

Problem Set 5

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To be solved on March 30

Short Questions

Exercise 1. An arbitrage opportunity exists if an investor can construct a _____ investment portfolio that will yield a sure profit.

- a. positive
- b. negative
- c. zero
- d. all the above
- e. none of the above

Exercise 2. Consider the multifactor APT with two factors. Stock A has an expected return of 14.4%, a beta of 1.2 on factor 1 and a beta of 1.5 on factor 2. The risk premium on the factor 1 portfolio is 2%. The risk-free rate of return is 6%, what is the risk-premium on factor 2 if no arbitrage opportunities exist?

- a. 2%
- b. 3%
- c. 4%
- d. 6%
- e. none of the above

Exercise 3. Consider the multifactor APT. There are two independent economic factors, F_1 and F_2 . The risk-free rate of return is 6%. The following information is available about two well-diversified portfolios:

Portfolio	β on F_1	β on F_2	Expected return
A	1.0	2.0	19%
B	2.0	0.0	12%

Assuming no arbitrage opportunities exist, the risk premium on the factor F_1 portfolio should be _____.

- a. 3%
- b. 4%
- c. 5%
- d. 6%
- e. none of the above

Exercise 4. The APT differs from the CAPM because the APT _____.

- a. places more emphasis on market risk
- b. minimizes the importance of diversification
- c. recognizes multiple unsystematic risk factors
- d. recognizes multiple systematic risk factors
- e. none of the above

Problem 1

Consider an economy with $N = 3$ risky assets whose returns are driven by $K = 3$ factors (this is not completely realistic, but it makes computations easier). The expected returns of the three assets are

$$\mu = \begin{bmatrix} 0.10 \\ 0.12 \\ 0.11 \end{bmatrix} \quad (1)$$

and the standard deviations of their returns 0.10, 0.11 and 0.15 per period, respectively. Suppose that the matrix of factor exposures is given by

$$B = \begin{bmatrix} 0.2 & 0.5 & 0 \\ 0.4 & 0.6 & 0 \\ 0 & 0 & 0.3 \end{bmatrix} \quad (2)$$

where each column corresponds to one asset and each row to one factor. The shocks on the three factors are uncorrelated, and their standard deviations are 0.1, 0.15 and 0.2 per period, respectively. Idiosyncratic risk is uncorrelated with the factor realizations and across assets.

1. Determine the systematic risk and the unique risk of each asset in standard deviation terms.
2. Determine the correlation between the returns on each pair of assets.
3. What is the exposure to each of the three factors of the portfolio $w = [0.25 \ 0.45 \ 0.3]'$?
4. Determine the composition of the factor portfolio for each factor, i.e. of a portfolio that has unit exposure to that factor and zero exposure to each of the other factors. What is the expected return on each of the factor portfolios? Their risk premium?
5. What is, in standard deviation terms, the systematic risk and the unique risk of the factor portfolio for the first factor?
6. Suppose that there is a fourth asset with an exposure of $b_4 = [-0.3 \ 0.8 \ 1.2]'$ and an expected return of 18%. Can this be an equilibrium? If so, explain why. If not, describe precisely the arbitrage strategy you would use to make money, including the position in the riskless asset (ignore unique risk for that purpose). How much is your arbitrage profit?

Problem 2

The following is a scenario for three stocks constructed by the security analysts of Pf Inc.

Stock	Price (\$)	Recession	Average	Boom
A	10	-15	20	30
B	15	25	10	-10
C	50	12	15	12

1. Construct an arbitrage portfolio using these stocks.
2. How might these prices change when equilibrium is restored? Give an example where a change in Stock C's price is sufficient to restore equilibrium, assuming that the dollar payoffs to Stock C remain the same.

Problem 3

Assume that stock market returns have the market index as a common factor, and that all stocks in the economy have a beta of 1 on the market index. Firm-specific returns all have a standard deviation of 30%. Suppose that an analyst studies 20 stocks, and finds that one-half have an alpha of 2%, and the other half an alpha of -2%. Suppose the analyst buys \$1 million of an equally weighted portfolio of the positive alpha stocks, and shorts \$1 million of an equally weighted portfolio of the negative alpha stocks.

1. What is the expected profit (in dollars) and standard deviation of the analyst's profit?
2. How does your answer change if the analyst examines 50 stocks instead of 20 stocks? 100 stocks?

Problem 4

Assume that security returns are generated by the single-index model,

$$R_i = \alpha_i + \beta_i R_M + e_i \quad (3)$$

where R_i is the excess return for security i and R_M is the market's excess return. The risk-free rate is 2%. Suppose also that there are three securities A, B, and C, characterized by the following data

Security	β_i	$E(R_i)$	$\sigma(e_i)$
A	0.8	10%	25%
B	1.0	12%	10%
C	1.2	14%	20%

1. If $\sigma_M = 20\%$, calculate the variance of returns of securities A, B, and C.

2. Now assume that there are an infinite number of assets with return characteristics identical to those of A, B, and C respectively. If one forms a well-diversified portfolio of type A securities, what will be the mean and variance of the portfolio's excess returns? What about portfolios composed only of type B or C stocks?
3. Is there an arbitrage opportunity in this market? What is it? Analyze the opportunity graphically.

Problem 5

Consider the following multifactor (APT) model of security returns for a particular stock.

Factor	Factor Beta	Factor Risk Premium
Inflation	1.2	6%
Industrial production	0.5	8%
Oil prices	0.3	3%

1. If T-bills currently offer a 6% yield, find the expected rate of return in this stock if the market views the stock as fairly priced.
2. Suppose that the market expected the values for the three macro factors given in column 1 below, but that the actual values turn out as given in column 2. Calculate the revised expectations for the rate of return on the stock once the "surprises" become known.

Factor	Expected Rate of Change	Actual Rate of Change
Inflation	5%	4%
Industrial production	3%	6%
Oil prices	2%	0%