Investments
Final Exam - 2 hours
Closed-book exam
Pocked calculator authorized

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True-False Questions (Answer by True or False. You do not need to justify your answer.)

1. Suppose that the yield-to-maturity on a 3-year coupon Treasury-bond is equal to 5.75%, the yield to-maturity on a one-year zero-coupon bond is 5.50%, and the 2-year sport rate is 5.65%. Then, the 3-year spot rate must be lower than 5.75%. True or False? **FALSE.**

2. Two corporate bonds with same maturity and same rating have the same yield to maturity. True or False? **FALSE.**

3. For premium bonds, bonds selling above par, the coupon rate is lower than the yield-to-maturity. True or False? **FALSE.**

4. If the term structure of interest rates is upward sloping then the 5-year spot rate is higher than the forward rate corresponding to the period between year 4 and 5. True or False? **TRUE.**

5. Prices of long-term bonds tend to be more sensitive to interest rate changes than prices of short-term bonds. True or False? **TRUE.**

6. The duration is a good approximation of the volatility of a corporate bond. The volatility of a corporate bond reflects its risk. Thus it is correct to say that a bond with a higher duration is riskier than a bond with a lower duration. True or False? **FALSE.**

7. Consider two bonds with the same time to maturity and the same value, but whose only difference is the coupon rates. Then, the bond with the higher coupon rate will have a lower duration. True or False? **TRUE.**

8. The duration of a coupon bond is higher when the bond's yield to maturity is lower. True or False? **TRUE.**

9. The price of a callable bond is always greater than the price of a straight bond. True or False? **FALSE.**

10. The price of a puttable bond is always greater than the price of a straight bond. True or False? **TRUE.**

Problem 1

Assume that the short rate r_t has the following dynamics:

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

where a, b, and σ are all positive constants and W_t is a standard Wiener process.

1. Using Itô's lemma, solve the above equation.
2. Compute $\mu_t \equiv E[r_t]$.
3. Compute $\sigma_t^2 \equiv V[r_t]$.

Solution

The short rate dynamics is given by:

$$dr_t = a(b - r_t)dt + \sigma dW_t$$

where a, b, and σ are all positive constants. You can solve this equation by applying Itô formula on a new variable y:

$$y = f(r,t) = e^{at}r$$

where we have $\frac{\partial f}{\partial r} = e^{at}$, $\frac{\partial^2 f}{\partial r^2} = 0$, and $\frac{\partial f}{\partial t} = ae^{at}r$. Now we want to find the dynamics of y. Using Itô formula we can write:

$$\begin{aligned}
dy &= \tfrac{\partial f}{\partial t}dt + \tfrac{\partial f}{\partial r}dr + \tfrac{1}{2}\tfrac{\partial^2 f}{\partial r^2}(dr)^2 \\
&= ae^{at}rdt + e^{at}\left[a(b-r)dt + \sigma dW\right]dt + 0 \\
&= ae^{at}rdt + e^{at}a(b-r)dt + e^{at}\sigma dW \\
&= ae^{at}rdt + e^{at}abdt - e^{at}ardt + e^{at}\sigma dW \\
&= abe^{at}dt + e^{at}\sigma dW
\end{aligned}$$

Now we integrate:

$$\begin{aligned}
&\textstyle\int_0^t dy_s = \int_0^t (abe^{au}du + e^{au}\sigma dW_u) \\
&\textstyle y_t - y_0 = \int_0^t abe^{au}du + \int_0^t e^{au}\sigma dW_u \\
&\textstyle e^{at}r_t - r_0 = \int_0^t abe^{au}du + \int_0^t e^{au}\sigma dW_u \\
&\textstyle r_t = e^{-at}\left(r_0 + \int_0^t abe^{au}du + \int_0^t e^{au}\sigma dW_u\right)
\end{aligned}$$

Using Itô's formula, we have shown that:

$$r_t = e^{-at}\left[r_0 + \int_0^t abe^{au}du + \sigma \int_0^t e^{au}dW_u\right]$$

Note further that:

$$\begin{aligned} r_t &= e^{-at}\left[r_0 + b(e^{at} - 1) + \sigma \int_0^t e^{au}dW_u\right] \\ &= \mu_t + \sigma \int_0^t e^{a(u-t)}dW_u \end{aligned}$$

where μ_t is a deterministic function. Clearly, $E[r_t] = \mu_t$. Similarly, we have:

$$\begin{aligned} \sigma_t^2 \equiv V[r_t] &= E\left[\left(\sigma e^{-at}\int_0^t e^{au}dW_u\right)^2\right] \\ &= \sigma^2 e^{-2at}E\left[\int_0^t e^{2au}du\right] \text{ by Itô's isometry} \\ &= \sigma^2\left(\frac{1-e^{-2at}}{2a}\right) \end{aligned}$$

Problem 2

A financial institution has raised $5 million by selling a number of 3-year zero-coupon bonds to individuals. These bonds have a yield-to-maturity of 7.5%. The institution has used the proceeds to buy a number of long-term coupon bonds. These bonds have a duration of 15 years and a yield-to-maturity of 9.25%. Assume that yields of all bonds decrease by 25 basis points.

1. Compute the change in value of the zero-coupon bonds.
2. Compute the change in value of the long term bonds.
3. Compute the change in value of the overall position of the financial institution.
4. What would be the change in value of the overall position of the financial institution if yields of all bonds increase by 25 basis points?

Solution

The initial investment is of 5 millions (this is 100% of the investment). On the liability side we have 3-year zero-coupon bonds with yield to maturity of 7.5%. On the assets side we have long term bonds with a duration of 15 years and a yield to maturity of 9.25%.

If yields of all bonds decrease by 25 basis points then we have:
- change in value of zero coupon bonds: $\frac{\Delta P}{P} = -\frac{3}{1+0.075}(-0.0025) = 0.6978\%$
- change in value of long term bonds: $\frac{\Delta P}{P} = -\frac{15}{1+0.0925}(-0.0025) = 3.4325\%$.

Then the overall position is as follows:

$$V = V_0 + \Delta assets + \Delta liabilities = 100\% - 0.6978\% + 3.4325\% = 102.7374\%.$$

We observe that because the duration of assets is longer than the duration of liabilities, the overall position will increase when the yields decrease.

Similarly, if we observe a increase by 25 basis points of all bond yields, the overall position will be:

$$V = V_0 + \Delta assets + \Delta liabilities = 100\% + 0.6978\% - 3.4325\% = 97.2653\%.$$

In this case, because the duration of assets is linger than the duration of liabilities, the overall position will decrease when the yields increase.

Problem 3

Assume that the Modigliani-Miller framework holds. In this setup, you can use Merton's model to compute the value of equity. Suppose that firm A is characterized by the following parameter values: $V_t = 100$, $F = 60$, $r = 6\%$, $\sigma = 30\%$, and $T = 5$ years.

1. What are the market values of equity and debt for firm A?
2. What interpretation can you give to $\mathcal{N}(d_2)$ and $Fe^{-rT}\mathcal{N}(d_2)$?
3. Assume that firm A is planning to merge with firm B and that this merger is going to reduce the volatility from 30% to 15%. What is the impact of the change in volatility on the value of equity? What happens to the value of corporate debt?
4. What is the minimum synergy gain (variation in V) that would make the merger a positive NPV project for shareholders of firm A? (For this question, you can use the fact that if you change the value of the firm's assets by \$1, you change the value of equity by $\$\mathcal{N}(d_1)$) (**For simplicity, assume that $\mathcal{N}(d_1)$ is constant and use the value of $\mathcal{N}(d_1)$ computed above in question 2.**)

Solution

Solution Q1

The value of equity is given by the value of a call option written on V with exercise price the face value of debt D. Using Black and Scholes formula we get

$$E = V_t\mathcal{N}(d_1) - Fe^{-r(T-t)}\mathcal{N}(d_2)$$

with

$$\begin{aligned}\mathcal{N}(d_1) &= 0.9387 \\ \mathcal{N}(d_2) &= 0.8087\end{aligned}$$

or

$$\begin{aligned}E &= 100 \times 0.9387 - 60 \times 0.74082 \times 0.8087 \\ &= 57.92\end{aligned}$$

According to the Modigliani-Miller theorem, the value of debt is then given by

$$\begin{aligned}D &= V_t - E \\ &= 42.08\end{aligned}$$

$\mathcal{N}(d_2)$ is the probability that the firm will not default on its debt obligations. $Fe^{-rT}\mathcal{N}(d_2)$ is the expected present value of the debt repayment.

Solution Q2

When volatility is more important, we have

$$\begin{aligned} \mathcal{N}(d_1) &= 0.9951 \\ \mathcal{N}(d_2) &= 0.9878 \end{aligned}$$

and the values of equity and debt are respectively given by

$$\begin{aligned} E &= 55.60 \\ D &= 44.40 \end{aligned}$$

The value of equity decreases. The value of corporate debt rises. Therefore, for this merger to be a positive NPV projects, there must be strictly positive synergy gains.

Solution Q3

The minimum synergy gain ensuring that the merger is a positive NPV is such that the merger satisfies

$$dE = 0.$$

Therefore, because a change of \$1 in V results in a change of \$$\mathcal{N}(d_1)$ in E, the change in the value of V must satisfies

$$dV \geq \frac{Equity\,(\sigma = 0.30) - Equity\,(\sigma = 0.15)}{\mathcal{N}(d_1)} = \frac{2.32}{0.9951} = 2.33$$

since we assume that $\mathcal{N}(d_1)$ is constant. (Note that because d_1 is high, the non-linearity in the cumulative standard normal distribution function does not affect the result.)

Problem 4

Denote by r the risk-free rate. Denote by X_t the operating cash flow of the firm. Under the risk-neutral measure, the dynamics of X_t are as follows:

$$dX_t = \mu X_t dt + \sigma X_t dW_t$$

where μ and σ are constants and W_t is the standard Wiener process. The firm has to pay corporate income taxes. The tax rate is τ. Because there are taxes, the firm has an incentive to issue debt. Debt is characterized by a perpetuity with coupon payments c. If the firm is unable to meet its contactual debt obligations, the firm enters into default. The firm defaults on its debt obligations the first time its cash flows reach a constant threshold X_B. In this setup, default can lead either to renegotiation of the debt contract or to liquidation of firm assets. Denote the proportional cost of renegotiation by κ and the proportional cost of liquidation by α. Because liquidation is socially more costly than reorganization ($\alpha > \kappa$), there is a surplus $(\alpha - \kappa)\Pi(X_B)$ associated with renegotiation, where $\Pi(X_B)$ represents the present value of a perpetual stream of cash flows $(1-\tau)X_t$ starting at $X_t = X_B$. Thus, at the time of default, claimholders bargain over the sharing of firm value and debt renegotiation leads to a debt-equity swap. Assume that shareholders can extract a fraction η of the surplus. Thus the value of equity at time of default is: $\eta(\alpha-\kappa)\Pi(X_B)$, while the value of debt at time of default is: $(1-\eta)(\alpha-\kappa)\Pi(X_B) + (1-\alpha)\Pi(X_B)$.

1. Determine the particular solution for the value of debt D.
2. State the boundary conditions for the value of debt D.
3. Derive the value of debt D.
4. Determine the particular solution for the value of equity E.
5. State the boundary conditions for the value of equity E.
6. Derive the value of equity E.
7. Derive the optimal default threshold X_B satisfying the condition:

$$E_X(X)\big|_{X=X_B} = \eta(\alpha-\kappa)\Pi_{X_B}(X_B)$$

where we have the notation $f_x = \frac{\partial f}{\partial x}$.

8. Derive the value of the firm $V = E + D$.
9. Derive the optimal coupon c^* satisfying the condition:

$$\frac{\partial V}{\partial c} = 0$$

10. Without doing any calculations, use your economic intuition in order to sign the following expressions: $\frac{\partial c}{\partial \tau}$, $\frac{\partial c}{\partial \kappa}$, and $\frac{\partial c}{\partial \eta}$. Motivate your answer.

Problem 5

Suppose that the risk-free zero curve is flat at 7% per annum with continuous compounding and that defaults can occur half way through each year in a new 3-year CDS. Suppose that the recovery rate is 30% and the default probability each year conditional on no earlier default is 3%. Assume payments are made annually.

1. Estimate the CDS spread.
2. Consider the same characteristics but for a binary CDS, i.e. paying \$1 for a notional principal of \$1 in case of default, irrespective of the recovery rate. What is the binary CDS spread?

Solution

The unconditional default and survival probabilities are:

Time	Default probability	Survival probability
1	0.0300	0.9700
2	0.0291	0.9409
3	0.0282	0.9127

The present value of the expected regular payments (payment rate is s per year) is:

Time	Proba. survival	Expected payment	Discount factor	PV of exp. payment
1	0.9700	$0.9700s$	0.9324	$0.9044s$
2	0.9409	$0.9409s$	0.8694	$0.8180s$
3	0.9127	$0.9127s$	0.8106	$0.7389s$
Total				$2.4613s$

The present value of the expected payoff with notional principal=1\$ is:

Time	Proba. default	Recovery rate	Expected payoff	Discount factor	PV exp. payoff
1	0.0300	0.3	0.0210	0.9656	0.0203
2	0.0291	0.3	0.0204	0.9003	0.0183
3	0.0282	0.3	0.0198	0.8395	0.0166
Total					0.0552

The present value of the accrual payments is:

Time	Proba. default	Expected accrual payment	Discount factor	PV exp. acc payment
1	0.0300	$0.0150s$	0.9656	$0.0145s$
2	0.0291	$0.0146s$	0.9003	$0.0131s$
3	0.0282	$0.0141s$	0.8395	$0.0118s$
Total				$0.0394s$

The CDS spread s is given by:

$$2.4613s + 0.0394s = 0.0552$$

Finally, we obtain: $s = 221$pbs.

Solution

In the case of a binary swap, the present value of the expected payoff with notional principal=1$ is:

Time	Proba. default	Expected payoff	Discount factor	PV exp. payoff
1	0.0300	0.0300	0.9656	0.0290
2	0.0291	0.0291	0.9003	0.0262
3	0.0282	0.0282	0.8395	0.0237
Total				0.0789

The binary CDS spread s is given by:

$$2.4613s + 0.0394s = 0.0789$$

Finally, we obtain: $s = 315$pbs.

Appendix A

The bond price change due to a small change in interest rates can be approximated by:

$$\frac{\Delta P}{P} = -D \frac{\Delta y}{1+y}$$

where P is the bond price, y is the yield to maturity, and D is the duration of the bond.

Appendix B

Assume that the Modigliani-Miller framework holds. In this setup, the Merton model gives you the value of equity at time t, denoted by E_t, as:

$$E_t = V_t \mathcal{N}(d_1) - Fe^{-r(T-t)} \mathcal{N}(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{V_t}{F}\right) + \left(r + \sigma^2/2\right)(T-t)}{\sigma\sqrt{T-t}}, \; d_2 = d_1 - \sigma\sqrt{T-t}.$$

and

V_t is the value of the firm's assets
F is the face value of debt
r is the continuously compounded risk free rate
σ is the volatility of the return on the firm's assets
T is the time to maturity of debt
$\mathcal{N}(.)$ is the standard normal cumulative distribution function

Appendix C

Denote by r the risk-free rate. Denote by X_t the operating cash flow of the firm. Under the risk-neutral measure, the dynamics of X_t are as follows:

$$dX_t = \mu X_t dt + \sigma X_t dW_t$$

where μ and σ are constants and W_t is the standard Wiener process. Denote by F_t a contingent claim on X_t. The claim F_t receives a payout flow of PF_t. We know that the claim F_t satisfies the following ODE:

$$rF = PF + \tfrac{1}{2}\sigma^2 X^2 F_{xx} + \mu X F_x$$

The solution to this ODE is given by:

$$F(X) = \underbrace{AX^{\beta_1} + BX^{\beta_2}}_{\text{General Solution}} + \underbrace{F_{PS}}_{\text{Particular Solution}}$$

where A and B are constants and β_1 and β_2 are the positive and negative solutions to the quadratic equation

$$\tfrac{1}{2}\sigma^2 \beta(\beta - 1) + \mu\beta - r = 0$$

The solution to this quadratic equation is:

$$\beta_1 = \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} + \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}}$$
$$\beta_2 = \tfrac{1}{2} - \tfrac{\mu}{\sigma^2} - \sqrt{\left(\tfrac{1}{2} - \tfrac{\mu}{\sigma^2}\right)^2 + \tfrac{2r}{\sigma^2}}$$

The constants A and B are determined by the boundary conditions of the ODE. The term F_{PS} represents a particular solution of the ODE.

In addition, we have the following useful relations:

$$E\left[\int_0^\infty (1-\tau)\, e^{-rt} X_t dt\right] = \frac{1-\tau}{r-\mu} X_0 = \Pi\left(X_0\right)$$
$$E\left[\int_0^\infty e^{-rt} a dt\right] = \frac{a}{r}, \text{ where } a \text{ is a constant}$$

Appendix D

Table of the Standard Normal Cumulative Distribution Function $\mathcal{N}(z)$.

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.001	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.001	0.001
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.002	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.003	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.004	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.006	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.008	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.011
-2.1	0.0179	0.0174	0.017	0.0166	0.0162	0.0158	0.0154	0.015	0.0146	0.0143
-2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.025	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.063	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.102	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.123	0.121	0.119	0.117
-1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.166	0.1635	0.1611
-0.8	0.2119	0.209	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.242	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.305	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.281	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.33	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.352	0.3483
-0.2	0.4207	0.4168	0.4129	0.409	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0	0.5	0.496	0.492	0.488	0.484	0.4801	0.4761	0.4721	0.4681	0.4641
0	0.5	0.504	0.508	0.512	0.516	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.591	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.648	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.67	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.695	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.719	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.758	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.791	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.834	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.877	0.879	0.881	0.883
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.898	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.937	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.975	0.9756	0.9761	0.9767
2	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.983	0.9834	0.9838	0.9842	0.9846	0.985	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.989
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.992	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.994	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.996	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.997	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.998	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.999	0.999
3.1	0.999	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998