.6) + \frac{1}{3}(1.0) + \frac{1}{3}(0.2) = 0.6 \end{aligned}$$

   Thus, the risk for portfolio D is identical to the risk of portfolio E.

3. Find the expected return on portfolio D in two different ways.
   Solution
   The expected return on portfolio D is

$$\frac{1}{3}(15\%) + \frac{1}{3}(14\%) + \frac{1}{3}(10\%) = 13\%$$

   Alternatively, since portfolio D must lie on the plane described above, we could have obtained its expected return from the equation of the plane:

$$\bar{R}_i = 0.0775 + 0.05\,(0.6) + 0.0375\,(0.6) = 0.13$$

4. Do you have an arbitrage strategy? If yes, construct the arbitrage strategy by using portfolios D and E. What is the riskless gain of the arbitrage strategy? What it will happen with the expected return of portfolio E?

Solution

In this situation it would pay arbitrageurs to step in and buy portfolio E while selling an equal amount of portfolio D short. This would guarantee a riskless profit with no investment and no risk. Assume the investor sells $100 worth of portfolio D short and buys $100 worth of portfolio E. The results are shown in the following table:

	Initial Cash Flow	End of Period Cash Flow	b_{i1}	b_{i2}
Portfolio D	+$100	-$113.0	-0.6	-0.6
Portfolio E	-$100	$115.0	0.6	0.6
Arbitrage Portfolio	$0	$2.0	0	0

The arbitrage portfolio involves zero investment, has no systematic risk (b_{i1} and b_{i2}), and earns $2. Arbitrage would continue until portfolio E lies on the same plane as portfolios A, B, and C.