Problem Set 2

Daniel Andrei

To be solved on March 9

Short Questions

Exercise 1. Which of the following statements regarding risk averse investors is true ?

a. They only care about the rate of return.
b. They accept investments that are fair games.
c. They only accept risky investments that offer risk premiums over the risk-free rate.
d. They are willing to accept lower returns and high risk.
e. a and b.

Exercise 2. In the mean-standard deviation graph an indifference curve has a ___________ slope.

a. negative
b. zero
c. positive
d. cannot be determined

Exercise 3. Elias is a risk averse investor. David is a less risk averse investor than Elias. Therefore,

a. for the same risk, David requires a higher rate of return than Elias.
b. for the same return, Elias tolerates higher risk than David.
c. for the same risk, Elias requires a lower rate of return than David.
d. for the same return, David tolerates higher risk than Elias.
e. cannot be determined.

Exercise 4. Consider a risky portfolio, A, with an expected rate of return of 0.15 and a standard deviation of 0.15, that lies on a given indifference curve. Which one of the following portfolios might lie on the same indifference curve ?

a. E (r) = 0.15; Variance = 0.20
b. E (r) = 0.15; Variance = 0.10
c. E (r) = 0.10; Variance = 0.10
d. E (r) = 0.20; Variance = 0.15

Exercise 5. Discuss the differences between investors who are risk averse, risk neutral, and risk loving.

–

Exercise 6. Discuss concepts covariance and correlation. How do the concepts differ in terms of calculation and interpretation ?

–

Problem 1

Consider a risk-averse investor with exponential utility

$$u(R_p) = -e^{-aR_p}, \quad a > 0. \tag{1}$$

Let the distribution of portfolio returns be normal with mean μ and variance σ^2. The pdf is

$$f(R_p; \mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{R_p-\mu}{\sigma^2}\right)^2}. \tag{2}$$

1. Compute the investor's degree of risk aversion
2. Write the investor's expected utility, $E[u(R_p)]$.
3. Show that maximizing expected utility is equivalent to maximizing $\mu - \frac{a}{2}\sigma^2$. Interpret your result.

Problem 2

Consider a portfolio that offers an expected rate of return of 12% and a standard deviation of 18%. T-bills offer a risk-free 7% rate of return. What is the maximum level of risk aversion for which the risky portfolio is still preferred to bills?

Problem 3

Draw the indifference curve in the expecteed return - standard deviation plane corresponding to a utility level of 5% for an investor with a risk aversion coefficient of 3. (*Hint: Choose several possible standard deviations, ranging from 5% to 25%, and find the expected rates of return providing a utility level of 5%. Then plot the expected return - standard deviation points so derived.*) Do the same exercise for a risk averison coefficient of 5, and for a risk-neutral investor.

Problem 4

Consider historical data showing that the average annual rate of return on the S&P 500 portfolio over the past 70 years has averaged about 8.5% more than the Treasury bill return and that the S&P 500 standard deviation has been about 20% per year. Assume these values are representative of investors' expectations for future performance and that the current T-bill rate is 5%.

1. Calculate the expected return and variance of portfolios invested in T-bills and the S&P 500 index with weights from the table below.

2. Calculate the utility levels of each portfolio of problem 10 for an investor with the risk aversion coefficient $A = 3$. What do you conclude?

3. Repeat the same exercise for an investor with $A = 5$. What do you conclude?

W_{bills}	W_{index}
0	1
0.2	0.8
0.4	0.6
0.6	0.4
0.8	0.2
1	0

Problem 5

Consider a portfolio consisting of two assets X and Y with the following characteristics:

State of the Economy	Probability	Return X	Return Y
1	0.2	18%	0%
2	0.2	5%	-3%
3	0.2	12%	15%
4	0.2	4%	12%
5	0.2	6%	1%

You are given as well the following portfolis:

W_X	W_Y
1.25	-0.25
1	0
0.5	0.5
0	1
-0.25	1.25

1. Calculate the expected return, the standard deviation of returns for each asset, and the correlation between the two assets.

2. Calculate the expected return and the standard deviation of returns for the given portfolios.

3. Draw the efficient frontier in the expecteed return - standard deviation plane.

4. Calclate the covariance between portfolio A which has 75% of X and portfolio B which has 25% of X.

Problem 6

Consider the expected returns and the volatilities of the two assets A and B.

Asset	Expected Return	Volatility
A	0.18	0.12
B	0.08	0.07

1. Calculate the composition of the minimum-variance portfolio, if the correlation between the returns of the assets is $\rho_{A,B} = -0.26$.
2. Calculate the expected return and the volatility of this portfolio.
3. Calculate the composition of the global minimum-variance portfolio if the correlation between the returns of the assets is $\rho_{A,B} = 0.5$.
4. Calculate the expected return and the volatility of this protfolio.
5. Comment your results.

Problem 7

Consider an economy with 3 risky assets with expected returns

$$\mu = \begin{bmatrix} 0.10 \\ 0.12 \\ 0.11 \end{bmatrix} \tag{3}$$

The variance-covariance matrix of returns is given by

$$\Sigma = \begin{bmatrix} 0.0100 & 0.0066 & 0 \\ 0.0066 & 0.0121 & 0 \\ 0 & 0 & 0.0225 \end{bmatrix} \tag{4}$$

1. What is the inverse of the variance-covariance matrix?
2. What is the composition of the global minimum-variance portfolio? What is its expected return and standard deviation of returns?
3. What is the composition of the minimum-variance portfolio that has an expected return of 0.12? Is it efficient or not? Why?
4. What is the correlation between the returns of the portfolio you computed under (3) and those of the global minimum-variance portflolio from (2)?

Problem 8

Consider an economy with 5 risky assets with expected returns and standard deviation of returns given below. The average correlation between the assets is $\rho = 0.5$.

$$\mu = \begin{bmatrix} 0.09 \\ 0.14 \\ 0.07 \\ 0.13 \\ 0.075 \end{bmatrix}, \quad \Sigma = \begin{bmatrix} 0.10 \\ 0.13 \\ 0.08 \\ 0.15 \\ 0.09 \end{bmatrix} \tag{5}$$

1. You want to analyze the impact of the number of assets on the minimum-variance set. Draw the minimum-variance set for the first 2 assets, then for the first 3 assets, and so on un to all the 5 assets.
2. Compute in each case the composition of the global minimum variance portfolio. What is its expectet return and standard deviation of returns?
3. Suppose you want an expected return of 0.11. Compute for each case the minimum-variance portfolio with an expected return of 0.11. Compare your results.

Problem 9

Consider the same economy as in Problem 9. You want to analyze the impact of the correlation on the minimum-variance set. Draw the minimum-variance set for the 5 assets, for the following values of correlations: $\rho = -0.1$, $\rho = 0$, $\rho = 0.25$, $\rho = 0.5$, $\rho = 0$, $\rho = 0.8$. Compare and interpret the results.