

Problem Set 6

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The solution is available online

Problem 1

Consider an economy with 3 risky assets with expected returns

$$\mu = \begin{bmatrix} 0.10 \\ 0.12 \\ 0.11 \end{bmatrix} \quad (1)$$

The variance-covariance matrix of returns is given by

$$\Sigma = \begin{bmatrix} 0.0100 & 0.0066 & 0 \\ 0.0066 & 0.0121 & 0 \\ 0 & 0 & 0.0225 \end{bmatrix} \quad (2)$$

Suppose that there is no riskless asset. The government has decided to impose constraints on portfolio holdings. Investors are required to invest at least 20% of their wealth in each of the 3 assets and can invest no more than 40% of their wealth in any given asset. We will consider an investor that has mean-variance utility $V = \mu_p - (a/2) \sigma_p^2$ and a risk aversion coefficient of 5.

1. Starting with an initial portfolio of $w = [1/3 \ 1/3 \ 1/3]$, determine the investor's optimal portfolio. What is its expected return and standard deviation of returns?
2. What is the value of marginal utility of each of the three assets at the optimal portfolio?
3. What is the value of λ and ϕ_L and ϕ_U for each of the three assets at the optimal portfolio? Explain in words why the multipliers take these values and what these values mean.
4. How much utility does the existence of the constraints cost the investor?
5. How much would an investor with an initial wealth of 1 million agree to pay to avoid being subject to the government's constraints? (For simplicity, assume that such a payment would relax the constraint for one year and can be made at the end of the year.) Explain why.