Problem Set 4

Daniel Andrei

To be solved on March 23

Short Questions

Exercise 1. The market portfolio has a beta of

a. 0.
b. 1.
c. -1.
d. 0.5.
e. none of the above.

Exercise 2. Which statement is not true regarding the market portfolio ?

a. It includes all assets of the universe.
b. It lies on the efficient frontier.
c. All securities in the market portfolio are held in proportion to their market values.
d. It is the tangency point between the capital market line and the indifference curve.
e. all of the above are true.

Exercise 3. According to the Capital Asset Pricing Model (CAPM), fairly priced securities

a. have positive betas.
b. have zero alphas.
c. have negative betas.
d. have positive alphas.
e. none of the above.

Exercise 4. According to the Capital Asset Pricing Model (CAPM),

a. a security with a positive alpha is considered overpriced.
b. a security with a zero alpha is considered to be a good buy.
c. a security with a negative alpha is considered to be a good buy.
d. a security with a positive alpha is consider to be underpriced.
e. none of the above.

Exercise 5. Security X has an expected rate of return of 0.13 and a beta of 1.5. The risk-free rate is 0.05 and the market expected rate of return is 0.10. According to the Capital Asset Pricing Model, this security is

a. underpriced.
b. overpriced.
c. fairly priced.
d. cannot be determined from data provided.
e. none of the above.

Exercise 6. Security A has an expected rate of return of 0.10 and a beta of 1.1. The market expected rate of return is 0.08 and the risk-free rate is 0.05. The alpha of the stock is

a. 1.7%.
b. -1.7%.
c. 8.3%.
d. 5.5%.
e. none of the above.

Problem 1

Consider a financial market with 2 assets, X and Y. Firm X is smaller and accounts for 1/3 of the total market capitalization. Firm Y accounts for 2/3 of the total market capitalization. Following analyst estimates, the expected return for X is 14%, while for Y is 20%. Standard deviations of returns are estimated at 15% and 30% respectively. Finally, the correlation between the returns of X and Y should be 0.3.

1. Calculate the expected return of the market portfolio.
2. Calculate the standard deviation of the returns for the market portfolio.
3. Compute the β of X wrt to the market portfolio.
4. Use the previous result to find the β of Y.
5. Use the CAPM relationship $\mu = r_f + \beta\,(\mu_M - r_f)$ to find the risk free rate.
6. Knowing that $\tilde{r}_X = \beta\tilde{r}_M + e_t$, where e_t is the idiosyncratic part of the return and $\beta\tilde{r}_M$ is the systematic part, find the variance of the systematic risk and the variance of the idiosyncratic risk.
7. Explain briefly why the idiosyncratic risk of X is not priced.

Problem 2

The market price of a security is \$50. Its expected rate of return is 14%. The risk-free rate is 6% and the market risk premium is 8.5%. What will be the market price of the security if its correlation coefficient with the market portfolio doubles (and all other variables remain unchanged)? Assume that the stock is expected to pay a constant dividend in perpetuity.

Problem 3

Assume that the risk-free rate of interest is 6% and the expected rate of return on the market is 16%.

1. A share of stock sells for \$50 today. It will pay a dividend of \$6 per share at the end of the year. Its beta is 1.2. What do investors expect the stock to sell for at the end of the year?
2. I am buying a firm with an expected perpetual cash flow of \$1'000 but am unsure of its risk. If I think the beta of the firm is 0.5, when in fact the beta is really 1, how much more will I offer for the firm than it is trully worth?
3. A stock has an expected rate of return of 4%. What is its beta?

Problem 4

In 1999 the rate of return on short-term government securities (perceived to be risk-free) was about 5%. Suppose the expected rate of return required by the market for a portfolio with a beta of 1 is 12%. According to the capital asset pricing model (security market line):

1. What is the expected rate of return on the market portfolio?
2. What would be the expected rate of return on a stock with $\beta = 0$?
3. Suppose you consider buying a share of stock at \$40. The stock is expected to pay \$3 dividends next year and you expect it to sell then for \$41. The stock risk has been evaluated at $\beta = -0.5$. Is the stock overpriced or underpriced?

Problem 5

Suppose that you can invest risk-free at rate r_f but can borrow only at a higher rate, r_f^B.

1. Draw a minimum-variance frontier. Show on the graph the risky portfolio that will be selected by defensive investors. Show the portfolio that will be selected by aggresive investors.
2. What portfolio will be selected by investors who neither borrow nor lend?
3. Where will the market portfolio lie on the efficient frontier?
4. Will the zero-beta CAPM be valid in this scenario? Explain. Show graphically the expected return on the zero-beta portfolio.

Problem 6

Suppose that borrowing is restricted so that the zero-beta version of the CAPM holds. The expected return on the market portfolio is 17%, and on the zero-beta portfolio it is 8%. What is the expected return on a portfolio with a beta of 0.6?

Problem 7

We analyze the returns of security X under CAPM. Today is $t = 0$ and the market price of the security is P_0. In one year from now, at $t = 1$, there are two possible scenarios, a and b. The price P_1 of the security at $t = 1$, the dividend D_1 paid at the end of the year, and the return of the market portfolio in each scenario, are shown in the table below.

Scenario	a	b
Probability	1/3	2/3
Market return r_M	15%	6%
P_1	19	17
D_1	3	2

The risk-free rate is 5%.

1. What is the expected return of the market portfolio? What is its standard deviation of returns?
2. Express the return of X as a function of the price P_0.
3. Compute the β of X as a function of the price P_0, by using the relationship $\beta_i = \sigma_{im}/\sigma_M^2$.
4. Use the CAPM relationship to compute the price P_0. What do you conclude?

Problem 8

Today is $t = 0$ and the market price of the security X is P_0. The price at date 1 is a random variable P_1. We suppose that CAPM holds and that the beta of X is β.

1. Show that $P_0 = \frac{E[P_1]}{1+r_f+\beta(\mu_M - r_f)}$.
2. Show how you can obtain the similar formula $P_0 = \frac{E[P_1]-a\rho\sigma}{1+r_f}$, where ρ is the correlation between the asset return and the market return, and σ is the volatility of the asset returns. Find a.
3. Interpret your result.