The Redistributive Effects of Monetary Policy*

Daniel Andrei† Bernard Herskovic‡ Olivier Ledoit§

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Abstract

We introduce a model of the economy as a social network. Two agents are linked to the extent that they transact with each other. This generates well-defined topological notions of location, neighborhood and closeness. We investigate the implications of our model for monetary economics. When a central bank increases the money supply, it must inject the money *somewhere* in the economy. The agent closest to the location where money is injected is better off, and the one furthest is worse off. Symmetrically, any decrease in the money supply redistributes purchasing power in the other direction. This redistribution channel is independent from other previously studied channels. Our model's theoretical predictions are supported by the data.

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†UCLA, Anderson School of Management, 110 Westwood Plaza, Suite C420, Los Angeles, CA 90095, USA, daniel.andrei@anderson.ucla.edu, danielandrei.info

‡UCLA, Anderson School of Management, 110 Westwood Plaza, Suite C420, Los Angeles, CA 90095, USA, bernard.herskovic@anderson.ucla.edu, bernardherskovic.com

§University of Zürich, Department of Economics, Zürichbergstrasse 14, 8032 Zürich, Switzerland, olivier.ledoit@econ.uzh.ch, www.ledoit.net

1 Introduction

Monetary policy unevenly affects economic agents. For example, the expansion of the Fed's balance sheet during the great recession affected investors differently, depending on the type of securities they held at that time. Arguably, this economic intervention changed the welfare distribution in the economy. In this paper, we study the redistributive effects of monetary policy in a economy with interconnected agents and show that monetary policy has heterogeneous effects in the cross section. We develop a theoretical framework in which firms are part of a trade network. We use this framework to measure the *economic distance from the Fed* and to identify economic agents most and least affected by monetary policies.

We model the economy as a network of trading relations. Agents' connections are based on how much they trade with each other. However, agents are different from each other because they hold different trading relations. Some may have strong trading relationships with one another, while others may have trading relationships that are not as strong. This network structure introduces notions of location, neighborhood and distance. More importantly, how and where monetary policies take place matters.

The implications of monetary policies for redistribution is source of much debate.[1] The typical argument lies on agents' differences in cash holdings. Money supply expansion transfers real consumption from agents with large cash balances to those with lower balances. This happens because money is worth less after the intervention, hurting those with large cash balances. Our economic channel of redistribution is different. We assume that a monetary intervention directly affects some economic agents. However, it indirectly affects economic agents close to those directly affected, which in turn will affect other economic agents close to them, and so on. This propagation mechanism is similar to the propagation of productivity shocks across sector linkages in Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). In our setting, monetary policy shocks propagate along economic linkages between agents, leading to real redistributive effects in the cross section.

Monetary policies have to be initiated *somewhere* in economy, and, moreover, some economic agents ought to be directly affected. For institutional and practical reasons, the central bank is not able uniformly implement a policy as a Friedman helicopter drop. In fact, only a few select institutions have direct dealings with the central bank. Most investors do not participate in the primary market, and securities go through a chain of intermediaries before reaching the final investor. The money injected into the economy by the central bank

[1] See, among others, D'Amico and King (2010), Doh (2010), Fuster and Willen (2010), Neely (2010) Gagnon, Raskin, Remache, and Sack (2011), Krishnamurthy and Vissing-Jorgensen (2011), Gabriel and Lutz (2014), and the references therein. For various redistribution channels of monetary policy, we refer the reader to the comprehensive study by Coibion, Gorodnichenko, Kueng, and Silvia (2012). See also Saiki and Frost (2014) and our literature review section.

percolates through the network linkages, leading to heterogeneous real effects in the cross section. Economic agents are affected by monetary policies differently, depending on their economic distance from the central bank.

Our model features a pure exchange economy with multiple goods. Each agent is endowed with one unit of a certain consumption good and with a quantity of money. To mute the redistribution channel of devaluation of cash balances, we assume that all agents are endowed with the same quantity of money. In addition, we assume that all agents have the same degree of preference for money. We formalize agents' heterogeneous trading relationships with a symmetric network, which we represent as a circle. Hence, every agent in the circle is closely related two immediate neighbors: one on the left and another on the right. Furthermore, trades become less intense as agents are further away from each other in the circle.

In our model, the central bank increases the money supply by purchasing one particular good, say the good of agent 1. We prove that this leads to an increase in the relative price of good 1. A direct implication is that such intervention benefits agent 1: he is strictly better off as a result of being the only agent who deals directly with the central bank. We also show that the agent who is the furthest from the central bank, i.e., the one diametrically opposite from agent 1 on the circle, is strictly worse off. Therefore, monetary expansion redistributes real consumption from the agent who is furthest from the central bank to the agent who is closest to the central bank.

Our model provides a method to empirically quantify agents' economic distance from the central bank. In the data, we assume that each sector is an economic agent. Rather than imposing which agents are close to the central bank, we estimate which sectors of the economy are most affected by monetary policy shocks by following our theoretical predictions. Formally, we estimate a measure of economic distance from the Fed (EDF). A sharp prediction of the model is that relative prices of sectors closer to the central bank are more sensitive to unanticipated monetary policy shocks. We obtain our EDF measure in two steps. First, we estimate unanticipated monetary policy shocks. Then, for each sector, we regress changes in relative prices at the sector level on these unanticipated shocks. The coefficient of this regression is our measure of economic distance from the Fed.

We test three major predictions of our theoretical framework. First, we test the model implications for the principal component structure of innovations in relative prices. We show that there is a strong factor structure in the changes in sectoral relative prices, which is consistent with our model predictions. In addition, the first principal component of changes in sectoral relative prices is directly related to monetary policy shocks, both in the model and in the data.

More importantly, we show that the first principal component weights are closely related to our measure of economic distance from the Fed. This strong evidence is consistent with our model. We calculate the first principal component weights from relative prices data, while the EDF is the sensitivity of changes in relative prices with respect to monetary policy shocks. In the model, a monetary policy shock leads to changes in relative prices in sectors closest to the Fed. These changes themselves affect prices of other economically close sectors, and the shock percolates through the network, affecting more sectors. The propagation of the monetary policy shock dies off as agents are further away from the central bank. As a result, changes in prices of sectors closest to the Fed drive most of the variance in relative prices. For this reason, the first principal component weights should align with our EDF measure. This is what we find in the data: by regressing the first principal component weights on our measure of economic distance from the Fed, we find an R-squared above 95 percent, and a t-statistic above 15 for the slope coefficient.

The second prediction we verify in the data is the model implication for the correlation structure of relative prices. As a monetary policy shock propagates through the network, changes in relative prices of sectors economically close to each other should be more correlated. We rank sectors based on their distance from the Fed, and we show that the correlation matrix resembles a block diagonal matrix, which is consistent with our model. The average correlation between changes in relative prices decline monotonically from 70% for neighboring sectors to about -20% for sectors furthest away from each other.

Finally, the third prediction we test in the data is the model implication for agents' welfare. According to our model, positive monetary policy shocks benefit the sector closest to the Fed, while they hurt the sector furthest away. Using excess returns of industry portfolios, we show that indeed excess returns of sectors closest to the Fed are more sensitive to unanticipated monetary shocks that sectors that are further away. These results are consistent with our theoretical prediction.

Next, we discuss the related literature. In Section 2, we present our model, and, in Section 3, we discuss its theoretical predictions. In Section 4, we discuss our empirical evidence, and we conclude in Section 5.

Related Literature While the redistribution channel we describe does not seem to have been studied in modern monetary economics, the pre-classical economist Richard Cantillon (1680-1734) alluded to it. He wrote in Chapter 6 of his book "Essai sur la Nature du Commerce en Général":[2]

> *If the increase of actual money comes from mines of gold or silver in the state the owner of these mines, the adventurers, the smelters, refiners, and all the other workers will increase their expenses in proportion to their gains. They will*

[2] "Essay on the Nature of Trade in General," written around 1730 and published in French in 1755.

> *consume in their households more meat, wine, or beer than before, will accustom themselves to wear better cloths, finer linen, to have better furnished houses and other choicer commodities. They will consequently give employment to several mechanics who had not so much to do before and who for the same reason will increase their expenses: all this increase of expense in meat, wine, wool, etc. diminishes of necessity the share of the other inhabitants of the state who do not participate at first in the wealth of the mines in question.*

This passage describes the percolation of money from the point where it is injected through a chain of economic agents, pushing prices up along the way. Until now, it was not known whether this pre-classical intuition had any validity within the neo-classical paradigm.

In a related paper, Williamson (2008) presents a theoretical model in which goods market segmentation slows down the percolation of new money across the population and generates a redistribution of wealth. Our framework does not have market segmentation itself, and thus it is frictionless in that sense. Another related work is by Ozdagli and Weber (2016), but they focus on the effects of monetary policy on stock returns in the short horizon. Using a production networks framework, they decompose the direct and indirect effect of monetary policy. We present a theoretical model in which monetary policy can have redistributive effects when agents are heterogeneous in their trading relations. In addition, we empirically test several predictions of our model.

Other articles on the redistributive effects of monetary policy focus on a different channel: the devaluation of cash balances. For example, in the overlapping generations model of Bhattacharya, Haslag, and Martin (2005), monetary expansion redistributes real wealth from old agents (who hold large amounts of money) to young agents (who do not). For Palivos (2005), monetary expansion redistributes real wealth from altruistic agents (who hold large amounts of money because they want to bequeath it to their children) to selfish agents (who hold less money because they do not care about their children). Romer and Romer (1999) point out that inflation redistributes wealth from creditors to debtors. In the turnpike model of Shi (1999), monetary expansion redistributes real wealth from the rich (i.e. agents with a large endowment) to the poor (agents with a small endowment). To the contrary, Erosa and Ventura (2002) find that monetary expansion redistributes real wealth from the poor (who hold a large percentage of their wealth in cash) to the rich (who hold a large percentage of their wealth as capital instead of cash). Since all the agents in our model have the same cash balance and the same appetite for cash, our redistributive effect is completely independent from these.

This line of research belongs to the broader topic of money neutrality (see Lucas, 1996). The redistributive effect that we identify argues strongly for the non-neutrality of money. This question is also a subject of considerable interest in political economy. For example, Albanesi (2007) views the choice of monetary policy as the outcome of a political conflict over redistribution between low income households and high income households. Ireland (2005) argues that policy makers face a choice between engineering redistribution through monetary policy or through fiscal policy, and tries to discern which one of the two options is better.

While this is beyond the scope of the paper, modelling the economy as a social network could prove useful in other areas of economics (e.g., Jackson, 2010). A closely related reference is Acemoglu et al. (2012), who show that the interconnections between different sectors function as a propagation mechanism of sectoral shocks. Our focus is different in that we are interested in the propagation of monetary policy shocks trough the economic network and the percolation of money in the economy.

2 Model

Agents Consider a pure exchange economy with an even number N of agents: $N = 2n$ for some integer $n > 1$.[3] Each agent can be interpreted as the representative agent for a certain class of individuals. These N agents are identical, except in one respect: the nature of the goods with which they are endowed and which they demand. The agents exchange money and N distinct consumption goods. Agent j is endowed with money and with good j (for all $j = 1, \ldots, N$). Let M denote the quantity of money with which agent j is endowed. Since we wish to make the agents as similar to one another as possible, M is assumed to be the same across all agents. Furthermore, agent j is endowed with one unit of good j. This assumption can be made without loss of generality, provided that we redefine the measurement unit of each good accordingly.

Objective Function Each agent is a price-taker with Cobb-Douglas utility function. The objective function of agent j ($j = 1, \ldots, N$) is:

$$\begin{aligned} &\max_{m_j; x_{1j}, \ldots, x_{Nj}} \left(\frac{m_j}{\sum_{k=1}^{N} m_k} \right)^{\beta} \times \prod_{i=1}^{N} {x_{ij}}^{\alpha_{ij}} \\ &\text{subject to:} \quad m_j + \sum_{i=1}^{N} p_i \, x_{ij} \leq M + p_j \end{aligned} \tag{1}$$

The price of good i is p_i, and by convention the price of a unit of money is 1. The budget available to agent j is $M + p_j$. The quantity m_j represents the amount of money demanded

[3]The assumption that N is even is not essential to our conclusions, but it facilitates the exposition of the model.

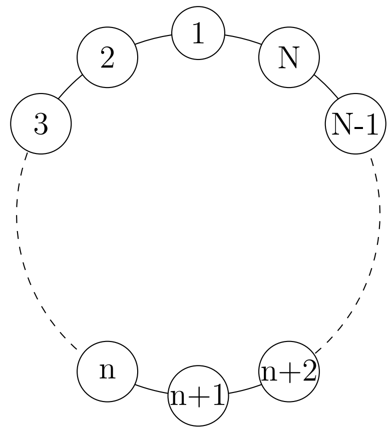

Figure 1: **Graphical representation of the circular structure of the economy.** Since $N = 2n$, the agent diametrically opposite Agent 1 is Agent $n + 1$.

by agent j, and x_{ij} represents the amount of good i demanded by agent j (for $i = 1, \dots, N$). As this is a one-period model, demand for money must be interpreted as reduced-form for holding a liquid asset that can easily be sold in order to finance any needs that may arise in the future (Sidrauski, 1967). Normalizing the cash holding of agent j by the sum of all cash holdings is necessary to avoid "money illusion": you cannot make everybody happier simply by multiplying everybody's cash holdings by a factor of ten. The Cobb-Douglas exponent β, which captures the intensity of demand for money, is assumed to be the same across all agents. We need $\beta > 0$ and $\forall i = 1, \dots, N \quad \alpha_{ii} > 0$. The other Cobb-Douglas exponents α_{ij} are all non-negative.

Circular Symmetry The agents are organized in a circle. See Figure 1 for an illustration.

We assume that the relationship of agent i to agent j depends only on the number of nodes that separates them: $\min(|i - j|, |N - i + j|)$. This assumption of circular symmetry implies a particular structure for the matrix $\mathbf{A} = (\alpha_{ij})_{i,j=1,\dots,N}$. There must exist $a_0 > 0$ and $N - 1$ non-negative coefficients $a_1, \dots, a_{N-1}$ such that:

$$\mathbf{A} = (\alpha_{ij})_{i,j=1,\dots,N} = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_{N-1} \\ a_{N-1} & a_0 & a_1 & \ddots & \vdots \\ a_{N-2} & a_{N-1} & a_0 & \ddots & a_2 \\ \vdots & \ddots & \ddots & \ddots & a_1 \\ a_1 & \cdots & a_{N-2} & a_{N-1} & a_0 \end{bmatrix}.$$

In mathematical terms, $\mathbf{A}$ is a *circulant* matrix. A circulant matrix is a matrix where each row vector is rotated one element to the right relative to the preceding row vector. Furthermore, $\mathbf{A}$ must be a *symmetric* circulant matrix, i.e. the following additional condition must be verified:

$$\forall i = 1, \ldots, N-1 \qquad a_{N-i} = a_i. \tag{2}$$

Thus, the Cobb-Douglas exponent of the demand for good i in the objective function of agent j should only depend on the number of nodes that separates i and j. This can be described as a *rotation-invariant* economy, since rotating the names of the agents by one or more notches, clockwise or counter-clockwise, does not modify any of the assumptions.

It will be convenient to denote by $S_C\left(z_0, z_1, \ldots, z_n\right)$ the symmetric circulant matrix whose first row vector is $[z_0 \;\; z_1 \;\; z_2 \ldots z_{n-1} \;\; z_n \;\; z_{n-1} \ldots z_2 \;\; z_1]$. With this notation, we can write: $\mathbf{A} = S_C\left(a_0, a_1, \ldots, \;\; a_n\right)$.

A convenient way to rewrite the objective function is to adopt the convention that the indices for goods and agents are not elements of the set of all integers $\mathbb{Z}$, but of the cyclic group $\mathbb{Z}/N\mathbb{Z}$ generated by the number of agents N. Thus, in terms of indices, $i + N = i - N = i$, i.e. all the indices are interpreted *modulo* N. With this convention, Equation (1) becomes:

$$\begin{gathered} \max_{m_j; x_{1j}, \ldots, x_{Nj}} \left(\frac{m_j}{\sum_{k=1}^N m_k}\right)^{\beta} {x_{jj}}^{a_0} \left[\prod_{k=1}^{n-1} \left(x_{j+k,j} \; x_{j-k,j}\right)^{a_k}\right] x_{j+n,j}^{a_n} \\ \text{subject to:} \quad m_j + \sum_{i=1}^N p_i \, x_{ij} \leq M + p_j. \end{gathered} \tag{3}$$

Central Bank The only institution that violates the rotation-invariance of the economy is the central bank. A central bank cannot have the same economic relationship to all the agents in the economy. Indeed, members of the general public are not usually allowed to deal with the central bank directly, but only through a sequence of intermediaries. One of the key roles of the head of the central bank is to control the money supply. She is authorized to inject money into the economy by buying certain assets and putting them on the central bank's balance sheet, or to reduce the amount of money outstanding by selling assets that were previously on its balance sheet. Either way, when the head of the central bank injects money, she must inject it *somewhere*, and when she reduces the money supply, she must take the money from *somewhere*. The notion of location implicit in the use of the word "somewhere" is well-defined in our economy because we have modelled the market as a social network, which generates proper topological notions of neighborhood and distance.

We assume that the head of the central bank injects money into the economy by buying one specific good at the prevailing market price, say good 1. Let Q denote the quantity of money injected by the central bank. The quantity of good 1 that is taken away from the economy and placed onto the central bank's balance sheet is Q/p_1. Q does not need to be strictly positive: if $Q < 0$ then the central bank is reducing the money supply and selling

$-Q/p_1$ units of good 1 from its balance sheet; finally, if $Q = 0$ then the central bank is inactive, which is the only case where the rotation-invariance of the economy is respected.

2.1 Equilibrium

First-Order Conditions Let us start from the objective function of agent j ($j = 1, \ldots, N$) given in Equation (1). Note that $\sum_{k=1}^{N} m_k = NM + Q$, which is constant, so it can be dropped. Taking the logarithm of the Cobb-Douglas utility function yields:

$$\begin{aligned} \max_{m_j; x_{1j}, \ldots, x_{Nj}} \quad & \beta \log(m_j) + \sum_{i=1}^{N} \alpha_{ij} \log(x_{ij}) \\ \text{subject to:} \quad & m_j + \sum_{i=1}^{N} p_i \, x_{ij} \leq M + p_j. \end{aligned}$$

We can assume without loss of generality that:

$$\beta + \sum_{i=1}^{N} \alpha_{ij} = 1. \tag{4}$$

The Lagrangian is:

$$\mathcal{L} = \beta \log(m_j) + \sum_{i=1}^{N} \alpha_{ij} \log(x_{ij}) - \lambda \left(m_j + \sum_{i=1}^{N} p_i \, x_{ij} - M - p_j \right),$$

where λ is the Lagrange multiplier. The first-order condition with respect to the demand for money m_j is:

$$\beta - \lambda \, m_j = 0. \tag{5}$$

The first-order condition with respect to x_{ij}, the demand for good i, is:

$$\alpha_{ij} - \lambda \, p_i \, x_{ij} = 0 \qquad \forall i = 1, \ldots, N. \tag{6}$$

Substituting Equation (5) into Equation (6) yields:

$$p_i \, x_{ij} = \frac{\alpha_{ij}}{\beta} \, m_j \qquad \forall i = 1, \ldots, N. \tag{7}$$

Market-Clearing Conditions The market-clearing conditions for goods 1 through N are:

$$\sum_{j=1}^{N} x_{ij} = 1 - \delta_{i1} \frac{Q}{p_1} \qquad \forall i = 1, \ldots, N,$$

where δ denotes the Kronecker symbol, i.e. δ_{i1} is equal to one if $i = 1$ and zero otherwise. Multiplying both sides by the price yields:

$$\sum_{j=1}^{N} p_i x_{ij} = p_i - \delta_{i1} Q \qquad \forall i = 1, \ldots, N.$$

Using Equation (7), we get:

$$\sum_{j=1}^{N} \frac{\alpha_{ij}}{\beta} m_j = p_i - \delta_{i1} Q \qquad \forall i = 1, \ldots, N. \tag{8}$$

Remember that we have defined the matrix $\mathbf{A} = (\alpha_{ij})_{i,j=1,\ldots,n}$. We can rewrite Equation (8) more synthetically in matrix form as:

$$\beta^{-1}\mathbf{A} \begin{bmatrix} m_1 \\ \vdots \\ m_N \end{bmatrix} = \begin{bmatrix} p_1 \\ \vdots \\ p_N \end{bmatrix} - Q\mathbf{e_1},$$

where $\mathbf{e_1}$ is a conformable vector with one in the first row and zeros everywhere else.[4] Define N-dimensional column vectors for money demand $\mathbf{m} = [m_1 \ldots m_N]'$ and for prices $\mathbf{p} = [p_1 \ldots p_N]'$. Then we have:

$$\beta^{-1}\mathbf{A}\mathbf{m} = \mathbf{p} - Q\mathbf{e_1}. \tag{9}$$

The market-clearing condition for money does not need to be examined as it will trivially be satisfied.

Budget Constraints Due to non-satiation, the budget constraint for a given agent j ($j = 1, \ldots, N$) is an equality constraint:

$$m_j + \sum_{i=1}^{N} p_i \, x_{ij} = M + p_j.$$

Substituting Equation (7) into the budget constraint yields:

$$m_j + \sum_{i=1}^{N} \frac{\alpha_{ij}}{\beta} m_j = M + p_j.$$

[4]Uppercase bold denotes matrices, and lowercase bold denotes vectors.

Thanks to Equation (4), this simplifies into:

$$\beta^{-1} m_j = M + p_j.$$

If we vectorize this expression and multiply both sides from the left by the matrix $\mathbf{A}$, we obtain:

$$\beta^{-1}\mathbf{Am} = M\mathbf{A1} + \mathbf{Ap},$$

where $\mathbf{1}$ denotes a conformable vector of ones. Since $\mathbf{A}$ is a symmetric circulant matrix, $\mathbf{1}$ is one of its eigenvectors:

$$\mathbf{A1} = \left(\sum_{j=1}^{N} \alpha_{1j}\right)\mathbf{1} = \left(\sum_{i=0}^{N-1} a_i\right)\mathbf{1} = (1-\beta)\mathbf{1}.$$

So we get:

$$\beta^{-1}\mathbf{Am} = (1-\beta)M\mathbf{1} + \mathbf{Ap}.$$

Comparing with Equation (9) yields:

$$\beta^{-1}\mathbf{Am} = \mathbf{p} - Q\mathbf{e_1} = (1-\beta)M\mathbf{1} + \mathbf{Ap}.$$

After some algebraic manipulations, we finally obtain:

$$\left(\mathbf{I} - \mathbf{A}\right)\mathbf{p} = (1-\beta)M\mathbf{1} + Q\mathbf{e_1}, \tag{10}$$

where $\mathbf{I}$ denotes a conformable identity matrix. In order to go further, we need to prove a technical lemma.

Lemma 1 *For any real square matrix $\mathbf{Z}$, let $\|\mathbf{Z}\|$ denote its spectral norm, which is defined as the square root of the largest eigenvalue of $\mathbf{ZZ}'$. Under the assumptions of Theorem 1, $\|\mathbf{A}\| < 1$.*

Proof of Lemma 1 All circulant matrices are diagonalizable. Therefore, in order to prove the lemma, it is sufficient to prove that all the eigenvalues of $\mathbf{A}$ have modulus strictly lower than 1. Since $\mathbf{A}$ is a circulant matrix, its j^{th} eigenvalue ($j = 1, \dots, n$) is given by the well-known formula (see e.g. Gray (2006), Chapter 3):

$$l_j = \sum_{k=0}^{N-1} e^{-2jk\pi i/N} a_k, \tag{11}$$

where $i = \sqrt{-1}$. Therefore its modulus is bounded by:

$$|l_j| = \left| \sum_{k=0}^{N-1} e^{-2jk\pi i/N} a_k \right| \leq \sum_{k=0}^{N-1} \left| e^{-2jk\pi i/N} a_k \right| \leq \sum_{k=0}^{N-1} \left| e^{-2jk\pi i/N} \right| \cdot |a_k| \leq \sum_{k=0}^{N-1} |a_k| = \sum_{k=0}^{N-1} a_k = 1-\beta < 1.$$

This completes the proof of Lemma 1. □

Solution The bound $\|\mathbf{A}\| < 1$ guarantees that the matrix $\mathbf{I} - \mathbf{A}$ is invertible. Therefore Equation (10) yields:

$$\mathbf{p} = (1-\beta) M \left(\mathbf{I} - \mathbf{A}\right)^{-1} \mathbf{1} + Q \left(\mathbf{I} - \mathbf{A}\right)^{-1} \mathbf{e_1}. \tag{12}$$

Define the matrix:

$$\mathbf{\Lambda} = \left(\lambda_{ij}\right)_{i,j=1,\ldots,N} = \left(\mathbf{I} - \mathbf{A}\right)^{-1}.$$

Since $\mathbf{1}$ is one of the eigenvectors of $\mathbf{A}$, it is also an eigenvector of $\mathbf{I} - \mathbf{A}$ and of $\mathbf{\Lambda}$. Therefore we have:

$$\begin{aligned} \left(\mathbf{I} - \mathbf{A}\right) \mathbf{1} &= \beta\, \mathbf{1} \\ \mathbf{\Lambda}\, \mathbf{1} &= \frac{1}{\beta}\, \mathbf{1}. \end{aligned}$$

This enables us to rewrite Equation (12) as:

$$\forall i = 1, \ldots, N \quad p_i = \frac{1-\beta}{\beta}\, M + \lambda_{i1}\, Q. \quad \square$$

We can summarize these results by the following theorem:

Theorem 1 *If the following assumptions are satisfied:*

(a) there are N agents and N real consumable goods (in addition to money), where N is an even number: $N = 2n$ for some integer $n > 1$;

(b) agent j is endowed with the quantity of money $M > 0$ and with one unit of good j;

(c) for all $j = 1, \ldots, N$, agent j has the objective function:

$$\left(\frac{m_j}{\sum_{k=1}^{N} m_k} \right)^{\beta} {x_{jj}}^{a_0} \left[\prod_{k=1}^{n-1} \left(x_{j+k,j}\; x_{j-k,j} \right)^{a_k} \right] {x_{j+n,j}}^{a_n},$$

where m_j is his demand for money, and x_{ij} is his demand for good i $(i = 1, \ldots, N)$;

(d) $\beta > 0$, $a_0 > 0$ *and* a_1,...a_{N-1} *are all non-negative;*

(e) *the exponents of the objective function are normalized so that* $\beta + a_0 + 2\sum_{k=1}^{n-1} a_k + a_n = 1$;

(f) Q *is the change in the money supply that occurs as the central bank buys (if* $Q \geq 0$*) or sells (if* $Q \leq 0$*) good 1 at the market price;*

then the equilibrium price for good i *is:*

$$p_i = \frac{1-\beta}{\beta} M + \lambda_{i1} Q \qquad \forall i = 1, \ldots, N, \tag{13}$$

where $\mathbf{\Lambda} = (\lambda_{ij})_{i,j=1,\ldots,N} = (\mathbf{I} - \mathbf{A})^{-1}$ *and* $\mathbf{A} = S_C(a_0, a_1, \ldots, a_n)$.

The meaning of Equation (13) is intuitive: if the central bank does not intervene ($Q = 0$), then the rotation-invariance of the economy is respected and the prices of all real consumable goods are equal to the baseline price $\frac{1-\beta}{\beta} M$. However, as soon as the central bank intervenes ($Q \neq 0$), rotation-invariance is violated, prices are distorted away from the baseline, and the size of the distortion is a linear function of the change in money supply Q.

3 Theoretical Predictions

Neighborhood Effects We assume that agents have closer economic ties to their immediate neighbors than to distant neighbors. This is necessary in order to induce topological notions of location and closeness in the economy. It translates into the condition:

$$a_0 \geq a_1 > a_2 \geq a_3 \geq \cdots \geq a_n. \tag{14}$$

In other words, agent j has strictly more intense need for the goods in the initial endowment of agents $j-1$ and $j+1$ than for those of agents $j-2$ and $j+2$. In turn, agent j has more intense (or the same) need for the goods in the initial endowment of agents $j-2$ and $j+2$ than for those of agents $j-3$ and $j+3$; and so forth. Of all the goods in the economy, the one for which agent j has the least intense need is the one in the initial endowment of agent $j+n$, who is located diametrically opposite him on the circle.

The most simplistic case is: $a_1 > 0$ and $a_2 = \cdots = a_{n/2} = 0$, i.e. an agent only transacts with the two neighbors on either side of him. But our results hold for the more general case described in Equation (14).

In order to perform comparative statics for the influence of the change in the money supply Q on equilibrium prices, we need to find out more about the λ_{i1}'s. This is achieved by the following theorem.

Theorem 2 *Under the assumptions of Theorem 1 and the neighborhood effects assumption (14),*

$$\forall i = 2, \ldots, N \qquad \lambda_{N+2-i,1} = \lambda_{i1}$$
$$\lambda_{11} > \lambda_{21} > \ldots > \lambda_{n1} \geq 0.$$

The proof of Theorem 2 is somewhat technical, so it is relegated to Appendix A.

Corollary 1 *Under the assumptions of Theorem 1 and the neighborhood effects assumption (14),*

$$\forall i = 2, \ldots, N \qquad p_{N+2-i} = p_i$$

and:

(i) $Q = 0 \implies p_1 = p_2 = \cdots = p_{n+1}$*;*

(ii) $Q > 0 \implies p_1 > p_2 > \cdots > p_{n+1}$*;*

(iii) $Q < 0 \implies p_1 < p_2 < \cdots < p_{n+1}$*.*

Proof of Corollary 1 This corollary follows immediately from Theorem 2 and Equation (13). □

Corollary 2 *Assume that the hypotheses of Theorem 1 and the neighborhood effects assumption (14) are satisfied. Relative to the baseline case where the money supply is constant (*$Q = 0$*):*

(i) if $Q > 0$ *then Agent* 1 *is strictly better off and Agent* $n+1$ *is strictly worse off;*

(ii) if $Q < 0$ *then Agent* 1 *is strictly worse off and Agent* $n+1$ *is strictly better off.*

Proof of Corollary 2 If $Q > 0$ ($Q < 0$) then the prices of all the goods Agent 1 consumes go down (up) relative to the price of good 1, with which this agent is endowed. As a result, his budget constraint becomes strictly less (more) binding. Therefore he is strictly better (worse) off.

If $Q > 0$ ($Q < 0$) then the prices of all the goods Agent $n + 1$ consumes go up (down) relative to the price of good $n+1$, with which this agent is endowed. As a result, his budget constraint becomes strictly more (less) binding. Therefore he is strictly worse (better) off. □

For any other agent (2 through n and $n+2$ through N), the impact of monetary intervention ($Q \neq 0$) is ambiguous because, relative to the price of the good he is endowed with, some of the goods he consumes become more expensive, and others become cheaper.

4 Evidence

In this section, we investigate several testable predictions of our theory. According to Theorem 1, economic distance from the monetary authority is the sensitivity of relative prices with respect to unanticipated monetary policy shocks. We exploit this model prediction to estimate a measure of the Economic Distance from the Fed (EDF). We describe our data sources in Subsection 4.1 and detail our empirical procedure to estimate EDF in Subsection 4.2. In Subsection 4.3, we test several predictions of the model and find supporting evidence for our mechanism for the transmission of monetary policy shocks. We discuss robustness analysis regarding the choice of our variables and our regression specifications at the end of the section.

4.1 Data

Producers' Price Index (PPI) is obtained from the Bureau of Economic Analysis (BEA), from 2005 to 2015, for a total of 15 sectors based on the two-digit North American Industry Classification System (NAICS) and the Bureau of Economic Analysis (BEA) industry classification. We use Consumer Price Index (CPI),[5] Industrial Production Index (IP), Gross Domestic Product (GDP), and money supply data from the Federal Reserve Bank of Saint Louis Economic Data (FRED). Unemployment rate is obtained from the Bureau of Labor Statistics (BLS). We deflate PPI, GDP, IP, and money supply using CPI. In the analysis, we use log changes of GDP, PPI, money supply, and unemployment rate.

For stocks returns, we consider all stocks from the Center for Research in Security Prices (CRSP) with share codes 10 and 11. Penny stocks are removed from the sample and delisting returns are taken into account. We form industry portfolios based on the same industry classification as the BEA's PPI data. Since the GDP data is available at quarterly frequency, we set all our data to quarterly frequency. The PPI data availability limits our sample to be from 2005 to 2015.

4.2 Economic Distance from the Fed

All testable implications of our theory hinge on econometrician's ability to construct an index that measures the economic distance from the Fed (EDF). We would have to assign an EDF index to every price in a given dataset. This can cover consumer prices, producer prices, wages, or even asset prices. To the best of our knowledge, this type of endeavor has never been attempted in the literature. It requires the same kind of information that goes into

[5] Consumer Price Index for All Urban Consumers: All Items.

building Leontief's (1941) input-output matrix, which models the structure of inter-industry relations.

An alternative approach is to exploit our model's predictions and build a proxy for the EDF index. Specifically, given an EDF index, Theorem 1 implies that prices will behave differently, depending on the index level they correspond to. We can therefore use money supply and Producers' Price Index data in order to build an EDF proxy. We start by defining a first proxy for monetary shocks as quarterly log changes in money supply

$$\Delta m_t = m_t - m_{t-1}, \tag{15}$$

where m is the log of the monetary base M1. Because these changes can be partly anticipated, we construct a second proxy for monetary shocks by regressing log changes in money supply on its own lags, lags of unemployment, and lags of industrial production,

$$\Delta m_t = a + \sum_{l=1}^{L} b_l \Delta m_{t-l} + \sum_{l=1}^{L} c_l \Delta u_{t-l} + \sum_{l=1}^{L} d_l \Delta ip_{t-l} + e_t^m, \tag{16}$$

where u is the log of unemployment, ip is the log of industrial production, and L is the number of lags, which in the above regression is fixed at four quarters. The residual e_t^m represents then our second, unanticipated measure of monetary shocks.

Using the PPI data for each one of the 15 sectors of the economy, we then estimate the following relationship between price changes at sectoral level and monetary shocks:

$$\Delta p_{i,t} = \alpha_i + \beta_{i,0} \Delta M_t + \beta_{i,1} \Delta M_{t-1} + \gamma_{i,1} \Delta p_{i,t-1} + e_{i,t}^p, \tag{17}$$

where p_i is the log PPI for sector i and ΔM is one of our two proxies for monetary shocks: Δm obtained from (15) or e^m obtained from (16). Notice that we add the lagged price change in the specification (17) in order to control for price-stickiness.

In what follows, we choose to describe our results using the unanticipated monetary shocks, i.e., $\Delta M_t = e_t^m$; the results are virtually the same if we use simple log changes in money supply, Δm_t. The quantity of interest is the sensitivity of price changes in each sector with respect to changes in money supply, which we interpret as the Economic Distance from the Fed. This sensitivity is directly measured by the coefficient $\beta_{i,0}$ estimated from the regression (17). It is, however, plausible that changes in money supply may have a persistent effect on prices; therefore, we build a second EDF proxy by calculating the sum of the beta coefficients:

$$\tilde{\beta}_i \equiv \beta_{i,0} + \beta_{i,1}. \tag{18}$$

A large and positive coefficient $\tilde{\beta}_i$ (or $\beta_{i,0}$) signals a low-EDF industry, whereas a large and negative coefficient signals a high-EDF industry. A coefficient close to zero signals an industry whose prices of goods are not significantly affected by by monetary shocks.

Table 1 presents the results of the regression specification (17). There is significant variation in sectors' distance from the central bank. The coefficients $\beta_{i,0}$ vary from 0.37 (Retail trade) to -1.56 (Mining). The table is ordered by the coefficients $\tilde{\beta}_i$. To capture potential delayed effects of monetary policy, we choose $\tilde{\beta}_i$ to be our measure of economic distance from the Fed (EDF) for the remaining of our analysis. Nevertheless, our results are robust to use $\beta_{i,0}$ as EDF measure as well.

4.3 Testable Implications

4.3.1 Principal Components

According to Theorem 1, in our model prices are driven by a unique common factor: the monetary shock Q. This implies that the variance-covariance matrix of price changes across sectors has only one nonzero eigenvalue, whose corresponding eigenvector is given by[6]

$$\bar{\lambda} = \begin{bmatrix} \lambda_{11} \\ \vdots \\ \lambda_{N1} \end{bmatrix}. \tag{19}$$

If our theory is a good description of reality, then (*i*) we expect to observe a strong factor structure in the data, and (*ii*) the most important factor explaining variation in price changes across sectors should indeed be highly correlated with any of our two proxies for monetary shocks. We thus perform an eigendecomposition of the covariance matrix of price changes across the 15 sectors in the economy. This eigendecomposition reveals that sectoral price changes have a strong factor structure, with the first principal component explaining nearly 80% of the variation.[7]

We plot in Figure 2 time series of our proxy of unexpected monetary shocks and of the one-quarter ahead first principal component obtained above. Both time series are normalized in order to make them comparable. The plot shows a clear positive relationship between the two series (the coefficient of correlation between the two series is 0.55; a linear regression of the one-quarter ahead principal component on this quarter's monetary shock results in a

[6]To see this, notice that the variance-covariance matrix of price changes resulting from (13) equals $\bar{\lambda}\bar{\lambda}'\sigma_Q^2$, where we denote by σ_Q^2 the variance of changes in the money supply. It follows that the unique nonnegative eigenvalue is $\bar{\lambda}'\bar{\lambda}$ and its corresponding eigenvector is $\bar{\lambda}$.

[7]This is an considerable value, given that we use deflated PPIs—without adjusting for inflation, this number would be even larger.

Table 1: **Economic Distance from the Fed (EDF).** Results of the regression specification

$$\Delta p_{i,t} = \alpha_i + \beta_{i,0}\Delta M_t + \beta_{i,1}\Delta M_{t-1} + \gamma_{i,1}\Delta p_{i,t-1} + e^p_{i,t},$$

in which price changes in each sector are regressed on unanticipated monetary shocks and their lagged values. Column (1) shows the estimated coefficients $\beta_{i,0}$, whereas column (2) shows the estimated values of $\tilde{\beta}_i$ (the EDF), defined in (18) as the sum of $\beta_{i,0}$ and $\beta_{i,1}$.

	(1) $\beta_{i,0}$	(2) $\tilde{\beta}_i \equiv \beta_{i,0} + \beta_{i,1}$	(3) Adj. R^2
Information	0.25***	0.19**	0.31
	[0.000]	[0.011]	
Retail trade	0.37***	0.16	0.32
	[0.000]	[0.106]	
Wholesale trade	0.32***	0.15*	0.33
	[0.000]	[0.090]	
Educational services	0.25***	0.11*	0.35
	[0.000]	[0.057]	
Other services	0.24***	0.11*	0.31
(except public administration)	[0.000]	[0.073]	
Arts and entertainment	0.24***	0.09	0.30
	[0.000]	[0.167]	
Professional, scientific,	0.25***	0.08	0.33
and technical services	[0.000]	[0.157]	
Finance and insurance	0.24***	0.04	0.38
	[0.000]	[0.431]	
Construction	0.30***	0.03	0.40
	[0.000]	[0.660]	
Public administration	0.17***	-0.02	0.36
	[0.000]	[0.622]	
Transportation and warehousing	0.20***	-0.19**	0.36
	[0.026]	[0.037]	
Utilities	-0.04	-0.36*	0.13
	[0.811]	[0.057]	
Manufacturing	-0.03	-0.49***	0.29
	[0.840]	[0.001]	
Agriculture, forestry,	0.05	-0.57*	0.17
fishing and hunting	[0.858]	[0.051]	
Mining, quarrying,	-1.56***	-2.45***	0.38
and oil and gas extraction	[0.009]	[0.000]	

p-values in square brackets

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

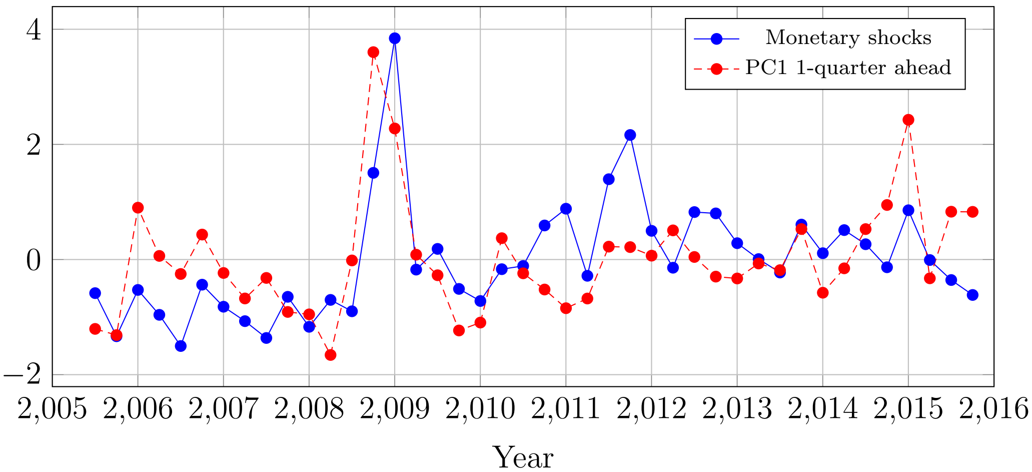

Figure 2: **Monetary Shocks and the First Principal Component of Price Changes.** The solid line represents a time-series of unexpected monetary shocks, defined as e_t^m in (16). The dashed line represents the time-series of the first principal component of sectoral-level price changes (one-quarter ahead).

strongly statistically significant coefficient, with a t-stat of 4.14). As in the model, the data shows that monetary policy shocks are a strong driver of the observed variation in prices across sectors of the economy.

Our model not only predicts that sectoral-level price changes obey a strong factor structure, but also identifies changes in money supply as the main factor driving price changes. More specifically, according to Theorem 1, regressions of price changes on monetary shocks identify exactly the coefficients λ_{i1}. Therefore, the empirical counterparts of the coefficients λ_{i1} are the coefficients $\tilde{\beta}_i$ resulting from the regression specification (17). The same coefficients λ_{i1} are also identified as the weights of the first principal component in price changes (Equation 19). This generates a testable theoretical prediction: the coefficients $\tilde{\beta}_i$ and the weights of the first principal component of sectoral-level price changes, which we denote by $\tilde{\lambda}_i$, are proportional:

$$\tilde{\beta}_i \propto \tilde{\lambda}_i. \tag{20}$$

Figure 3 depicts the relationship between $\tilde{\beta}_i$ and $\tilde{\lambda}_i$. The x-axis shows the coefficients $\tilde{\beta}_i$ resulting from the regression specification (17) and the y-axis shows the weights of the first principal component of sectoral-level price changes. Because one of the sectors (Mining) has considerably larger values for both $\tilde{\beta}_i$ and $\tilde{\lambda}_i$, we also show the same relationship in panel (*b*) after excluding this sector. The two panels show a strong positive relation in each case, in line with the theoretical prediction (20).

The two panels in Figure 3 show the linear fit lines between $\tilde{\beta}_i$ and $\tilde{\lambda}_i$. Ideally, if the

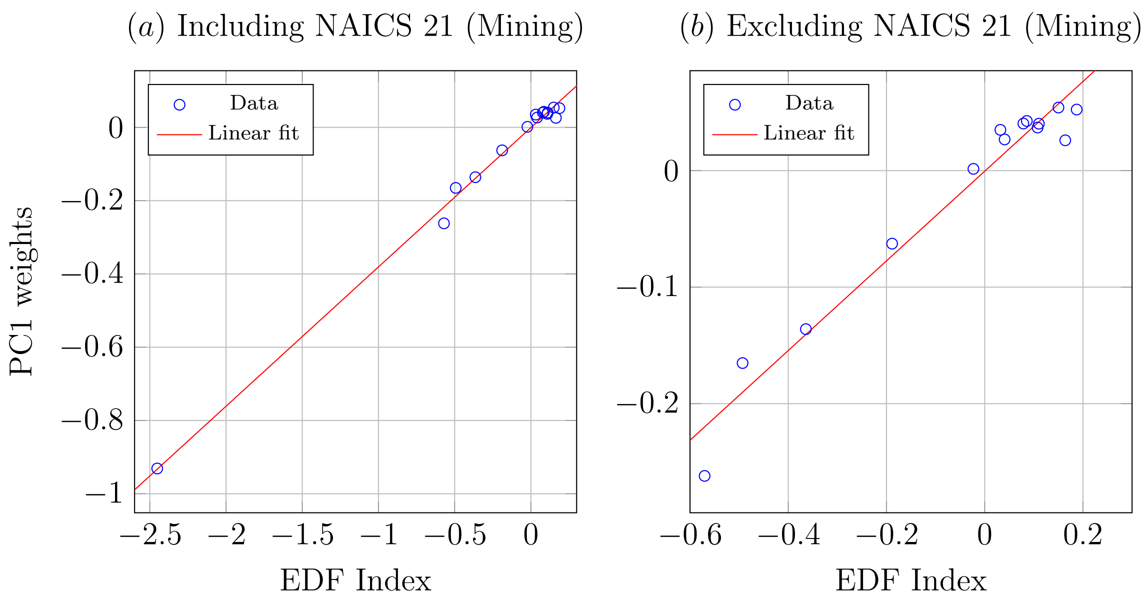

Figure 3: **EDF Index vs First Principal Component of Price Changes.** The EDF index is defined as the sum of coefficients $\beta_{i,0}$ and $\beta_{i,1}$ from the regression (17), whereas the first principal component is obtained through an eigendecomposition of the covariance matrix of sectoral-level price changes.

proportionality relation (20) is satisfied, we should expect a strong positive coefficient but also an insignificant constant coefficient. Table 2 shows the results of the regressions, with columns (1) and (2) corresponding respectively to panels (a) and (b) of Figure 3. The R-squared coefficients are above 95 percent in each case (with or without Mining). The slope coefficients are strongly statistically significant, with t-stats of 47.02 and 16.72 respectively.[8] Furthermore, the slope coefficient does not change much after removing Mining from the regression. Finally, the constant coefficients are both very close to zero and not statistically significant, as the theoretical relationship (20) predicts.

Figure 3 and Table 2 show overall support for our main theoretical prediction. Monetary shocks affect differently economic sectors: prices in sectors "closer" to the monetary authority are more sensitive to monetary shocks than sectors furthest away. In the data, as in the model, monetary shocks are the main driver of fluctuations in sector-level price changes and percolate through the economic network.

4.3.2 Correlations

We turn now to a second prediction of the model. If indeed the economic distance from the monetary authority matters, then we expect the covariance structure of prices to reflect

[8]Notice that the weights of the first principal component of sectoral-level price changes can be adjusted arbitrarily. Therefore, we do not necessarily expect a slope coefficient equal to one.

Table 2: **EDF vs Principal Component Weights.** Results of the regression specification

$$\text{PC1 weight}_i = \eta_0 + \eta_1 \tilde{\beta}_i + \varepsilon,$$

in which the weights of the first principal component of sectoral-level price changes are regressed on the EDF index. Columns (1) and (2) correspond to panels (*a*) and (*b*) of Figure 3.

	(1) With Mining	(2) Without Mining
Constant η_0	-0.001	-0.001
	(-0.115)	(-0.091)
Slope η_1	0.380***	0.385***
	(47.02)	(16.72)
R^2	0.994	0.955
Nb. Obs.	15	14

t-statistics in round brackets

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

this effect. Price changes in sectors with large and positive coefficients $\tilde{\beta}_i$ (low-EDF sectors) should covary more with each other than with price changes in other sectors. The same holds for price changes in sectors with large and negative coefficients $\tilde{\beta}_i$ (high-EDF sectors), which should covary more with each other than with price changes in other sectors. In other words, the correlation matrix of price changes across sectors should show stronger correlations among price changes in close-by sectors.

Panel (*a*) of Figure 4 draws a heatmap of the correlation matrix of price changes across sectors of the economy. We order sectors by their EDF index, as we did in Table 1. The correlation matrix indeed depicts a strong factor structure: price changes among sectors along the diagonal (close-by sectors) are more correlated with each other than price changes among sectors that are furthest away from each-other. This suggest that our proposed EDF index is indeed capturing a notion of "economic distance from the Fed."

In panel (*b*), we plot averages across the main diagonal and the upper diagonals of the correlation matrix. That is, the average of the main diagonal is one, which is the first point in the plot. Then, the second point represents the average of the first diagonal above the main diagonal; this value represents the average correlations between price changes in sectors that are "neighbors of degree one." The third point represents the average correlations between price changes in sectors that are "neighbors of degree two," and so on. Our theory predicts that these averages should go down as we move away from the main diagonal, and panel (*b*)

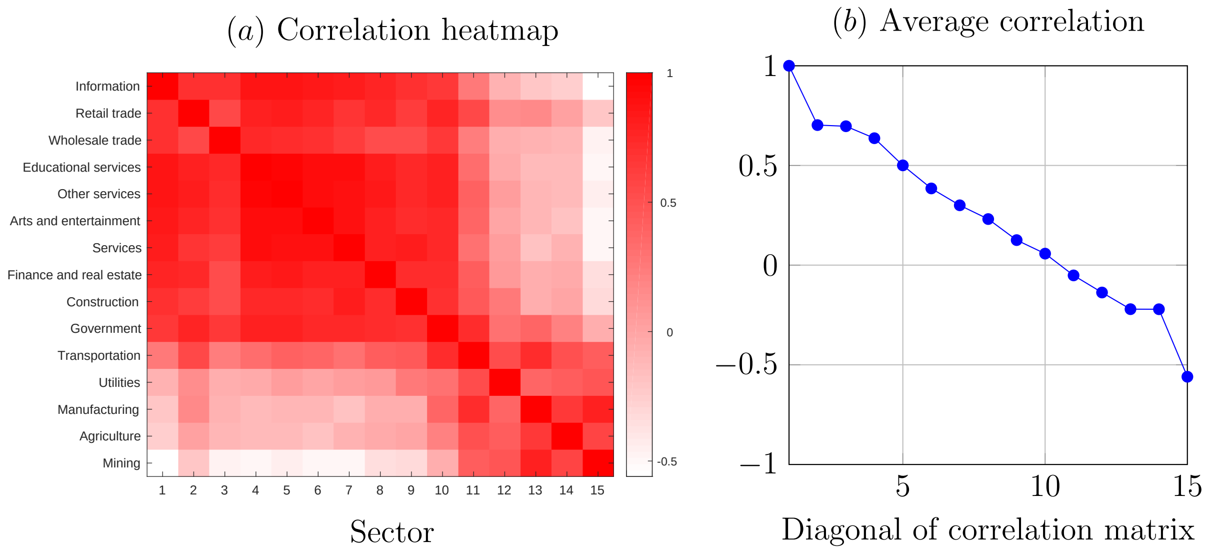

Figure 4: **Heatmap of the Correlation Matrix of Price Changes.** In panel (a), sectors are ranked by their EDF Index (coefficient $\tilde{\beta}_i$). Panel (b) computes the average correlation on the main diagonal and on the upper diagonals of the correlation matrix.

of Figure 4 shows that this is indeed the case.[9]

4.3.3 Industry Returns

The third and final testable prediction of our model results from Corollary 2 of Theorem 2. According to our model, the net effect of an expansionary monetary policy is to redistribute real consumption from the Agent $n+1$ to the Agent 1, making Agent 1 better off and Agent $n+1$ worse off. Symmetrically, a contractionary monetary policy redistributes purchasing power in the other direction, reversing the welfare implication.

In order to identify this effect, we first measure sector-level excess returns. Assuming that higher excess returns for a particular sector of the economy indicates that the industry is better off, Corollary 2 then implies that the excess returns of the sector closest to the Fed should be positively correlated with monetary shocks. Conversely, the sector furthest away should be strictly worse off when the central bank increases the money supply; hence we expect the excess returns in this sector to be negatively correlated to monetary shocks. Accordingly, in a first step, we perform the following regressions for each sector:

$$r_{i,t} = \alpha_i^r + \beta_{i,0}^r \Delta M_t + \beta_{i,1}^r \Delta M_{t-1} + \gamma_{i,1}^r r_{i,t-1} + e_{i,t}^r, \tag{21}$$

[9] The average correlations obtained in panel (b) can help in calibrating the theoretical model of Section 2. More specifically, the vector of average correlations can be mapped in a vector of non-negative Cobb-Douglas exponents α_{ij}.

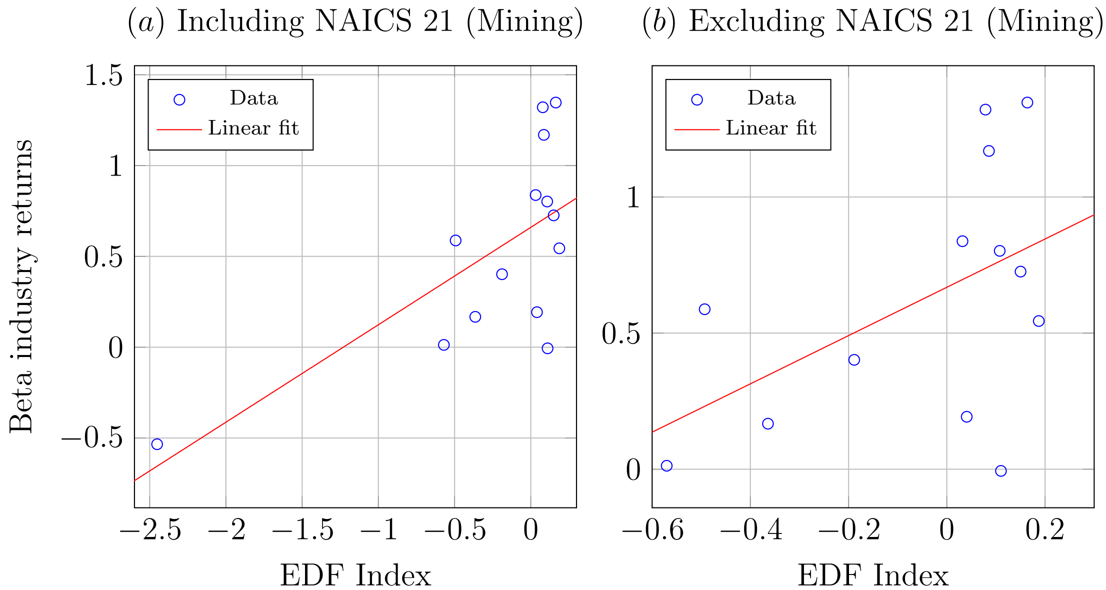

Figure 5: **EDF Index vs Betas of Industry Returns.** The EDF index is defined as the sum of coefficients $\beta_{i,0}$ and $\beta_{i,1}$ from the regression (17), whereas the betas of industry returns are defined as the sum of coefficients $\beta^r_{i,0}$ and $\beta^r_{i,1}$ from the regression (21).

where $r_{i,t}$ denotes the excess return of sector i. These regressions have the same structure as the regressions in (17) but are based on different data. We also eliminate from these regressions the sector "Public Administration" (NAICS code 92). The coefficients $\beta^r_{i,0}$ and $\beta^r_{i,1}$ capture how the returns of sector i respond to monetary shocks. Consistent with our definition of EDF, we focus on the sum of these coefficients, namely $\tilde{\beta}^r_i = \beta^r_{i,0} + \beta^r_{i,1}$, to measure which sectors benefit more from monetary shocks.

Figure 5 plots the return-based coefficients against the EDF measure. There is a positive relation between the two varaibles. If we regress $\tilde{\beta}^r_i$ on the EDF index, we obtain an statistically significant positive relationship, which holds even after excluding the mining sector. We report the results of this regressions in Table 3. These results are consistent with our theoretical prediction in Corollary 2 of Theorem 2.

Robustness We conduct a series of robustness exercises. First, our analysis is robust to different measures of money supply available from Fred: using M2 instead of M1, or other standard measures of money supply, does not change our results. Second, varying the number of lags L in the regression (16) does not change the results. Interestingly, if we add more lags to the regression (17), then results for the principal components of price get stronger. Third, if we do not deflate the PPI data then using CPI, then our results still hold. Finally, for industry returns, our findings to do change much if we use returns instead of excess returns.

Table 3: **EDF vs Betas of Industry Returns.** Results of the regression specification

$$\tilde{\beta}_i^r = \eta_0 + \eta_1 \tilde{\beta}_i + \varepsilon,$$

in which the industry returns betas obtained from (21) are regressed on the EDF index. Columns (1) and (2) correspond to panels (*a*) and (*b*) of Figure 5.

	(1) With Mining	(2) Without Mining
Constant η_0	0.660***	0.668***
	(5.620)	(5.589)
Slope η_1	0.536***	0.887*
	(3.201)	(1.909)
R^2	0.416	0.181
Nb. Obs.	14	13

t-statistics in round brackets

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

5 Conclusion

This paper introduces a model of the economy as a social network. The basic point is that everybody does not trade equally with everybody else. This fact induces topological notions of distance, neighborhood and closeness on the set of economic agents. One of the key roles of a central bank is to control the quantity of money. In order to do so, the central bank must inject money somewhere into the social network that is the economy. The effects of monetary policy will percolate through the whole network, but not uniformly so. Whoever is closest to the location where money is injected will be more strongly impacted.

Our model shows that whoever stands closest to (furthest from) the point where the central bank intervenes will benefit most (least) from unanticipated expansionary monetary policy shocks. Monetary policy redistributes consumption goods from the agents who are furthest from the central bank to those who interact directly with the central bank. We test several implications of the theory and find empirical support.

A Proof of Theorem 2

In order to prove the theorem, we must first prove 3 lemmas in succession. The most important of these lemmas is the first one.

Lemma 2 *Let* $\mathbf{X} = S_C(x_0, \ldots, x_n)$ *and* $\mathbf{Y} = S_C(y_0, \ldots, y_n)$ *be two symmetric circulant matrices of dimension* $N \times N$*. Then* $\mathbf{Z} = \mathbf{XY}$ *is also a symmetric circulant matrix:* $\mathbf{Z} = S_C(z_0, \ldots, z_n)$*. If* $x_0 \geq x_1 \geq \cdots \geq x_n \geq 0$ *and* $y_0 \geq y_1 \geq \cdots \geq y_n \geq 0$ *then* $z_0 \geq z_1 \geq \cdots \geq z_n \geq 0$*.*

Proof of Lemma 2 All circulant matrices are diagonalizable on the same set of eigenvectors (Gray (2006), Chapter 3). It is well known that the product of two symmetric matrices that share the same set of eigenvectors is also a symmetric matrix. Furthermore, the product of two circulant matrices is a circulant matrix. Therefore $\mathbf{Z} = \mathbf{XY}$ is a symmetric circulant matrix. By writing down the formula for the product of two matrices, we find that:

$$\begin{aligned} \forall k = 0, \ldots, n \qquad z_k &= \sum_{j=0}^{N-1} x_j y_{j-k} &(22)\\ z_{k+1} - z_k &= \sum_{j=0}^{N-1} x_j \left(y_{j-k-1} - y_{j-k}\right). &(23) \end{aligned}$$

From Equation (22) it is obvious that when all the x_i's and all the y_i's are greater than or equal to zero, so are the z_i's. Now let us define: $y'_j = y_j - y_{j-1}$ for all $j = 0, \ldots, N-1$. It is easy to verify that $y'_{N+1-j} = y'_{1-j} = -y'_j$. Furthermore, by the assumptions of Lemma 2, $y'_j \geq 0$ for $j \in \{0, n+1, n+2, \ldots, N-1\}$ and $y'_j \leq 0$ for $j \in \{1, 2, \ldots, n\}$. We can now rewrite Equation (23) more concisely as:

$$\forall k = 0, \ldots, n \qquad z_{k+1} - z_k = -\sum_{j=0}^{N-1} x_j\, y'_{j-k}. \tag{24}$$

We shall prove that this quantity is nonpositive by considering two cases separately. We use the symbol $\lfloor\,\rfloor$ to denote the floor function.

First Case: $k \in \{0, 1, 2, \ldots, \lfloor \frac{n-1}{2} \rfloor\}$ In this case, we can split the summation on the right-hand side of Equation (24) into:

$$\begin{aligned} z_{k+1} - z_k &= -\sum_{j=0}^{k} x_j\, y'_{j-k} - \sum_{j=k+1}^{2k+1} x_j\, y'_{j-k} - \sum_{j=n}^{k+n} x_j\, y'_{j-k} \\ &\quad - \sum_{j=k+n+1}^{2k+n+1} x_j\, y'_{j-k} - \sum_{j=2k+2}^{n-1} x_j\, y'_{j-k} - \sum_{j=2k+n+2}^{N-1} x_j\, y'_{j-k}. \end{aligned}$$

Let us make the change of variable $i = 2k + 1 - j$ in the second summation, and the change of variable $i = 2k + N + 1 - j$ in the fourth and sixth summations. This yields:

$$\begin{aligned} z_{k+1} - z_k &= -\sum_{j=0}^{k} x_j\, y'_{j-k} - \sum_{i=0}^{k} x_{2k+1-i}\, y'_{k+1-i} - \sum_{j=n}^{k+n} x_j\, y'_{j-k} \\ &\quad - \sum_{i=n}^{k+n} x_{2k+N+1-i}\, y'_{k+N+1-i} - \sum_{j=2k+2}^{n-1} x_j\, y'_{j-k} - \sum_{i=2k+2}^{n-1} x_{2k+N+1-i}\, y'_{k+N+1-i} \\ &= -\sum_{j=0}^{k} x_j\, y'_{j-k} + \sum_{i=0}^{k} x_{2k+1-i}\, y'_{i-k} - \sum_{j=n}^{k+n} x_j\, y'_{j-k} \\ &\quad + \sum_{i=n}^{k+n} x_{2k+N+1-i}\, y'_{i-k} - \sum_{j=2k+2}^{n-1} x_j\, y'_{j-k} + \sum_{i=2k+2}^{n-1} x_{i-2k-1}\, y'_{i-k} \\ &= \sum_{j=0}^{k} (x_{2k+1-j} - x_j)\, y'_{j-k} + \sum_{j=n}^{k+n} (x_{2k+N+1-j} - x_j)\, y'_{j-k} \\ &\quad + \sum_{j=2k+2}^{n-1} (x_{j-2k-1} - x_j)\, y'_{j-k}. \end{aligned} \tag{25}$$

Let us consider these three summations separately:

- When $j \in \{0, 1, 2, \ldots, k\}$, we have:

$$\begin{aligned} 0 \le j \le 2k + 1 - j \le n &\implies x_{2k+1-j} - x_j \le 0 \\ \text{and} \quad 1 - n \le j - k \le 0 &\implies y'_{j-k} \ge 0, \end{aligned}$$

  therefore the 1st summation is less than or equal to zero.

- When $j \in \{n, n+1, \ldots, n+k\}$, we have:

$$\begin{aligned} n \le j \le 2k + N + 1 - j \le N &\implies x_{2k+N+1-j} - x_j \ge 0 \\ \text{and} \quad 1 \le j - k \le n &\implies y'_{j-k} \le 0, \end{aligned}$$

therefore the 2nd summation is less than or equal to zero.

- When $j \in \{2k+2, 2k+3, \dots, n-1\}$, we have:

$$\begin{aligned} 0 \le j-2k-1 \le j \le n &\implies x_{j-2k-1} - x_j \ge 0 \\ \text{and} \quad 1 \le j-k \le n &\implies y'_{j-k} \le 0, \end{aligned}$$

therefore the 3rd summation is less than or equal to zero.

Together, these results establish that $z_{k+1} - z_k \le 0$ for all $k \in \{0, 1, 2, \dots, \lfloor \frac{n-1}{2} \rfloor\}$.

Second Case: $k \in \{\lfloor \frac{n-1}{2} \rfloor + 1, \lfloor \frac{n-1}{2} \rfloor + 2, \dots, n-1\}$ In this case, we can split the summation on the right-hand side of Equation (24) into:

$$\begin{aligned} z_{k+1} - z_k \;\; = \;\; & -\sum_{j=2k-n+1}^{k} x_j\, y'_{j-k} - \sum_{j=k+1}^{n} x_j\, y'_{j-k} - \sum_{j=2k+2}^{k+n} x_j\, y'_{j-k} \\ & -\sum_{j=k+n+1}^{N-1} x_j\, y'_{j-k} - \sum_{j=0}^{2k-n} x_j\, y'_{j-k} - \sum_{j=n+1}^{2k+1} x_j\, y'_{j-k}. \end{aligned}$$

Let us make the change of variable $i = 2k+1-j$ in the second and sixth summations, and the change of variable $i = 2k+N+1-j$ in the fourth summation. This yields:

$$\begin{aligned} z_{k+1} - z_k \;\; = \;\; & -\sum_{j=2k-n+1}^{k} x_j\, y'_{j-k} - \sum_{i=2k-n+1}^{k} x_{2k+1-i}\, y'_{k+1-i} - \sum_{j=2k+2}^{k+n} x_j\, y'_{j-k} \\ & -\sum_{i=2k+2}^{k+n} x_{2k+N+1-i}\, y'_{k+N+1-i} - \sum_{j=0}^{2k-n} x_j\, y'_{j-k} - \sum_{i=0}^{2k-n} x_{2k+1-i}\, y'_{k+1-i} \\ = \;\; & -\sum_{j=2k-n+1}^{k} x_j\, y'_{j-k} + \sum_{i=2k-n+1}^{k} x_{2k+1-i}\, y'_{i-k} - \sum_{j=2k+2}^{k+n} x_j\, y'_{j-k} \\ & +\sum_{i=2k+2}^{k+n} x_{2k+N+1-i}\, y'_{i-k} - \sum_{j=0}^{2k-n} x_j\, y'_{j-k} + \sum_{i=0}^{2k-n} x_{N-2k-1+i}\, y'_{i-k} \\ = \;\; & \sum_{j=2k-n+1}^{k} (x_{2k+1-j} - x_j)\, y'_{j-k} + \sum_{j=2k+2}^{k+n} (x_{2k+N+1-j} - x_j)\, y'_{j-k} \\ & + \sum_{j=0}^{2k-n} (x_{N-2k-1+j} - x_j)\, y'_{j-k}. \end{aligned} \tag{26}$$

Let us consider these three summations separately:

- When $j \in \{2k-n+1, 2k-n+2, \ldots, k\}$, we have:

$$\begin{aligned} 0 \leq j \leq 2k+1-j \leq n &\implies x_{2k+1-j} - x_j \leq 0 \\ \text{and} \quad 1-n \leq j-k \leq 0 &\implies y'_{j-k} \geq 0, \end{aligned}$$

  therefore the 1st summation is less than or equal to zero.

- When $j \in \{2k+2, 2k+3, \ldots, k+n\}$, we have:

$$\begin{aligned} n \leq j \leq 2k+N+1-j \leq n &\implies x_{2k+N+1-j} - x_j \geq 0 \\ \text{and} \quad 1 \leq j-k \leq n &\implies y'_{j-k} \leq 0, \end{aligned}$$

  therefore the 2nd summation is less than or equal to zero.

- When $j \in \{0, 1, 2, \ldots, 2k-n\}$, we have:

$$\begin{aligned} 0 \leq j \leq N-2k-1+j \leq n &\implies x_{N-2k-1+j} - x_j \leq 0 \\ \text{and} \quad 1-n \leq j-k \leq 0 &\implies y'_{j-k} \geq 0, \end{aligned}$$

  therefore the 3rd summation is less than or equal to zero.

Together, these results establish that $z_{k+1} \leq z_k$ for all $k \in \left\{\lfloor\frac{n-1}{2}\rfloor + 1, \lfloor\frac{n-1}{2}\rfloor + 2, \ldots, n-1\right\}$.

Bringing together the two cases, we conclude that $z_0 \geq z_1 \geq \cdots \geq z_n$. This completes the proof of Lemma 2. □

Lemma 3 *Let* $\mathbf{D}$ *be a symmetric circulant matrix:* $\mathbf{D} = S_C(d_0, d_1, \ldots, d_n)$*, where* $d_0 \geq d_1 \geq \ldots \geq d_n \geq 0$*. Then for all* $m \in \mathbb{N}$ $\mathbf{D}^m$ *is also symmetric circulant matrix:* $\mathbf{D}^m = S_C(d_{0m}, d_{1m}, \ldots, d_{nm})$*. Furthermore, we have:* $d_{0m} \geq d_{1m} \geq \ldots \geq d_{nm} \geq 0$*.*

Proof of Lemma 3 We shall prove the lemma by induction. For $m = 0$, $\mathbf{D}^0 = \mathbf{I}$, so the statement is trivially true. Now let us suppose that the statement is true for some power $m \in \mathbb{N}$. We have the decomposition: $\mathbf{D}^{m+1} = \mathbf{D}^m\mathbf{D}$. Notice that the hypotheses of Lemma 2 are verified when we take $\mathbf{X} = \mathbf{D}^m$ and $\mathbf{Y} = \mathbf{D}$. So Lemma 2 implies that the statement is true for the power $m+1$. Therefore, by induction, the statement is true for all $m \in \mathbb{N}$. □

Lemma 4 *Under the assumptions of Lemma 3, if* $d_1 > d_2$ *then for all* $m = 0, \ldots, n-1$ $d_{mm} > d_{m+1,m}$*.*

Proof of Lemma 4 We shall prove this lemma by induction. Since $\mathbf{D}^0 = \mathbf{I}$, we have $d_{00} = 1$ and $d_{10} = 0$. Therefore the statement is true for $m = 0$. Now let us suppose that the statement is true for some $m \in \{0, \ldots, n-2\}$. Consider the decomposition: $\mathbf{D}^{m+1} = \mathbf{D}^m \mathbf{D}$. The hypotheses of Lemma 2 are verified when we take $\mathbf{Z} = \mathbf{D}^{m+1}$, $\mathbf{X} = \mathbf{D}^m$ and $\mathbf{Y} = \mathbf{D}$. From Equations (25-26) in the proof of Lemma 2, we know that:

$$\forall k = 0, \ldots, n-1 \qquad z_{k+1} - z_k \leq \sum_{j=\max(0,2k-n+1)}^{k} \left(x_{2k+1-j} - x_j\right) y'_{j-k},$$

where all the terms in the summation are less than or equal to zero. Taking $k = m+1$ and $j = m$, we obtain:

$$z_{m+2} - z_{m+1} \leq \left(x_{m+3} - x_m\right) y'_{-1}.$$

Since

$$\begin{aligned} x_{m+3} - x_m \leq x_{m+1} - x_m &< 0 \\ \text{and} \quad y'_{-1} = y'_2 = y_2 - y_1 &< 0 \end{aligned}$$

by assumption, it follows that: $z_{m+2} - z_{m+1} < 0$, i.e. $d_{m+2,m+1} - d_{m+1,m+1} < 0$. Therefore the statement is true for $m+1$. By induction, this means that it is true for all $m = 0, \ldots, n-1$. This completes the proof of Lemma 4. $\square$

Proof of the Theorem We are now ready to complete the proof of Theorem 2. Since the matrix $\mathbf{A}$ satifies the assumptions of Lemma 3, we know that for all $m \in \mathbb{N}$ $\mathbf{A}^m$ is a symmetric circulant matrix: $\mathbf{A}^m = S_C\left(a_{0m}, a_{1m}, \ldots, a_{nm}\right)$, and that: $a_{0m} \geq a_{1m} \geq \ldots \geq a_{nm} \geq 0$.

The bound $\|\mathbf{A}\| < 1$ established in Lemma 1 guarantees that the matrix $\mathbf{I} - \mathbf{A}$ is invertible, i.e. that $\boldsymbol{\Lambda} = (\mathbf{I} - \mathbf{A})^{-1}$ exists. Since $\mathbf{A}$ is a symmetric circulant matrix, $\mathbf{I} - \mathbf{A}$ is also a circulant matrix, and so is $\boldsymbol{\Lambda} = (\mathbf{I} - \mathbf{A})^{-1}$. Thus, we can write: $\boldsymbol{\Lambda} = S_C\left(l_0, l_1, \ldots, l_n\right)$. The layout of a symmetric circulant matrix is such that:

$$\begin{aligned} \forall i = 1, \ldots, n+1 \qquad \lambda_{i1} &= l_{i-1} \\ \forall i = n+2, \ldots, N \qquad \lambda_{i1} &= l_{N+1-i}. \end{aligned}$$

This implies that $\forall i = 2, \ldots, N \quad \lambda_{N+2-i} = \lambda_i$, which was one of the assertions of Theorem 2 that needed to be proven.

Thanks to Lemma 1, we know that the series $\sum_{m=0}^{\infty} \mathbf{A}^m$ is convergent. Standard arguments (see e.g. Szidarovszky and Molnár (2002), pp. 375–376) show that the sum of this series is: $\sum_{m=0}^{\infty} \mathbf{A}^m = (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{\Lambda}$. In particular, the entries of the matrix $\mathbf{\Lambda}$ satisfy:

$$\forall j = 0, \ldots, n \qquad l_j = \sum_{m=0}^{\infty} a_{jm}.$$

Therefore they inherit their ordering from the a_{jm}'s, and we have:

$$l_0 \geq l_1 \geq \ldots \geq l_n \geq 0.$$

The fact that these inequalities are actually strict:

$$l_0 > l_1 > \ldots > l_n$$

is a straightforward application of Lemma 4, given the assumption $a_1 > a_2$. Therefore:

$$\lambda_{11} > \lambda_{21} > \ldots > \lambda_{n+1,1} \geq 0. \quad \square$$

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