

Problem Set 3

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Short Questions

Exercise 1. Given the capital allocation line, an investor's optimal portfolio is the portfolio that

- a. maximizes his/her expected profit.
- b. maximizes his/her risk.
- c. maximizes both his/her risk and return.
- d. maximizes her expected utility.
- e. none of the above.

Exercise 2. The change from a straight to a kinked capital allocation line is a result of :

- a. reward-to-volatility ratio increasing.
- b. borrowing rate exceeding lending rate.
- c. investors risk tolerance decreasing.
- d. increase in the portfolio proportion of the risk-free asset.
- e. none of the above.

Exercise 3. Passive investing

- a. may be accomplished by investing in index mutual funds.
- b. involves considerable security selection.
- c. involves considerable transaction costs.
- d. a and c.
- e. b and c.

Exercise 4. Market risk is also referred to as

- a. systematic risk, diversifiable risk.
- b. systematic risk, nondiversifiable risk.
- c. unique risk, nondiversifiable risk.
- d. unique risk, diversifiable risk.
- e. none of the above.

Exercise 5. Beta is the measure of

- a. firm specific risk.
- b. diversifiable risk.
- c. market risk.
- d. unique risk.
- e. none of the above.

Exercise 6. The risk that can be diversified away is

- a. firm specific risk.
- b. beta.
- c. systematic risk.
- d. market risk.
- e. none of the above.

Exercise 7. The Capital Allocation Line provided by a risk-free security and N risky securities is

- a. the line that connects the risk-free rate and the global minimum-variance portfolio of the risky securities.
- b. the line that connects the risk-free rate and the portfolio of the risky securities that has the highest expected return on the efficient frontier.
- c. the line tangent to the efficient frontier of risky securities drawn from the risk-free rate.
- d. the horizontal line drawn from the risk-free rate.
- e. none of the above.

Exercise 8. Consider an investment opportunity set formed with two securities that are perfectly negatively correlated. The global minimum variance portfolio has a standard deviation that is always

- a. greater than zero.
- b. equal to zero.
- c. equal to the sum of the securities' standard deviations.
- d. equal to -1.
- e. none of the above.

Exercise 9. A measure of how the returns of two risky assets move together is:

- a. variance.
- b. standard deviation.
- c. covariance.
- d. correlation.
- e. c and d.

Exercise 10. In a two-security minimum variance portfolio

- a. the security with the higher standard deviation will be weighted more heavily.
- b. the security with the higher standard deviation will be weighted less heavily.
- c. the two securities will be equally weighted.
- d. the risk will be zero.
- e. the return will be zero.

Problem 1

Consider an economy with 3 risky assets with expected returns

$$\mu = \begin{bmatrix} 0.10 \\ 0.12 \\ 0.11 \end{bmatrix} \quad (1)$$

The variance-covariance matrix of returns is given by

$$\Sigma = \begin{bmatrix} 0.0100 & 0.0066 & 0 \\ 0.0066 & 0.0121 & 0 \\ 0 & 0 & 0.0225 \end{bmatrix} \quad (2)$$

1. What is the optimal portfolio for an investor that has a mean-variance utility $V = \mu_P - (a/2)\sigma_P^2$ and a risk aversion coefficient of 5?
2. What is its expected return and standard deviation of returns?

Problem 2

Consider an economy with 5 risky assets with expected returns and standard deviation of returns given below. The average correlation between the assets is $\rho = 0.5$.

$$\mu = \begin{bmatrix} 0.09 \\ 0.14 \\ 0.07 \\ 0.13 \\ 0.075 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 0.10 \\ 0.13 \\ 0.08 \\ 0.15 \\ 0.09 \end{bmatrix} \quad (3)$$

You want to analyze the impact of the number of assets on the optimal portfolio of an investor that has a mean-variance utility $V = \mu_P - (a/2)\sigma_P^2$ and a risk aversion coefficient of 5, when he includes in his decision the first 2 assets, then the first 3 assets, etc. Build a table with all the results and try to interpret the modifications arising in the composition each time you add one more asset.

Problem 3

Consider again the Problem 1. Suppose now that a riskless asset with rate of return of 5% is also available for investment.

1. What is the optimal portfolio for the investor with a risk aversion of 5? Does the investor lend or borrow? How much?
2. What is the composition of the market portfolio? Its expected return and standard deviation of returns?
3. How much is the market price of risk? Explain in words what the value you obtain means.
4. How much is the economy-wide aggregate risk aversion implicit in the market portfolio? What does this value mean?

5. What are the betas of the three risky assets?
6. What is the correlation of the returns on the second asset with those on the market portfolio? How much is, in standard deviation terms, the systematic risk and the unique risk of the second asset?

Problem 4

Consider an economy with 5 risky assets with expected returns and standard deviation of returns given below. The average correlation between the assets is 0.3.

$$\mu = \begin{bmatrix} 0.11 \\ 0.15 \\ 0.12 \\ 0.20 \\ 0.17 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 0.10 \\ 0.20 \\ 0.12 \\ 0.15 \\ 0.12 \end{bmatrix} \quad (4)$$

1. Plot the set of minimum variance portfolios in the standard deviation - return space.
2. What is the composition of the global minimum-variance portfolio? What is its expected return and standard deviation of returns? Add this portfolio in the graph plotted at point (1).
3. The risk-free rate is $r_f = 5\%$. What is the composition of the tangency portfolio? What is its expected return and standard deviation of returns? Add this portfolio in the graph plotted at point (1).
4. Consider a mean-variance investor with risk aversion $a = 8$, seeking to determine his optimal portfolio. What is the composition of this portfolio? Does the investor lend or borrow? How much?
5. Add this portfolio in the graph plotted at point (1). Draw the capital market line and the indifference curve of the investor corresponding to its maximized utility.

Problem 5

The financial market is composed by three assets, A, B and C. The table below gives you two minimum-variance portfolios, P1 and P2:

	P1	P2
A	15%	40%
B	55%	17%
C	30%	43%

The expected returns and the standard deviation of returns for these two portfolios are

	P1	P2
μ_i	5%	17%
σ_i	8%	14%

The correlation of returns for these two portfolios is $\rho = -0.1$. The risk-free rate is $r_f = 2\%$. The tangency portfolio (P_T) has a $\delta = 44\%$ share of P1 and a $1 - \delta = 56\%$ share of P2.

1. What is the composition of the tangency portfolio, in terms of the original assets A, B, C? What is its expected return (μ_x) and standard deviation of returns (σ_x)?
2. What is the slope of the capital market line?
3. Compute analytically and numerically $d\mu_x/d\delta$ and $d\sigma_x/d\delta$.
4. Compute the slope of the minimum variance frontier at P_T . What do you conclude? (round answers to one decimal point)
5. You are a mean-variance investor with a risk aversion $a = 20$. What is the composition of the optimal portfolio, in terms of P_T and the risk-free asset? Do you lend or borrow? How much?
6. What is the composition of the optimal portfolio, in terms of $P1$, $P2$, and the risk-free asset?
7. What is the composition of the optimal portfolio, in terms of the original assets A, B, C, and the risk-free asset?

Problem 6

(This problem is drawn from Chapter 7, BKM. Reading that chapter carefully could help a lot). You manage a risky portfolio with an expected rate of return of 18% and a standard deviation of 28%. The T-bill rate is 8%.

1. Your client chooses to invest 70% of a portfolio in your fund and 30% in a T-bill money market fund. What is the expected value and standard deviation of the rate of return on his portfolio?
2. Suppose that your risky portfolio includes the following investments in the given proportions: Stock A 25%, Stock B 32%, Stock C 43%. What are the investment proportions of your client's overall portfolio, including the position in T-bills?
3. What is the reward-to-variability ratio (S) of your risky portfolio? Your client's?
4. Draw the CAL of your portfolio on an expected return - standard deviation diagram. What is the slope of the CAL? Show the position of your client on your fund's CAL.

5. Suppose that your client decides to invest in your portfolio a proportion y of the total investment budget so that the overall portfolio will have an expected rate of return of 16%. What is the proportion y ? What are your client's investment proportions in your three stocks and the T-bill fund? What is the standard deviation of the rate rate of return on your client's portfolio?
6. Suppose that your client prefers to invest in your fund a proportion y that maximizes the expected return on the complete portfolio subject to the constraint that the complete portfolio's standard deviation will not exceed 18%. What is the investment proportion, y ? What is the expected return on the complete portfolio?
7. Your client's degree of risk aversion is $a = 3.5$. What proportion, y , of the total investment should be invested in your fund? What is the expected value and standard deviation of the rate of return on your client's optimized portfolio?
8. You estimate that a passive portfolio, that is, one invested in a risky portfolio that mimics the S&P 500 stock index, yields an expected rate of return of 13% with a standard deviation of 25%. Draw the CML and your fund's CAL on an expected return - standard deviation diagram. What is the slope of the CML? Characterize in one short paragraph the advantage of your fund over the passive fund.
9. Your client ponders whether to switch the 70% that is invested in your fund to the passive portfolio. Explain to your client the disadvantage of the switch. Show him the maximum fee you could charge (as a percentage of the investment in your fund, deducted at the end of the year) that would leave him at least as well off investing in your fund as in the passive one.