Schumpeterian Competition and Financial Markets*

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ABSTRACT

Creative destruction not only involves bringing new technology to market, it imposes higher risk on the future of existing assets. We characterize the asset pricing implications of creative destruction when Schumpeterian competition takes place. Compared to first best, the quest for oligopoly rents leads to over-investment in uncertain projects, spikes in the price-dividend ratio, and an aftermath in which prices fall steeply as uncertainty resolves. If competition for rents is sufficiently intense, the elevated price-dividend ratio predicts negative future expected excess returns. This may provide a rational explanation for why periods in which investors race to market during technological change may promote investment bubbles. Our analysis yields novel empirical predictions and we discuss how creative destruction affects the term structure of risk.

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1 Introduction

According to Schumpeter (1934), the propensity for creative destruction depends on the rents availble for extraction and the number of competitors who race to the new paradigm technology. This has spawned a large literature that characterizes Schumpeterian competition (e.g., Futia, 1980), with a focus on competitive industry analysis (e.g., Williamson, 1965; Reinganum, 1983; Aghion, Bloom, Blundell, Griffith, and Howitt, 2005)[1] and quantifying how creative destruction changes social welfare (e.g., Witt, 1996; Aghion, Akcigit, Bergeaud, Blundell, and Hémous, 2015; Komlos, 2014).[2]

The purpose of this paper is to study the effect of Schumpterian competition on financial markets. Competition in general is associated with overinvestment in new technology and asset over-valuation (DeMarzo, Kaniel, and Kremer, 2007). But, creative destruction is different in two respects. First, new technology makes the future of existing assets more risky (Kung and Schmid, 2015). Gârleanu, Kogan, and Panageas (2012) refer to this as "displacement risk" and show that it can rationalize both the existence of the growth-value factor in returns, as well as the equity premium. Because creative destruction may divert scarce resources away from existing assets or alter their growth options and capabilities, it is different from simply adding an exogenous new asset to a well-diversified portfolio. Creative destruction may endogenously change the variance-covariance matrix of existing assets, which may affect the ability of investors to bear risk. As such, it appears to be associated with a risk premium (Grammig and Jank, 2015).

The second difference is that creative destruction involves learning by doing and has an observer effect. When capital gets allocated to a new opportunity, learning occurs via *experimentation* (e.g., Arrow, 1962; Grossman, Kihlstrom, and Mirman, 1977; Aghion, Bolton, Harris, and Jullien, 1991; Rob, 1991), which affects expectations about existing assets in the rest of the market. This "perturbs" the system of asset prices and expectations, so learning has feedback effects to the rest of the market and is costly through its effect on risk. Such learning by doing contrasts with the standard learning processes that are typically analyzed in the asset pricing literature,

[1] See also Swan (1970), Loury (1979), Reinganum (1985), Aghion and Howitt (1992), Boldrin and Levine (2008), Aghion and Griffith (2008).

[2] See also Carree and Thurik (2005), Bampoky, Prieger, Blanco, and Liu (2016), Acemoglu and Restrepo (2017), and Akcigit, Grigsby, and Nicholas (2017).

whereby agents update their beliefs from a time-series of signals that they receive for free.[3]

Taking these two differences into account, we characterize a model which marries non-cooperative, game-theoretic competition for industry rents with a Lucas (1978) endowment economy where agents have access to a dividend-consumption stream. Investment in creative destruction has two effects on the consumption stream: it changes the growth rate of consumption—hopefully for the better—and it amplifies the magnitude of its diffusion. The former captures the fact that new technologies have uncertain benefits for economic growth; the latter captures the fact that new technologies have unintended consequences and make the future of existing assets more uncertain. Strategic interaction is modeled as an aggregative game (i.e., Cournot competition) in which competitors make simultaneous choices about how much to experiment with the new technology and the payoffs are a function of each investor's claim to an endogenous dividend stream.[4] That is, each investor's choice affects both the size of the pie as well as the share.

Compared to the first best benchmark, when competitors fight for market share, this leads to overinvestment, and the aggregate experimentation grows with the number of investors. In turn, with excessive experimentation, both the volatility of future consumption and the uncertainty about the expected growth rate of the economy are magnified. In equilibrium, this results in a spike in the price-dividend ratio because of Jensen's inequality effects—the asset has a *convex*, option-like payoff and becomes more valuable with over-experimentation. This spike increases with competition and with the level experimentation. Subsequently, as uncertainty resolves over time, there is an aftermath in which prices reverse and return to normal. Higher experimentation leads to faster learning and resolution of uncertainty, which causes prices to fall quickly after the run up.

Our work has implications for time-series return predictability and the term structure of risk. We show that when competition for rents is sufficiently intense, the elevated price-dividend ratio may in fact predict negative future expected excess returns. This may help to explain why periods in which investors race to market during

[3]The incomplete information literature starts with Williams (1977), Detemple (1986), Dothan and Feldman (1986), and Gennotte (1986). A comprehensive survey is provided in Ziegler (2003).

[4]In the aggregative game, the payoff to each player is a function of the sum of their own choice and that of all competitors. This not only affects each player's consumption share, but affects the nature of the consumption stream they get to enjoy.

technological change may promote investment bubbles (e.g., the Tronics boom, 1959-1962; the Biotech bubble of the 1980's; the Dotcom era, 1995-2001). Additionally, we show that the systematic risk imposed by creative destruction is not constant across maturities, but rather concentrated at the short end. Investors demand much higher compensation for the "destruction" that innovation imposes in the short-run.

Our model builds on the insights in DeMarzo et al. (2007), who analyze a model of competition among multiple agents. In their economy, a "keep up with the Joneses" concern arises endogenously as no agent wants to be left behind. This leads to over-investment that is predictably unprofitable. Such over-investment arises in our paper as well, but we focus on characterizing the explicit asset pricing quantities that evolve in the market. This allows us to show how elevated price-dividend ratios arise from the uncertainty that is imposed by the race to the new paradigm shift and how elevated prices may in fact induce negative future expected excess returns. We view the two papers as complementary.

Our paper is also related to Pastor and Veronesi (2009), but the channel through which innovation affects asset prices is different. There is no competition or over-investment in their framework. Prices rise and then fall as risk changes from idiosyncratic to systematic in nature. Further, whereas learning in our model impacts asset prices in real time (learning by doing), the agent in Pastor and Veronesi (2009) is able to experiment costlessly with a negligible amount of the new technology, which does not disrupt the status quo until the agent is sure that a paradigm shift is warranted.

The rest of the paper proceeds as follows. Section 2 poses the model, characterizes learning and first best experimentation, and contrasts the equilibrium behavior of competitive agents to first best. Section 3 characterizes the asset pricing implications of creative destruction. Section 4 concludes. All proofs are relegated to the Appendix.

2 Experimentation and Learning

Consider an exchange economy defined over a continuous-time finite horizon $[0, T]$. In the status quo, the aggregate output is

$$\frac{d\delta_t^S}{\delta_t^S} = \bar{f}dt + \sigma dW_t, \tag{1}$$

where the parameters $\bar{f}$ and σ are known. The economy is populated by an *active* investor who consumes a share θ of the output and a *passive* investor who consumes fraction $1-\theta$. For now, we fix θ exogenously so that the active investor cannot modify her consumption share. Later, we endogenize θ, which will reflect rent seeking behavior and strategic considerations among competitive active investors.

At time $t = 0$, the active investor has the choice to re-allocate existing capital $x_0 \geq 0$ to an *experimental* asset in the market (e.g., a new technology). This new technology is sufficiently important to affect the entire economy. It affects the aggregate consumption stream in two ways. First, the new asset has an unknown effect on the drift, which becomes $(\bar{f} + \beta x_0)$. The parameter β is unknown and captures both the adverse effect that creative destruction imposes on other assets and a possible benefit in higher future consumption. Thus, innovation can both fuel economic growth (Aghion and Howitt, 1992; Romer, 1990) and be destructive (e.g., Acemoglu and Restrepo, 2017), and β is similar to a Schumpeter "creativity ratio" as defined in Komlos (2014). We assume that the active investor has initial beliefs such that

$$\beta \sim N(\widehat{\beta}_0, \nu_0), \tag{2}$$

where $\widehat{\beta}_0 > 0$, so that the active investor starts with an initial prior that changing the status quo and investing in the experimental asset is a good idea.

Second, investment in the experimental asset amplifies the magnitude of the diffusion term to $(1 + kx_0)\sigma$, which captures the increased economic risk that creative destruction introduces into the economy (Kung and Schmid, 2015). Taken together, for any x_0, the dynamics for the aggregate output stream in the new economy is

$$\frac{d\delta_t}{\delta_t} = (\bar{f} + \beta x_0)dt + (1 + kx_0)\sigma dW_t. \tag{3}$$

In Appendix A.1.1, we provide a microfoundation for (3) in an economy with two differentiated goods. Here, we take (3) as a starting point for our analysis.[5]

[5] Johnson (2007) provides a discrete-time microfoundation in a production economy, which is consistent with our choice of allowing experimentation to affect the drift. However, we depart in that experimentation affects the uncertainty regarding the interaction between the new technology and existing assets.

The agents have preferences over lifetime consumption

$$U(c_i) = \mathbb{E}\left[\int_0^T e^{-\rho t}\frac{c_{i,t}^{1-\gamma}}{1-\gamma}dt\right], \tag{4}$$

where $c_{a,t} = \theta\delta_t$ for the active investor and $c_{p,t} = (1-\theta)\delta_t$ for the passive investor. Both agents have the same coefficient of risk aversion, $\gamma \geq 1$. This ensures that the agents' expected utility is higher under complete information than under incomplete information.[6] Furthermore, this is consistent with empirical evidence[7], and is frequently used in analyses like these (e.g., Pastor and Veronesi, 2009). Notwithstanding, in separate analysis we have explored an alternative model with stochastic differential utility (Epstein and Zin, 1989); the results presented in this paper remain valid as long as the elasticity of intertemporal substitution is below one.[8]

Only the active investor can choose x_0. At $t = 0$, the active investor commits to an experimentation level x_0 that remains constant from time 0 to T. As such, the active investor chooses how far to open Pandora's box at $t = 0$ and then both investors live with the consequences. Because θ is fixed and investors are otherwise identical, the choice of the active investor is the first best choice (it simultaneously maximizes the lifetime utility of both investors—see Proposition 2).

If the active investor chooses $x_0 = 0$, then the economy remains in the status quo. Once the active investor chooses $x_0 > 0$, both investors observe the total output δ_t and learn over time how the new asset impacts future expected growth. But the experiment comes at a cost: it disturbs the process by increasing the magnitude of the diffusion. This implies that there is an observer effect, which is distinct from what is typically modeled in the asset pricing literature with incomplete information. Usually, agents update their beliefs by observing signals about the drift of the dividend process for free. Instead, in our case learning occurs *by doing*, as in Arrow (1962),

[6] More precisely, for any time-additive utility function which is increasing and concave in current consumption, better information will increase expected utility whenever the second derivative of the utility with respect to the natural logarithm of consumption is negative. In the case of power utility, this condition is satisfied when $\gamma > 1$. See Chapter 2 in Ziegler (2003) for a discussion.

[7] Friend and Blume (1975) estimate an average coefficient of relative risk aversion well in excess of one and perhaps in excess of two. Dreze (1981) finds even higher values using an analysis of deductibles in insurance contracts. See also Mehra and Prescott (1985, p. 154).

[8] These separate calculations are available upon request. Empirical studies disagree about reasonable values for the EIS. Some studies find EIS greater than one (Vissing-Jørgensen and Attanasio, 2003), other studies find EIS smaller than one (Campbell, 1999; Vissing-Jørgensen, 2002).

Grossman et al. (1977), and Rob (1991). However, unlike these papers, the cost of experimentation is the added disturbance introduced into the diffusion term through the parameter $k > 0$ and the added uncertainty about the true expected growth of the experimental asset.

Both investors observe the aggregate output stream δ_t, whose changes are informative about the unknown parameter β. Since the choice of x_0 is common knowledge, both investors form expectations under the same probability measure.

Proposition 1 ***(Learning)*** *From investors' viewpoint, this partially observed economy is equivalent to a perfectly observed economy with consumption process*

$$\frac{d\delta_t}{\delta_t} = (\bar{f} + \widehat{\beta}_t x_0)dt + \sigma(1 + kx_0)d\widehat{W}_t, \tag{5}$$

where

$$d\widehat{\beta}_t = \frac{x_0}{\sigma(1 + kx_0)}\nu_t d\widehat{W}_t, \tag{6}$$

$$d\nu_t = -\frac{x_0^2}{\sigma^2(1 + kx_0)^2}\nu_t^2 dt, \tag{7}$$

and $d\widehat{W}_t \equiv dW_t + \frac{x_0(\beta - \widehat{\beta}_t)}{\sigma(1+kx_0)}dt$ *represents the "surprise" component of the change in total consumption.*

Investors revise their estimate of β in the direction of the output surprises they observe (Brennan, 1998). We define ν_t as the Bayesian uncertainty about β at time t (i.e., posterior variance). The expression in (7) together with the initial condition ν_0 implies a deterministic path for ν_t:

$$\nu_t = \frac{1}{\frac{x_0^2}{\sigma^2(1+kx_0)^2}t + \frac{1}{\nu_0}}. \tag{8}$$

The posterior variance starts at ν_0 but then decays to zero as t goes to infinity. One benefit of experimentation is that investors can learn about the new technology and lower the future Bayesian uncertainty. Moreover, uncertainty decreases faster when x_0 is high. However, when $k > 0$, experimentation also has a negative effect on learning because it disturbs the economy. In fact, consider the limit of the speed of learning (the term multiplying time t in the denominator of (8)) as $x_0 \to \infty$:

$$\frac{1}{k^2\sigma^2}. \tag{9}$$

For any $k > 0$, the speed of learning cannot go above (9) in any finite time. Because experimentation disturbs the economy, it indeed applies a brake to learning.

2.1 First Best Experimentation

The active investor's problem is to choose a level of experimentation that balances between information gains coupled with the chance of a good experiment and higher disturbance to future consumption coupled with the chance of a bad experiment. This tradeoff affects the expected value of future dividends (i.e., $\mathbb{E}_0[\delta_t]$) and the future volatility of dividends (i.e., $\mathrm{Var}_0[\ln\delta_t]$), which we derive in Appendix A.1.3. There, we show that both $\mathbb{E}_0[\delta_t]$ and $\mathrm{Var}_0[\ln\delta_t]$, for any $t > 0$, unambiguously increase with experimentation, consistent with the idea that creative destruction increases expected growth but also makes the future more uncertain.

The above tradeoff affects the active investor's lifetime expected utility of consumption at time t, which is defined as

$$J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \mathbb{E}_t\left[\int_t^T e^{-\rho s}\frac{(\theta\delta_s)^{1-\gamma}}{1-\gamma}ds\right] = \frac{\theta^{1-\gamma}}{1-\gamma}\int_t^T e^{-\rho s}\mathbb{E}_t\left[\delta_s^{1-\gamma}\right]ds. \tag{10}$$

In Appendix A.1.4, we show that

$$J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \frac{e^{-\rho t}(\theta\delta_t)^{1-\gamma}}{1-\gamma}F(\widehat{\beta}_t, \nu_t, t), \tag{11}$$

where $F(\widehat{\beta}_t, \nu_t, t)$ is the price-dividend ratio in this economy (to be defined and fully characterized in Proposition 5). We further show that the value function unambiguously increases with $\mathbb{E}_0[\delta_t]$ and unambiguously decreases with $\mathrm{Var}_0[\ln\delta_t]$, for any $t > 0$. This tradeoff implies an optimal level of experimentation.

Proposition 2 ***(Optimal Experimentation)*** *At $t = 0$, the active investor chooses* *an optimal level of experimentation that maximizes welfare. This level is*

$$x_S^* = \begin{cases} \frac{(\widehat{\beta}_0 - \gamma k \sigma^2)\mathbb{D}_0}{\gamma k^2 \sigma^2 \mathbb{D}_0 + (\gamma - 1)\nu_0 \mathbb{C}_0}, & \text{if } \widehat{\beta}_0 - \gamma k \sigma^2 > 0 \\ 0, & \text{otherwise,} \end{cases} \tag{12}$$

where $\mathbb{D}_0$ is the equity duration (i.e., the weighted average maturity of discounted cash-flows) and $\mathbb{C}_0$ is the equity convexity (i.e., the weighted average squared maturity of discounted cash-flows).[9]

The optimal level of experimentation in (12) resembles a mean-variance portfolio. The expression (12) is implicit in x_0^*, however, because the duration and convexity of the cash flows are functions of x_0^*. Experimentation has more impact when the duration of cash-flows is higher (both in terms of higher growth and higher future consumption volatility) because the active investor's choice has a longer-lasting impact on the economy. When the margin between the expected benefit of experimentation and the penalty from disturbing the economy is higher, the active investor's incentive to experiment increases further with the duration of cash flows. On the other hand, uncertainty lowers the active investor's incentive to experiment. This effect is stronger when the convexity of future cash flows is high.

Going forward, we assume that the first best level of experimentation x_S^* is strictly positive, i.e., the condition $\widehat{\beta}_0 - \gamma k \sigma^2 > 0$ is satisfied.

2.2 Experimentation with Rent Seeking Behavior

The amount of Schumpeterian competition that arises in the economy is a function of the industry market structure and the cost of new entry and imitation (e.g., Futia, 1980). As such, the degree of creative destruction that results is driven by the potential monopoly rents that are available and it is often associated with a wealth transfer from the owners of existing assets to people who innovate. It is also affected by the degree to which cannabalism of existing technologies occurs when creative destruction takes place. Investors who engage in creative destruction often tradeoff between the wealth gains they enjoy from new knowledge and the losses they endure because the existing assets they own will be rendered obsolete. This mirrors Schumpeter's idea that creative destruction, "incessantly revolutionizes the economic

[9] These positive quantities, defined in Appendix A.1.5, are omitted here for ease of exposition.

structure *from within*, incessantly destroying the old one, incessantly creating a new one" (Schumpeter, 1942, p. 83).[10]

To capture these forces in the context of our model, we consider an aggregative game in which $N \geq 1$ active investors simultaneously choose experimentation levels $x_i \geq 0$, for $i \in \{1, ..., N\}$ at $t = 0$ to maximize their cumulative utility over $\theta_i \delta_t$ according to (4). Both the share of the consumption stream and the value of the stream itself is affected by each investor's experimentation choice, given the actions of others. As such, the active investors not only affect the size of the pie, but also their share. Several necessary ingredients are therefore included in the model. First, the increased investment by any agent increases their potential market share, based on the rents available for extraction. Second, entrepreneurship may cannibalize some of the existing assets owned by the investor. Third, investment by competitors in the same technology adversely affects an agent's market share. Finally, and probably most importantly, the benefits of increased market share may adversely affect the quality of the consumption stream that results.

Define $\tilde{x}_C \equiv \sum_{i=1}^{N} x_i$ as the total level of experimentation in the economy and

$$\theta_i(x_i, \tilde{x}_C) = \frac{1}{N} e^{-\varphi + x_i(w - \tilde{x}_C)} \tag{13}$$

as the investor i's consumption share, which depends not only on her experimentation decision but also on the aggregate level of experimentation in the economy. We provide a microfoundation for (13) in Appendix A.1.1[11] The passive investor receives $c_{p,t} = (1 - \theta_C)\delta_t$, where $\theta_C \equiv \sum_{i=1}^{N} \theta_i(x_i, \tilde{x}_C)$, and we assume that $\varphi > 0$.

By construction, the fraction of the dividend stream claimed by agent i is increasing in wx_i, which is meant to capture the increase in the share of aggregate wealth that agent i can claim (monopoly rents) from the new technology.[12] Going forward,

[10]See also Aghion and Howitt (1992), where this tradeoff is made explicit by showing that more future research discourages current research by threatening to make it obsolete.

[11]Equation (13) arises from a Taylor approximation once we assume that the active investors capture the rents generated by the new technology.

[12]Innovation and top income inequality in the US and other developed countries tend to follow a parallel evolution. According to Aghion et al. (2015), 11 out of the 50 wealthiest individuals across US states in 2015 *"are listed as inventors in a US patent and many more manage or own firms that patent."*

we assume that

$$w > \frac{2(\widehat{\beta}_0 - k\sigma^2)}{k^2\sigma^2} > 0, \tag{14}$$

so that this incentive is sufficiently large. Indeed, we are interested in the eras with technology booms, with important sources of economic rents. However, the wealth share is also decreasing in the product $x_i\tilde{x}_C$, which captures how much wealth is lost to competition and also because agent i's existing assets are cannibalized or rendered obsolete (Reinganum, 1983; Aghion and Howitt, 1992).

The consumption stream that evolves based on the aggregate level of experimentation is

$$\frac{d\delta_t}{\delta_t} = (\bar{f} + \widehat{\beta}_t\tilde{x}_C)dt + \sigma(1 + k\tilde{x}_C)d\widehat{W}_t. \tag{15}$$

Each agent experiments less when their competitors experiment and disturb the economy, which increases the output volatility that everyone faces. Competition also lowers the share of consumption that each agent earns by experimenting. Last, experimentation by competitors increases the overall uncertainty in the economy. Given this, each agent chooses an optimal level of experimentation, taking into account the simultaneous choices of the other players.

Proposition 3 ***(Cournot Experimentation)*** *There exists a unique Nash equilibrium in which the aggregate level of experimentation under competition among N active investors is*

$$\tilde{x}^*_C = \frac{\widehat{\beta}_0\mathbb{D}_0 - \gamma k\sigma^2\mathbb{D}_0 + w}{\gamma k^2\sigma^2\mathbb{D}_0 + (\gamma - 1)\nu_0\mathbb{C}_0 + \frac{N+1}{N}}, \tag{16}$$

where each active agent experiments at a symmetric level $x^*_i = \tilde{x}^*_C/N$. The quantity $\tilde{x}^*_C$ is strictly increasing in N and $\tilde{x}^*_C > x^*_S$ $\forall N$.

Proposition 3 shows that rent seeking increases experimentation in the economy, and that competition exacerbates this. As competition intensifies, the aggregate level of experimentation moves further away from first best x^*_S , reaching a maximum when $N \to \infty$.

Such creative destruction and competition for rents causes the lifetime utility from the future consumption stream to deteriorate and lowers the welfare for the passive

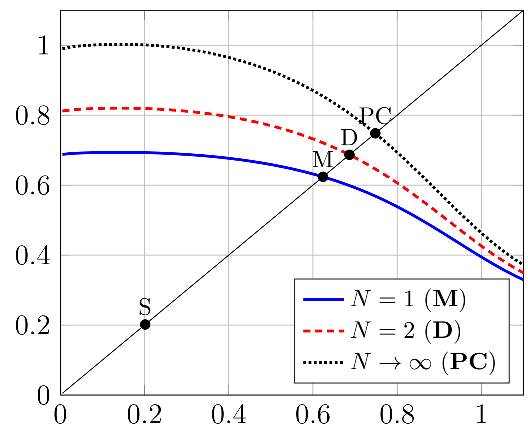

Figure 1: **Experimentation with Rent Seeking Behavior.** The fixed point solution for Eq. (16), for three different values of N: $N = 1$ (monopoly, **M**), $N = 2$ (duopoly, **D**), and $N \to \infty$ (perfect competition, **PC**). The point **S** represents the first best level of experimentation (Proposition 2). The calibration used is: $\gamma = 2$, $\bar{f} = 0.03$, $\widehat{\beta}_0 = 0.03$, $\nu_0 = 0.03^2$, $\sigma = 0.05$, $k = 2$, $\rho = 0.03$, $T = 100$, $\delta_0 = 1$, $k = 3$, $e^{-\varphi} = 0.1\%$, and $w = 2$.

investor,

$$U(c_i) = \frac{(1-\theta_C)^{1-\gamma}}{1-\gamma}\mathbb{E}\left[\int_0^T e^{-\rho t}\frac{\delta_t^{1-\gamma}}{1-\gamma}dt\right]. \tag{17}$$

This is because rent seeking by active investors not only reduces the passive investors' consumption share, it exposes them to too much risk and lowers the expected utility which they derive from the future consumption stream. Further, higher competition exacerbates this because experimentation rises and hurts residual claimants in the economy.

We illustrate the effect of rent seeking behavior and creative destruction on the aggregate level of experimentation in Figure 1. The parameters that we choose for this numerical example (provided in the caption of the Figure) are economically plausible and will be the same when we explore the implications for asset prices. The plot depicts three equilibria (**M**onopoly, **D**uopoly, and **P**erfect **C**ompetition). These equilibria are the fixed-point solutions of Eq. (16) for three different values of N. We add the first best level of experimentation (point **S**) for comparison. Confirming the results of Proposition 3, there is higher experimentation as competition rises in the market.

3 Implications for Asset Prices

We begin by characterizing the dynamics of the stochastic discount factor, the risk-free rate, and the market price of risk for a generic level of experimentation x_0.

Proposition 4 ***(Stochastic Discount Factor)*** *The stochastic discount factor, defined as $\xi_t \equiv e^{-\rho t}(\delta_t/\delta_0)^{-\gamma}$, follows*

$$\frac{d\xi_t}{\xi_t} = -\left[\rho + \gamma(\bar{f} + \widehat{\beta}_t x_0) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(1+kx_0)^2\right] dt - \gamma\sigma(1+kx_0)d\widehat{W}_t. \quad (18)$$

The equilibrium risk-free rate and the market price of risk are given by

$$r_t^f = \rho + \gamma(\bar{f} + \widehat{\beta}_t x_0) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(1+kx_0)^2 \quad (19)$$

$$\theta_t = \gamma\sigma(1+kx_0). \quad (20)$$

The equilibrium risk-free rate increases with the expected growth rate of consumption and decreases with the volatility of aggregate consumption. The level of experimentation amplifies both of these well-known asset pricing effects. Furthermore, experimentation increases the market price of risk. This arises because the process of creative destruction disturbs the economy. Neither the risk free rate nor the market price of risk depend on the uncertainty about the new technology.

Proposition 5 ***(Asset Prices)*** *For any experimentation level x_0, the equilibrium price-dividend ratio at time t, $F(\widehat{\beta}_t, \nu_t, t; x_0)$, is*

$$\frac{P(\widehat{\beta}_t, \nu_t, t; x_0)}{\delta_t} = \int_t^T \exp\left[\kappa(x_0, \widehat{\beta}_t)(s-t) + \frac{(1-\gamma)^2}{2}x_0^2\nu_t(s-t)^2\right] ds, \quad (21)$$

where the function $\kappa(x_0, \widehat{\beta}_t)$ is defined as

$$\kappa(x_0, \widehat{\beta}_t) \equiv -\rho - (\gamma-1)(\bar{f} + x_0\widehat{\beta}_t) + \frac{\gamma(\gamma-1)}{2}\sigma^2(1+kx_0)^2. \quad (22)$$

The equilibrium risk premium in the economy is

$$RP_t = \gamma\sigma^2(1+kx_0)^2 + \gamma(1-\gamma)x_0^2\nu_t\mathbb{D}_t, \quad (23)$$

and the equilibrium stock market volatility is given by

$$|\sigma_{P,t}| = \sigma(1+kx_0)\left|1+(1-\gamma)\frac{x_0^2\nu_t}{\sigma^2(1+kx_0)^2}\mathbb{D}_t\right|. \tag{24}$$

A key result of Proposition 5 is that the equilibrium price-dividend ratio increases with the uncertainty about β. This effect arises because, as uncertainty about the true expected growth of the new technology increases, the expected value of future income streams rises. To see this, suppose that β were known. Then, an application of Leibniz' integral rule confirms that the price-dividend ratio is convex in β:

$$\frac{\partial^2 F(\beta,t;x_0)}{\partial\beta^2} = \int_t^T x_0^2(\gamma-1)^2(s-t)^2 e^{\kappa(x_0,\beta)(s-t)}ds > 0. \tag{25}$$

Since β is a random variable, Jensen's inequality implies that that the price-dividend ratio under incomplete information must be *greater* than the one under complete information in the presence of uncertainty about growth rates (see also Pástor and Veronesi, 2003, 2006).

Over-valuation in presence of uncertainty arises because the asset is a good hedge against consumption shocks. For example, a negative consumption shock increases the likelihood of low consumption states in the future. Anticipating this, agents attempt to smooth their consumption by investing more in the asset and consuming less today, which increases the asset's valuation. When this income effect is strong enough, looking forward the asset's value is higher because it has an option-like feature that acts as a hedge against future shocks. This is amplified for high levels of experimentation when there is more uncertainty about the new technology.

The option-like feature follows directly from the valuation of the stock price,

$$P_t = \mathbb{E}_t\left[\int_t^T \frac{\xi_s}{\xi_t}\delta_s ds\right] = \delta_t\int_t^T e^{-\rho(s-t)}\mathbb{E}_t\left[\left(\frac{\delta_s}{\delta_t}\right)^{1-\gamma}\right]ds. \tag{26}$$

The function in the expectation is convex when $\gamma > 1$. Due to this convexity, the asset helps agents to smooth consumption across states. Hence, agents are willing to pay an insurance premium to hold the risky asset.

The hedging property described here is not a special feature of CRRA utility, but also arises in an Epstein-Zin framework in which the elasticity of intertemporal substitution (EIS) is lower than one (and agents have a preference for early resolution of uncertainty).[13] When the EIS is higher than one, the substitution effect dominates the income effect and the risky asset becomes a bad hedge against consumption shocks. However, an EIS larger than one implies that investors have high willingness to pay for early resolution of uncertainty (Epstein, Farhi, and Strzalecki, 2014). Given that the investors we wish to model are indeed entrepreneurs, it is likely that they have a weaker preference for early resolution of uncertainty.

Inspection of the equity risk premium in (23) also illustrates the potential for the risky asset to be a hedge. The expression is made up of two terms: the first is positive and the second is negative. Agents require a premium for holding the risky asset, but at the same time are willing to pay an insurance premium for its protection against future consumption shocks. The balance between these two effects is affected by the level of experimentation. Higher experimentation increases the uncertainty in the economy and with it the value of hedging. When experimentation and uncertainty are sufficiently high, the second term in (23) dominates, which may lead to a *negative* risk premium. We explore this further in Section 3.2.

The reaction of the risky asset to consumption shocks is dampened by experimentation, which affects the volatility of stock returns. Equation (24) has two terms, again with opposite effects of experimentation and uncertainty. Indeed, experimentation increases the volatility by disturbing the economy and amplifying macroeconomic fluctuations. However, it also decreases volatility through the insurance premium channel as described above.

Figure 2 provides an example that illustrates the effect of experimentation on the price-dividend ratio, the risk premium, and the volatility of stock returns. Panel (a) shows that the price-dividend ratio reaches a minimum at the first best level of experimentation of Proposition 2 (the dot labeled **S** on the graph). To understand why this is the case, we can re-write (11) as

$$F(\widehat{\beta}_0, \nu_0, 0) = (1-\gamma)(\theta\delta_0)^{\gamma-1} J_a(\delta_0, \widehat{\beta}_0, \nu_0, 0). \tag{27}$$

This shows that the price-dividend ratio is directly related to the value function of the active investor, with a negative sign when the coefficient of risk aversion is higher than one. Because the active investor chooses an experimentation level that maximizes the value function, it follows that the equilibrium price-dividend ratio reaches a minimum

[13]Further analysis using Epstein-Zin preferences are available upon request from the authors.

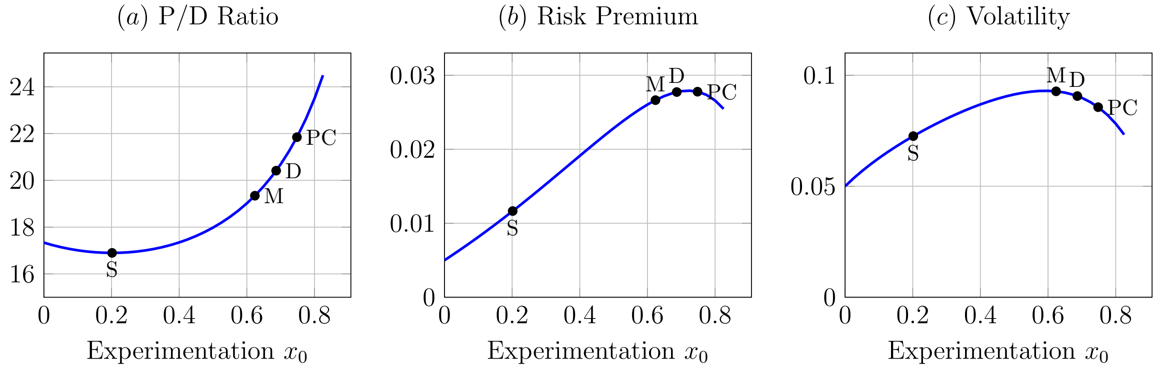

Figure 2: **Experimentation and Asset Prices.** The effect of experimentation on the price-dividend ratio, the risk premium, and stock market volatility. The dot labeled **S** represents the first best amount of experimentation, x_S^* (Proposition 2). The dot **M** represents the optimal amount of experimentation under monopoly, x_M^* (Proposition 3, $N = 1$). The dots **D** and **PC** represent the total amount of experimentation under Cournot competition, with $N = 2$ and $N \to \infty$ respectively (Proposition 3). All parameters are provided in Figure 1.

when experimentation is first best.

This inverse relationship implies that any level of experimentation that deviates from the optimum *always* increases the price-dividend ratio. Panel (a) shows that the price-dividend ratio increases under monopoly (Proposition 3, $N = 1$, dot labeled **M**), further increases under duopoly ($N = 2$, dot labeled **D**), and reaches a maximum under perfect competition ($N \to \infty$, dot labeled **PC**). The increase in the stock price for higher levels of experimentation then first best stems from the over-valuation channel described above.

Panels (b) and (c) of Figure 2 show that the risk premium and the volatility are hump-shaped in experimentation. The initial increase is driven by the first terms in (23)-(24), whereas the subsequent decrease is driven by the second, uncertainty terms.

Experimentation also affects the term structure of risk. In Corollary 5.1, we decompose the asset into dividend strips (i.e., assets that pay the aggregate consumption only at time $s > t$).

Corollary 5.1 *For any experimentation level x_0, the price of a dividend strip with* *maturity $s > t$ is*

$$P_{t,s} \equiv \frac{1}{\xi_t}\mathbb{E}_t[\xi_s \delta_s] = \delta_t \exp\left[\kappa(x_0, \widehat{\beta}_t)(s-t) + \frac{(1-\gamma)^2}{2} x_0^2 \nu_t (s-t)^2\right]. \tag{28}$$

The risk premium and the volatility for each maturity $s \geq t$ are given by

$$RP_{t,s} = \gamma\sigma^2(1+kx_0)^2 + \gamma(1-\gamma)x_0^2\nu_t(s-t) \tag{29}$$

$$|\sigma_{P,t,s}| = \sigma(1+kx_0)\left|1 + (1-\gamma)\frac{x_0^2\nu_t}{\sigma^2(1+kx_0)^2}(s-t)\right|. \tag{30}$$

Experimentation has two effects on the term structure of risk premia and volatilities. First, by increasing the disturbance associated with the introduction of new technologies, experimentation increases the risk premia and volatilities at all maturities. This can be seen from the first terms in (29)-(30). Second, experimentation introduces uncertainty about the expected growth of the new technology. This dampens the risk premia and volatilities because the asset offers insurance against this increase in uncertainty. Corollary 5.1 shows that the dampening effect positively depends on maturity. The volatility of dividend strips with longer maturities is lowered through learning, which induces lower risk premia. As a result, the term structures of risk premia and volatilities is downward sloping. Figure 3 illustrates this. It shows that in the status quo the term structure of risk premia and volatilities is flat, whereas with experimentation all term structures become downward sloping. Competition among active investors amplifies the steepening: the slopes are steepest in the perfect competition case, when the total experimentation in the economy equals $x_0 = x^*_{PC}$.

Overall, an economy with creative destruction is riskier now than it is expected to be in the future. That creative destruction is a source of systematic risk has been already acknowledged by Kung and Schmid (2015), Gârleanu et al. (2012) and tested empirically by Grammig and Jank (2015).[14] In our model, this higher risk is concentrated at short maturities, but dampened at long maturities. Investors demand a high compensation for the "destruction" that innovation imposes on the economy

[14] In Kung and Schmid (2015), agents are sensitive to long-run uncertainty induced by industrial innovation, which rationalizes a sizeable equity risk premium. In Gârleanu et al. (2012), innovation erodes human capital and creates "displacement risk," which cannot be perfectly shared and thus generates a high equity premium and a value premium. Extending this line of reasoning, Grammig and Jank (2015) show that cross-sectional differences across size and book-to-market sorted portfolios can be explained as premia for bearing creative destruction risk.

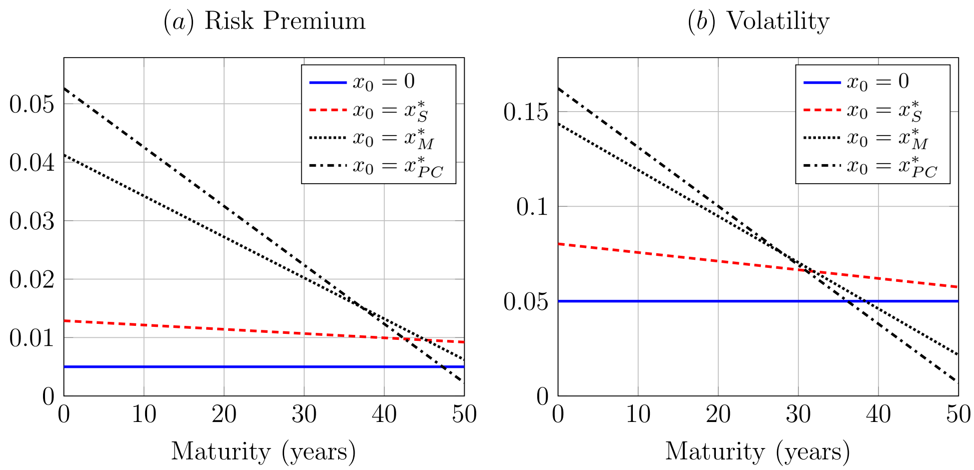

Figure 3: **Experimentation and the Term Structure of Risk.** The term structure of risk premia and volatilities in several cases. The solid lines depict the flat term structure that arise in the "status-quo" economy with no experimentation. The dashed lines depict the case of first best experimentation (Proposition 2). The remaining two lines (dotted and dash-dotted) depict term structures that arise with monopoly or with perfect competition (Proposition 3). All parameters are provided in Figure 1.

in the short-run, but at the same time recognize that the asset insulates them against the uncertainty associated with innovation in the long-run. This latter effect is what induces the over-valuation of the risky asset.

3.1 The Aftermath of Experimentation

We now consider the consequences for asset prices in the aftermath of the decision to experiment. We illustrate how the rent seeking behavior of active investors engaged in creative destruction amplifies the effect that competition has on stock price valuations and generates patterns that ex post can be identified as booms and busts.

To this end, we consider an economy with perfect competition ($N \to \infty$) and assume that $w = 5$, which implies an equilibrium price-dividend ratio of apprixi-mately 30. Panel (a) in Figure 4 depicts the price-dividend ratio in this economy as a function of the aggregate experimentation (solid line). The dot labeled **IC** ("Intense Competition") represents the equilibrium price-dividend ratio with perfect competition. In the same plot, the dashed line depicts the price-dividend ratio when uncertainty about β is fixed at $\nu_0 = 0$. The gap between the two lines provides a

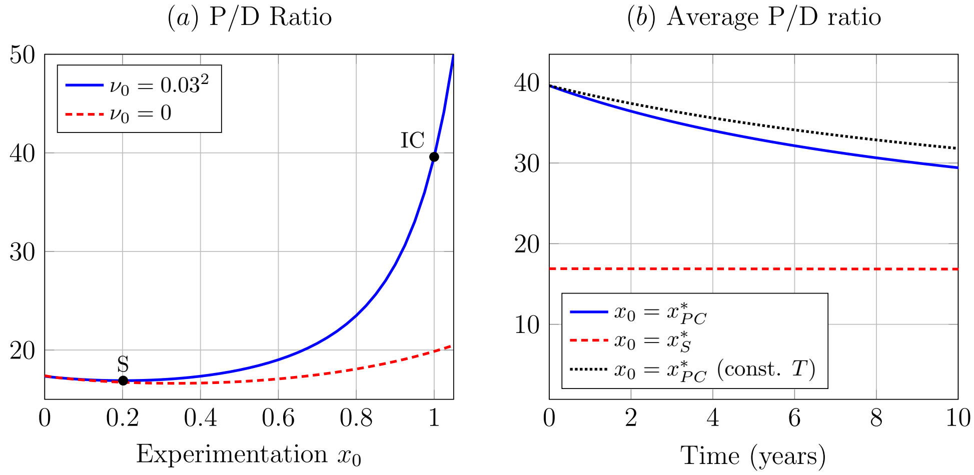

Figure 4: **Booms and Busts in Asset Prices.** Panel (a) shows the price-dividend ratio as a function of the aggregate experimentation level in the economy, x_0. The solid line includes uncertainty about β. Uncertainty is set at zero for the dashed line. The dot labeled **IC** ("Intense Competition") corresponds to the equilibrium aggregate level of experimentation in perfect competition. The dot labeled **S** corresponds to the first best level of experimentation. Panel (b) depicts average price paths over time starting from **IC** (solid and dotted lines) and from **S** (dashed line), starting from $\widehat{\beta}_0 = \beta$. The dotted line maintains the maturity of the asset constant at $T = 100$ years. With the exception of $w = 5$, all parameters are provided in Figure 1.

measure of the over-valuation due to uncertainty. We also show on the plot the point **S**, which corresponds to the first best experimentation level. Rent seeking behavior and competition lead to over-experimentation, adding uncertainty and increasing the the gap between the two lines. This gap grows from being almost negligible at **S** to roughly 100% at **IC**.

It is instructive to compare the dynamics of the price-dividend ratio once the points **IC** and **S** have been reached. To do so, let us assume that the prior $\widehat{\beta}_0$ is exactly equal to the true value of β, so that the agents are initially right—but still uncertain—about the expected growth of the new technology. As such, we are not imposing any behavioral bias on the future price paths. As it turns out, this assumption is extremely convenient because it ensures that simulations of the economy over time will result in an average $\widehat{\beta}_t$ that is equal to the true value of β.[15] Furthermore, because the

[15]This can be seen from Proposition 1. At time $t = 0$, $d\widehat{W}_0 = dW_t$ (because $\beta - \widehat{\beta}_0 = 0$). Technically, at time $t = 0$ the filter $\widehat{\beta}$ is a martingale. This ensures that the average of its future simulated values one step ahead, $\widehat{\beta}_{0+dt}$, is exactly β. Then apply the same reasoning at time $t = 0 + dt$.

remaining state variable that enters into the valuation of the price-dividend ratio, the uncertainty ν_t, decreases deterministically over time, we can directly plot the average price-dividend ratio over time by simply using (8) for the dynamics of ν_t, without resorting to simulations.

Panel (b) in Figure 4 depicts these dynamics. The solid line corresponds to the average path of the price-dividend ratio starting from **IC**, whereas the dashed line shows the average path of the price-dividend ratio starting from **S**. Because the remaining life of the asset diminishes, we also add to the plot the dotted line, which isolates the effect of the decrease in uncertainty by holding the remaining asset life constant at $T = 100$ years (this adjustment is unnecessary in the **S** case, where the decrease in the price-dividend ratio is almost imperceptible). The panel shows that, after reaching the point **IC**, asset prices have the tendency to decrease as uncertainty about the new technology resolves. In contrast, after experimentation at the first best level (point **S**), the average decrease in asset price due to resolution of uncertainty is negligible.

Our model therefore predicts that markets characterized by fierce competition for new technologies are not only prone to strong asset price inflation but also to subsequent declines through learning by doing and resolution of uncertainty. Because learning by doing decreases the uncertainty about β over time, stock prices adjust endogenously. The pace of this adjustment process depends on the rate at which uncertainty about the new technology is resolved. Equation (8) shows that the posterior variance decays faster with over-experimentation,[16] which accelerates the eventual decline in asset prices.

This highlights the importance of learning for the overvaluation of asset prices and their subsequent declines. The aftermath depicted in Panel (b) of Figure 4 can only take place through learning. If no uncertainty (and thus learning) about the true expected growth of the new technology were present, then prices would never fall and we would not observe an aftermath. Furthermore, because stock over-valuation is associated with high uncertainty, it is likely that valuation revisions will be large when market participants receive additional information about the expected growth of the new technology. Although we do not model in this paper information other than the consumption process, in reality it is likely that investors pay attention to other

[16] In Eq. (8), the coefficient multiplying time in the denominator can be interpreted as the *speed of learning*. It is a matter of algebra to show that the speed of learning increases with experimentation.

sources of information (e.g., earnings announcements, technology markets forecasts). Investors' heightened attention to news (Andrei and Hasler, 2014) can then constitute a potential trigger for steep market declines.

3.2 Time-Series Predictability

The previous analysis suggests that in our model a high price-dividend ratio predicts low future excess returns. That the price-dividend ratio negatively predicts returns is a well-known feature of the data (Fama and French, 1988; Hodrick, 1992; Cochrane, 2008) and is not our main object of interest. We are rather interested here in a particular form of this predictability. Namely, our model postulates that intense competition among active investors increases the amount of uncertainty in the markets, which in turn inflates the price-dividend ratio to arbitrarily large levels. Hence, the question emerges whether these episodes can be accompanied by *negative* future excess returns, during which investors are willing to *pay* a premium for holding the risky asset.

We emphasize here that the asset price over-valuation in our model is not generated by investors' "greed" (this line of reasoning would imply that high prices predict high future excess returns). We posit instead that creative destruction is associated with both high expected economic growth and amplified uncertainty about future consumption. As discussed above, because the asset offers valuable insurance against this uncertainty, it may command a sizeable insurance premium. Consequently, high prices when competition is intense may in fact predict negative future excess returns.

One way to illustrate this is to study the impact of competition on the time-series predictability of returns. An important input in the consumption share of each individual active investor is the rent-seeking parameter w, which controls how fierce is the competition for market share. As such, we perform simulations of our model for different values of the parameter w, and measure in each case the amount of return predictability implied by the model and the resultant expected future excess returns.

Using a discretization of our continuous-time setup (see Appendix A.2.3), we perform 10,000 simulations of the model at quarterly frequency for 50 years (while keeping the remaining life of the asset constant at $T = 100$ years, in order to eliminate any effect coming from the diminishing time to maturity). Then, we run the following predictive regression at quarterly frequency:

$$\sum_{k=1}^{K}(r_{t+k} - r_{f,t+k}) = a_K + b_K pd_t + \epsilon_{t+K}, \tag{31}$$

where the dependent variable is the K-period time-aggregated excess return and the independent variable is the log price-dividend ratio. We consider several values for the horizon K: 4, 12, 20, and 28 quarters.

Figure 5 reports simulation results for an horizon of five years ($K = 20$ quarters). Each one of the three rows of plots corresponds to a different value of the rent-seeking parameter w. In each case, we solve for the aggregate level of experimentation under perfect competition among the active investors and for the resulting equilibrium price path. The plots depict histograms for the 10,000 occurrences of the regression coefficients from (31), their t-stats, the R^2 coefficients of the regressions, and the expected 5-years excess returns on the stock market. The dashed lines depict medians, whereas the solid lines depict the numbers from US quarterly data, 1947-2008 (Beeler and Campbell, 2012, Table 4, row labeled "20 Q"). As the figure shows, return predictability is changing sign from positive to negative when the rent-seeking parameter w gets larger and the aggregate level of experimentation increases. While the median t-stat from simulations is clearly positive when $w = 5$ (and $\tilde{x}_C^* = 0.996$), it becomes negative and strongly statistically significant when $w = 15$ (and $\tilde{x}_C^* = 1.352$).

Negative return predictability arises in our model because both the price-dividend ratio and the risk premium fluctuate due to movements in the uncertainty ν_t.[17] Uncertainty simultaneously increases the price-dividend ratio and decreases the risk premium. If the impact of uncertainty is strong enough—as it is the case when w is high—return predictability becomes negative.

Hence, competition among active investors in the market for new technologies not only inflates the price-dividend ratio, but it also lowers future excess returns. The last column of panels in Figure 5 illustrates this. It depicts histograms of 5-year expected returns computed at time $t = 0$ by using the estimated coefficients from the regression specification (31) and the value of the price-dividend ratio determined by the parameter w. As panel (l) shows, the median five-years excess return on the market is mostly negative: investors are willing to hold the risky asset at a high price,

[17] This can be seen from Corollary 5.1, which shows that the equilibrium risk premium on each dividend strip is driven solely by the uncertainty ν_t.

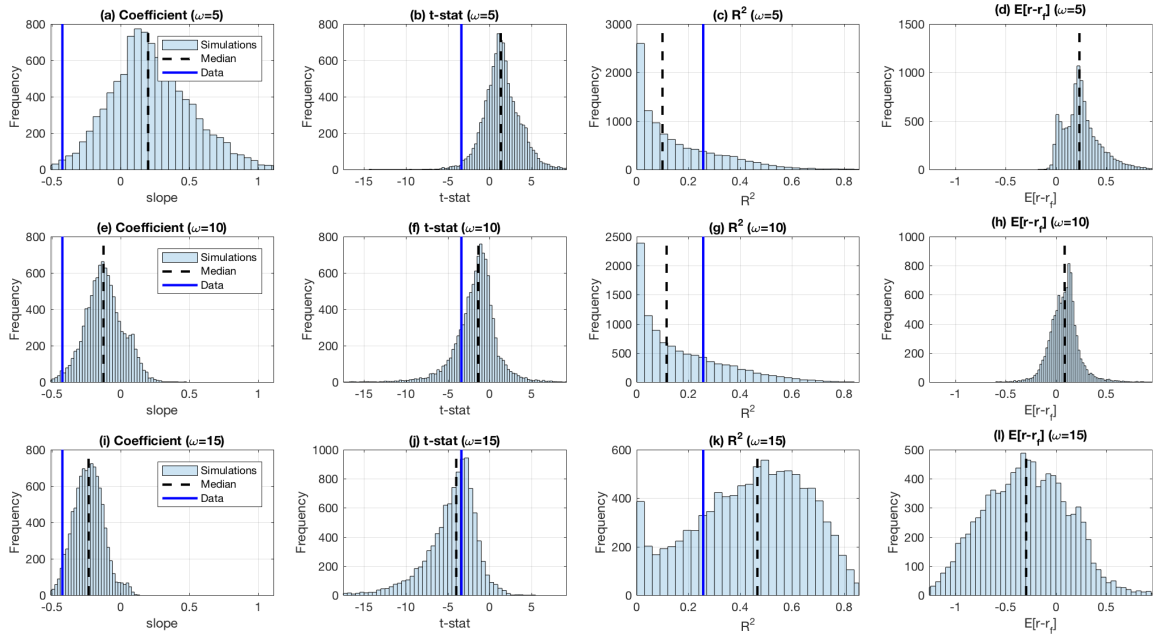

Figure 5: **Return predictability with the price-dividend ratio (simulations).** Panels (a)-(c) show the results of the regression specification (31) with $K = 20$ (5 years) on 10,000 finite-sample simulations of the model (50 years at quarterly frequency). There are three rows of plots, each one corresponding to a different value of w. From left to right, the plots show histograms for the regression coefficients, t-stats (computed with Newey and West (1987) standard errors with $2(K-1) = 38$ lags), R^2 coefficients, and expected 5 year returns at date $t = 0$ in the simulations. The dashed lines are the medians of the simulations. The solid lines are the numbers from US quarterly data, 1947-2008 (Beeler and Campbell, 2012, Table 4, row labeled "20 Q"). For ease of comparison, we maintain the x-axis bounds of the first row plots on the rows below.

yet they demand a negative risk premium for it.

Table 1 summarizes the return predictability results for all considered horizons: 4, 12, 20, and 28 quarters. The table has 3 panels, each one for a different value of w. The panels report medians from simulations for the regression coefficinents, the t-stats, and the R^2 coefficients. As the table shows, return predictability is negative for higher levels of the rent-seeking parameter w. Furthermore, investors indeed expect negative excess returns for all horizons in the future. Although this exercise is only illustrative, it is worth noting that the R^2 coefficients that we obtain are not far from the ones in the data, which reach values between 30 and 60%, depending on time

Predictability of future excess returns

Intense Competition I, $w = 5$, $\tilde{x}_C^* = 0.996$ (medians of 10,000 simulations)

	4Q	12Q	20Q	28Q
Coefficient of Log (P/D)	0.039	0.119	0.194	0.265
t-stat	1.057	1.134	1.294	1.467
R-squared	0.018	0.056	0.096	0.135
Expected Excess Returns (annualized)	4.6%	4.7%	4.8%	4.7%

Intense Competition II, $w = 10$, $\tilde{x}_C^* = 1.176$ (medians of 10,000 simulations)

	4Q	12Q	20Q	28Q
Coefficient of Log (P/D)	-0.036	-0.091	-0.129	-0.156
t-stat	-1.408	-1.424	-1.420	-1.436
R-squared	0.035	0.086	0.122	0.147
Expected Excess Returns (annualized)	0.4%	1.2%	1.8%	2.2%

Intense Competition III, $w = 15$, $\tilde{x}_C^* = 1.352$ (medians of 10,000 simulations)

	4Q	12Q	20Q	28Q
Coefficient of Log (P/D)	-0.057	-0.156	-0.238	-0.303
t-stat	-2.515	-3.280	-4.034	-4.719
R-squared	0.139	0.338	0.469	0.555
Expected Excess Returns (annualized)	-7.8%	-6.9%	-5.9%	-5.2%

Table 1: **Return predictability with the price-dividend ratio (simulations).** This table reports the predictability of excess stock returns with the log price-dividend ratio. There are three panels, each one corresponding to a different value of w. Each panel reports medians from 10,000 simulations for the regression coefficients, t-stats (computed with Newey and West (1987) standard errors with $2(K-1)$ lags), the R^2 coefficients, and the expected annualized excess returns at time $t = 0$. The panels show results for different horizons: 4, 12, 20, and 28 quarters.

period and estimation details (see, e.g., Cochrane, 2008).

4 Conclusion

This paper proposes a financial markets perspective on Schumpeter (1934)'s evolutionary economics ideas, according to which introduction of new technologies disturbs the flow of economic life and forces existing means of production to lose their position within the economy. It is then the task of the financier to decide how much of the new technology the economy should be willing to take.

From the financier's viewpoint, an optimum exists. This optimum balances the gains of economic development associated with new productive technologies against the disturbance imposed on the status quo. The process of reaching such an optimum involves learning by doing (i.e., experimentation), which has an observer effect and creates uncertainty in financial markets. Rent-seeking behavior and intensified competition for new markets leads to excessive experimentation, inflating asset prices. In hindsight, asset prices exhibit familiar boom and bust patterns observed during technological revolutions.

Rent seeking behavior by active investors has welfare consequences. It reduces the passive investors' (those who are not involved in entrepreneurial activity) consumption share, exposes them to too much risk and lowers the expected utility of their consumption stream. Higher competition exacerbates this because experimentation rises, which hurts the residual claimants in the economy.

We have assumed throughout this paper that investors share the same beliefs about the expected growth of a new technology. In reality, these markets might very well be characterized by strong divergence of opinion about the promise of new innovations. An interesting exercise would be to study the interaction between competition and difference of beliefs. Our guess is that active agents will decide to experiment more in order to preempt entry of optimistic competitors, in this way exacerbating the effect of over-experimentation on asset prices.

We have assumed that the decision to experiment is made only once. In reality, however, innovation is not a one-time decision, but often occurs in waves (Gort and Klepper, 1982). Dynamic experimentation provides the agents an additional option to abandon a new technology at any point in the future, which can have adverse consequences for the valuation of firms involved in entrepreneurial activity. Coupled with lack of perfect knowledge, dynamic experimentation might lead agents to conclude that a particular new technology is not productive and abandon it prematurely. This "abandonment risk" may have consequences for asset prices and for the cross section of stock returns.

A Appendix

A.1 Experimentation and Learning

A.1.1 Microfoundation

Consider two intermediate goods, a "status-quo" good and an "experimental" good:

$$\frac{d\delta_t^S}{\delta_t^S} = \bar{f}dt + \sigma dW_t \tag{32}$$

$$\frac{d\delta_t^E}{\delta_t^E} = \left(\Gamma - \frac{k^2\sigma^2}{2}x_0\right)dt + k\sigma dW_t. \tag{33}$$

The parameter Γ, which enters in the expected growth of the experimental good, is an unknown constant. The expected growth for the experimental good exhibits decreasing returns to scale: a higher initial level of experimentation x_0 decreases its expected growth by a term proportional to the instantaneous variance of δ_t^E, $k^2\sigma^2$. Define the consumption basket in the economy as[18]

$$\delta_t = \delta_t^S \left(\delta_t^E\right)^{x_0}. \tag{34}$$

If $x_0 = 0$, the economy remains in the status-quo and only the good δ_t^S is consumed. When $x_0 > 0$, the experimental good becomes a new variety in the consumption basket, with a weight given by $x_0/(1 + x_0)$. Aplying Itô's lemma on δ_t yields:

$$\frac{d\delta_t}{\delta_t} = \left(\bar{f} + \beta x_0\right)dt + \sigma(1 + kx_0)dW_t, \tag{35}$$

where $\beta \equiv \Gamma + \frac{1}{2}(2 - k)k\sigma^2$ is an unknown constant. This provides a microfoundation of Eq. (3) in the text.

From the Cobb-Douglas specification (34), the value of the experimental good δ^E in terms of the numeraire in the economy (which is the price of the aggregate good δ) is given by

$$p^E\delta^E = \frac{x_0}{1 + x_0}\delta, \tag{36}$$

where p^E is the price of the experimental good. Assuming that a (monopolist) active investor captures all the rents associated with the experimental good, then this is equivalent with receiving a share $x_0/(1 + x_0)$ of the total consumption stream. This share increases in x_0, and there are decreasing returns to scale (i.e., entrepreneourship may cannibalize some of the existing assets owned by the investor). The following second order approximation holds:

$$e^{\log\left(\frac{x_0}{1+x_0}\right)} \approx e^{-A_0 + A_1x_0 - A_2x_0^2}, \tag{37}$$

[18] This specification is commonly adopted in the international finance literature. See Helpman and Razin (2014), Cole and Obstfeld (1991), Zapatero (1995), and Pavlova and Rigobon (2007) among others.

where A_0, A_1, and A_2 are positive constants that depend on the approximation point. This approximation yields the same structure as the function (13) proposed in the text.

A.1.2 Proof of Proposition 1 (Learning)

The proof of Proposition 1 follows from direct application of this standard result in filtering theory:

Theorem 6 ***(Liptser and Shiryayev, 1977)*** *Consider an unobservable process u_t and an observable process s_t with dynamics*

$$du_t = [a_0(t, s_t) + a_1(t, s_t)u_t]\, dt + b_1(t, s_t)dZ_t^u + b_2(t, s_t)dZ_t^s \tag{38}$$
$$ds_t = [A_0(t, s_t) + A_1(t, s_t)u_t]\, dt + B_1(t, s_t)dZ_t^u + B_2(t, s_t)dZ_t^s. \tag{39}$$

All the parameters can be functions of time and of the observable process. Then, the filter evolves according to (we drop the dependence of coefficients on t and s_t for notational convenience):

$$d\widehat{u}_t = (a_0 + a_1\widehat{u}_t)dt + [(b \circ B) + \nu_t A_1^\top](B \circ B)^{-1}[ds_t - (A_0 + A_1\widehat{u}_t)dt] \tag{40}$$
$$\frac{d\nu_t}{dt} = a_1\nu_t + \nu_t a_1^\top + (b \circ b) - [(b \circ B) + \nu_t A_1^\top](B \circ B)^{-1}[(b \circ B) + \nu_t A_1^\top]^\top, \tag{41}$$

where

$$b \circ b = b_1 b_1^\top + b_2 b_2^\top \tag{42}$$
$$B \circ B = B_1 B_1^\top + B_2 B_2^\top \tag{43}$$
$$b \circ B = b_1 B_1^\top + b_2 B_2^\top. \tag{44}$$

In the present setup, the unobservable variable is the constant β. Hence,

$$a_0 = a_1 = b_1 = b_2 = 0. \tag{45}$$

Furthermore, the observable process is δ_t. Applying Itô's lemma on $\ln \delta_t$ yields

$$A_0 = \bar{f} - \frac{1}{2}\sigma^2(1 + kx_0)^2, \quad A_1 = x_0, \quad B_1 = 0, \quad B_2 = \sigma(1 + kx_0). \tag{46}$$

Direct application of Theorem 6 yields Proposition 1. □

A.1.3 Conditional moments of future output

We prove the following proposition using the theory of affine processes (Duffie, Filipović, and Schachermayer, 2003):[19]

[19] An alternative approach would be to follow Ziegler (2003, Appendix A) and Bryson and Ho (1975, Section 11.4) and compute $\mathbb{E}_t[\delta_s]$ and $\mathrm{Var}[\ln \delta_s]$ in one step. Both approaches are equally tedious.

Proposition 7 *For any $s > t$, the expected value of future dividends and the future volatility of dividends are respectively given by*

$$\mathbb{E}_t[\delta_s] = \delta_t \exp\left[\left(\bar{f} + x_0\widehat{\beta}_t\right)(s-t) + \frac{x_0^2\nu_t}{2}(s-t)^2\right] \tag{47}$$

$$\mathrm{Var}_t[\ln\delta_s] = \sigma^2(1+kx_0)^2(s-t) + x_0^2\nu_t(s-t)^2. \tag{48}$$

Both quantities are increasing in the level of experimentation x_0.

Proof Apply first Itô's lemma on $\ln\delta_t$, with the process of δ_t provided in (5). Using the fact that $\widehat{\beta}_t$ is a martingale yields

$$\mathbb{E}_t[\ln\delta_s] = \ln\delta_t + \left(\bar{f} + x_0\widehat{\beta}_t - \frac{\sigma^2(1+kx_0)^2}{2}\right)(s-t). \tag{49}$$

Write the dynamics of the system of two state variables $\{\ln\delta_t, \widehat{\beta}_t\}$ under an affine form:

$$\begin{bmatrix} d\ln\delta_t \\ d\widehat{\beta}_t \end{bmatrix} = \left(\begin{bmatrix} \bar{f} - \frac{1}{2}\sigma^2(1+kx_0)^2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & x_0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \ln\delta_t \\ \widehat{\beta}_t \end{bmatrix}\right)dt + \begin{bmatrix} \sigma(1+kx_0) \\ A(t) \end{bmatrix} d\widehat{W}_t, \tag{50}$$

where $A(t)$ is a function of time (see Proposition 1):

$$A(t) \equiv \frac{\nu_0 x_0 \sigma(1+kx_0)}{\nu_0 x_0^2 t + \sigma^2(1+kx_0)^2}. \tag{51}$$

The following Lemma results immediately from differentiation of $A(t)$:

Lemma 8 *The function $A(t)$ satisfies*

$$A'(t) = -\frac{x_0}{\sigma(1+kx_0)}A(t)^2. \tag{52}$$

Notice that (50) is a *time-inhomogeneous* multifactor affine process (Filipović, 2005). This is because the diffusion of $\widehat{\beta}$ depends on time (but it does not depend on the two state variables). Define:

$$K_0 \equiv \begin{bmatrix} \bar{f} - \frac{1}{2}\sigma^2(1+kx_0)^2 \\ 0 \end{bmatrix} \tag{53}$$

$$K_1 \equiv \begin{bmatrix} 0 & x_0 \\ 0 & 0 \end{bmatrix} \tag{54}$$

$$H_0(t) \equiv \begin{bmatrix} \sigma(1+kx_0) \\ A(t) \end{bmatrix}\begin{bmatrix} \sigma(1+kx_0) \\ A(t) \end{bmatrix}' = \begin{bmatrix} \sigma^2(1+kx_0)^2 & \sigma(1+kx_0)A(t) \\ \sigma(1+kx_0)A(t) & A(t)^2 \end{bmatrix}. \tag{55}$$

In order to compute the expectation

$$\mathbb{E}_t[\delta_s] = \mathbb{E}_t\left[e^{\ln\delta_s}\right], \tag{56}$$

we conjecture an exponential-affine solution of the form

$$\mathbb{E}_t\left[\delta_s\right] = e^{\alpha_0(s-t)+\alpha_1(s-t)\ln\delta_t+\alpha_2(s-t)\widehat{\beta}_t}, \tag{57}$$

for some coefficient functions $\alpha_j(\cdot)$, $j = 0, 1, 2$. Since the system (50) is a multifactor affine diffusion, the coefficients $\alpha_j(\cdot)$, $j = 0, 1, 2$ satisfy a system of Riccati ODEs (Duffie et al., 2003). Defining $\tau \equiv s - t$, the system of ODE writes

$$\begin{bmatrix}\alpha_1'(\tau)\\ \alpha_2'(\tau)\end{bmatrix} = K_1^\top \begin{bmatrix}\alpha_1(\tau)\\ \alpha_2(\tau)\end{bmatrix} \tag{58}$$

$$\alpha_0'(\tau) = K_0^\top \begin{bmatrix}\alpha_1(\tau)\\ \alpha_2(\tau)\end{bmatrix} + \frac{1}{2}\begin{bmatrix}\alpha_1(\tau) & \alpha_2(\tau)\end{bmatrix} H_0(t) \begin{bmatrix}\alpha_1(\tau)\\ \alpha_2(\tau)\end{bmatrix}, \tag{59}$$

with boundary conditions $\alpha_0(0) = 0$, $\alpha_1(0) = 1$, and $\alpha_2(0) = 0$. The first Riccati Eq. (58) has a straightforward solution:

$$\alpha_1(\tau) = 1 \tag{60}$$

$$\alpha_2(\tau) = x_0\tau, \tag{61}$$

which can be now inserted in the remaining Riccati Eq. (59):

$$\alpha_0'(\tau) = \begin{bmatrix}\bar{f} - \tfrac{1}{2}\sigma^2(1+kx_0)^2 & 0\end{bmatrix}\begin{bmatrix}1\\ x_0\tau\end{bmatrix} + \frac{1}{2}\begin{bmatrix}1 & x_0\tau\end{bmatrix} H_0(s-\tau)\begin{bmatrix}1\\ x_0\tau\end{bmatrix}, \tag{62}$$

which leads to

$$\alpha_0'(\tau) = \bar{f} + \sigma(1+kx_0)x_0 A(s-\tau)\tau + \frac{x_0^2}{2}A(s-\tau)^2\tau^2 \tag{63}$$

$$= \bar{f} + \sigma(1+kx_0)x_0\left[A(s-\tau)\tau + \frac{1}{2}\frac{x_0}{\sigma(1+kx_0)}A(s-\tau)^2\tau^2\right] \tag{64}$$

and boundary condition $\alpha_0(0) = 0$. Using Lemma 8, we can write:

$$\alpha_0'(\tau) = \bar{f} + \sigma(1+kx_0)x_0\left[A(s-\tau)\tau + \frac{1}{2}\frac{\partial A(s-\tau)}{\partial\tau}\tau^2\right] \tag{65}$$

$$= \bar{f} + \sigma(1+kx_0)x_0\frac{\partial\left(\frac{1}{2}A(s-\tau)\tau^2\right)}{\partial\tau}, \tag{66}$$

which we can now integrate to get

$$\alpha_0(s-t) = \bar{f}(s-t) + \frac{1}{2}\sigma(1+kx_0)x_0\frac{x_0}{\sigma(1+kx_0)}\nu_t(s-t)^2 \tag{67}$$

$$= \bar{f}(s-t) + \frac{1}{2}x_0^2\nu_t(s-t)^2 \tag{68}$$

It then follows that

$$\mathbb{E}_t\left[\delta_s\right] = e^{\alpha_0(s-t)+\alpha_1(s-t)\ln\delta_t+\alpha_2(s-t)\widehat{\beta}_t} \tag{69}$$

$$= \exp\left[\ln\delta_t + \left(\bar{f} + x_0\widehat{\beta}_t\right)(s-t) + \frac{x_0^2\nu_t}{2}(s-t)^2\right] \tag{70}$$

$$= \delta_t \exp\left[\left(\bar{f} + x_0\widehat{\beta}_t\right)(s-t) + \frac{x_0^2\nu_t}{2}(s-t)^2\right], \tag{71}$$

which is increasing in the level of experimentation x_0. Write now

$$\mathbb{E}_t\left[\delta_s\right] = \mathbb{E}_t\left[e^{\ln\delta_s}\right] = \exp\left[\mathbb{E}_t[\ln\delta_s] + \frac{1}{2}\mathrm{Var}_t[\ln\delta_s]\right], \tag{72}$$

which, using (49) and (71) yields

$$\exp\left[\frac{\sigma^2(1+kx_0)^2}{2}(s-t) + \frac{x_0^2\nu_t}{2}(s-t)^2\right] = \exp\left[\frac{1}{2}\mathrm{Var}_t[\ln\delta_s]\right], \tag{73}$$

and thus

$$\mathrm{Var}_t\left[\ln\delta_s\right] = \sigma^2(1+kx_0)^2(s-t) + x_0^2\nu_t(s-t)^2, \tag{74}$$

which is increasing in the level of experimentation x_0. $\square$

A.1.4 Value Function of the Active Investor

Proposition 9 *The value function of the active investor unambiguously increases with expected future dividends* $\mathbb{E}_t[\delta_s]$ *and unambiguously decreases with the future variance* $\mathrm{Var}_t[\ln\delta_s]$*, for any* $s \geq t$*. Furthermore, the value function can be written*

$$J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \frac{e^{-\rho t}(\theta\delta_t)^{1-\gamma}}{1-\gamma} F(\widehat{\beta}_t, \nu_t, t), \tag{75}$$

where

$$F(\widehat{\beta}_t, \nu_t, t) \equiv \int_t^T \exp\left[\kappa(x_0, \widehat{\beta}_t)(s-t) + \frac{(1-\gamma)^2}{2} x_0^2\nu_t(s-t)^2\right] ds \tag{76}$$

and

$$\kappa(x_0, \widehat{\beta}_t) \equiv (1-\gamma)\left(\bar{f} + x_0\widehat{\beta}_t - \gamma\frac{\sigma^2(1+kx_0)^2}{2}\right) - \rho. \tag{77}$$

Proof In equilibrium, the active investor consumes a fraction θ of the entire output δ_t and thus her lifetime expected utility of consumption can be computed as

$$J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \mathbb{E}_t\left[\int_t^T e^{-\rho s}\frac{(\theta\delta_s)^{1-\gamma}}{1-\gamma} ds\right] = \frac{\theta^{1-\gamma}}{1-\gamma}\int_t^T e^{-\rho s}\mathbb{E}_t\left[\delta_s^{1-\gamma}\right] ds, \tag{78}$$

where the second equality results from application of Fubini's theorem. The expectation in Eq. (10) can be further expanded by using the property that for a normally distributed random variable $y = \ln(x)$, $\mathbb{E}[x^\alpha] = \exp(\alpha \mathbb{E}[y] + \alpha^2 \mathrm{Var}[y]/2)$:

$$J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \frac{\theta^{1-\gamma}}{1-\gamma} \int_t^T e^{-\rho s} \exp\left[(1-\gamma)\mathbb{E}_t[\ln \delta_s] + \frac{(1-\gamma)^2}{2} \mathrm{Var}_t[\ln(\delta_s)]\right] ds. \tag{79}$$

Replacing (49) and (74) yields

$$J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \frac{e^{-\rho t}(\theta \delta_t)^{1-\gamma}}{1-\gamma} \int_t^T \exp\left[\kappa(x_0, \widehat{\beta}_t)(s-t) + \frac{(1-\gamma)^2}{2} x_0^2 \nu_t (s-t)^2\right] ds, \tag{80}$$

with $\kappa(x_0, \widehat{\beta}_t)$ defined in (77). In order to show that $J_a(\delta_t, \widehat{\beta}_t, \nu_t, t)$ increases in $\mathbb{E}_t[\delta_s]$ and decreases in $\mathrm{Var}_t[\ln \delta_s]$, replace the relation

$$\mathbb{E}_t[\ln \delta_s] = \ln(\mathbb{E}_t[\delta_s]) - \frac{1}{2} \mathrm{Var}_t[\ln \delta_s] \tag{81}$$

in (79) to obtain

$$J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \frac{\theta^{1-\gamma}}{1-\gamma} \int_t^T e^{-\rho s} \mathbb{E}_t[\delta_s]^{1-\gamma} \exp\left[-\frac{\gamma(1-\gamma)}{2} \mathrm{Var}_t[\ln \delta_s]\right] ds. \tag{82}$$

From Eq. (82), it is a matter of algebra to show that $J(\delta_t, \widehat{\beta}_t, t)$ increases in $\mathbb{E}_t[\delta_s]$ and decreases in $\mathrm{Var}_t[\ln \delta_s]$, for any value of the risk aversion parameter γ. □

A.1.5 Proof of Proposition 2 (Optimal Experimentation)

The expected lifetime utility of the passive investor equals:

$$J_p(\delta_t, \widehat{\beta}_t, \nu_t, t) = \frac{e^{-\rho t}((1-\theta)\delta_t)^{1-\gamma}}{1-\gamma} F(\widehat{\beta}_t, \nu_t, t). \tag{83}$$

Because the consumption share θ is constant, the choice of the active investor also maximizes the expected lifetime utility of the passive investor and is therefore first best (in other words, different values of the parameter $\theta \in [0,1]$ will always yield the same amount of optimal experimentation). The first order condition for the active investor writes

$$\begin{aligned} 0 = \frac{\partial J_a(\delta_0, \widehat{\beta}_0, \nu_0, 0)}{\partial x_0} &= \frac{(\theta \delta_0)^{1-\gamma}}{1-\gamma} \int_0^T \left[(1-\gamma)\left(\widehat{\beta}_0 - \gamma k \sigma^2 (1 + k x_0)\right) t + (1-\gamma)^2 x_0 \nu_0 t^2\right] \\ &\quad \times \exp\left[\kappa(x_0, \widehat{\beta}_0) t + \frac{(1-\gamma)^2}{2} x_0^2 \nu_0 t^2\right] dt. \end{aligned} \tag{84}$$

Define the functions G and H as

$$G(\widehat{\beta}_t,\nu_t,t) \equiv \int_t^T (s-t)\exp\left[\kappa(x_0,\widehat{\beta}_t)(s-t)+\frac{(1-\gamma)^2}{2}x_0^2\nu_t(s-t)^2\right]ds \tag{85}$$

$$H(\widehat{\beta}_t,\nu_t,t) \equiv \int_t^T (s-t)^2\exp\left[\kappa(x_0,\widehat{\beta}_t)(s-t)+\frac{(1-\gamma)^2}{2}x_0^2\nu_t(s-t)^2\right]ds. \tag{86}$$

After replacing G and H and dividing by F, the first order condition becomes

$$\widehat{\beta}_0\mathbb{D}_0 - \gamma k\sigma^2(1+kx_0)\mathbb{D}_0 + (1-\gamma)x_0\nu_0\mathbb{C}_0 = 0, \tag{87}$$

where the two quantities $\mathbb{D}_t$ and $\mathbb{C}_t$ represent the equity duration and the equity convexity:

$$\mathbb{D}_t \equiv \frac{G(\widehat{\beta}_t,\nu_t,t)}{F(\widehat{\beta}_t,\nu_t,t)} \tag{88}$$

$$\mathbb{C}_t \equiv \frac{H(\widehat{\beta}_t,\nu_t,t)}{F(\widehat{\beta}_t,\nu_t,t)}. \tag{89}$$

The value $\mathbb{D}_t$ represents the weighted average maturity (i.e., the *equity duration*), whereas the value $\mathbb{C}_t$ represents the weighted average squared maturity (i.e., the *equity convexity*). Both $\mathbb{D}_t$ and $\mathbb{C}_t$ ar positive. Solving for x_0 in (87) yields (12). □

A.1.6 Proof of Proposition 3 (Experimentation under Competition)

Agent i faces the following choice in terms of consumption share and output:

$$\theta_i(x_i,\tilde{x}_C) = \frac{1}{N}e^{-\varphi+x_i(w-\tilde{x}_C)} \tag{90}$$

$$\frac{d\delta_t}{\delta_t} = (\bar{f}+\widehat{\beta}_t\tilde{x}_C)dt + \sigma(1+k\tilde{x}_C)d\widehat{W}_t. \tag{91}$$

The expected lifetime utility of consumption for agent i at time t is

$$J_i(\delta_t,\widehat{\beta}_t,\nu_t,t) = \mathbb{E}_t\left[\int_t^T e^{-\rho s}\frac{\left(\frac{1}{N}e^{-\varphi+x_i(w-\tilde{x}_C)}\delta_s\right)^{1-\gamma}}{1-\gamma}ds\right] \tag{92}$$

$$= \frac{\left(\frac{1}{N}e^{-\varphi+x_i(w-\tilde{x}_C)}\right)^{1-\gamma}}{1-\gamma}\int_t^T e^{-\rho s}\mathbb{E}_t\left[\delta_s^{1-\gamma}\right]ds \tag{93}$$

$$= \underbrace{\frac{e^{-\rho t}\left(\frac{1}{N}e^{-\varphi+x_i(w-\tilde{x}_C)}\delta_t\right)^{1-\gamma}}{1-\gamma}}_{\equiv f(x_i)}F(\widehat{\beta}_t,\nu_t,t), \tag{94}$$

where $F(\cdot)$ is defined as before, except that now it is a function of $\tilde{x}_C$ instead of x_0. We prove first the following result.

Proposition 10 *At $t=0$, the function $J_i(\delta_0,\widehat{\beta}_0,\nu_0,0)$ admits a unique maximum in x_i.*

Proof *Write the function $J_i(\delta_0, \widehat{\beta}_0, \nu_0, 0)$ as the product of a negative constant and another function $z(x_i)$:*

$$J_i(\delta_t, \widehat{\beta}_t, \nu_t, t) = \underbrace{\frac{e^{-\rho t}\left(\frac{1}{N}\delta_t\right)^{1-\gamma} e^{(1-\gamma)(-\varphi)}}{1-\gamma}}_{<0} z(x_i), \tag{95}$$

where the function $z(x_i)$ is defined as

$$z(x_i) \equiv e^{(1-\gamma)x_i(w-\tilde{x}_C)} F(\widehat{\beta}_0, \nu_0, 0). \tag{96}$$

The function $z(x_i)$ is a product of two strictly positive functions (an exponential function and the price-dividend ratio).[20] *Consider a strictly monotonic transformation of $z(x_i)$:*

$$\ln z(x_i) = (1-\gamma)x_i(w - x_i - \tilde{x}_C^-) + \ln F(\widehat{\beta}_0, \nu_0, 0) \tag{97}$$

$$= \underbrace{(w - \tilde{x}_C^-)(1-\gamma)x_i + (\gamma-1)x_i^2}_{\equiv A(x_i)} + \underbrace{\ln F(\widehat{\beta}_0, \nu_0, 0)}_{\equiv B(x_i)}, \tag{98}$$

where $\tilde{x}_C^- \equiv \tilde{x}_C - x_i$. The term $A(x_i)$ in (98) *is strictly convex when $\gamma > 1$. The term $B(x_i)$ represents the log price-dividend ratio. Close inspection of* (76)-(77) *reveals that the price-dividend ratio at time $t = 0$ has this particular form:*

$$F(\widehat{\beta}_0, \nu_0, 0) = \int_0^T y(\tilde{x}_C, s)ds. \tag{99}$$

It can be quickly verified that $y(\tilde{x}_C, s)$ is log-convex in x_i. This implies that the price-dividend ratio $F(\widehat{\beta}_0, \nu_0, 0)$ is log-convex in x_i (Boyd and Vandenberghe, 2004, p. 105). It then follows that the term $B(x_i)$ in (98) *is convex. Thus, $\ln z(x_i)$ is strictly convex and therefore the function $z(x_i)$ admits a unique minimum. This implies that $J_i(\delta_t, \widehat{\beta}_t, \nu_t, t)$ admits a unique maximum.* □

Starting from (94), write the first order condition for agent i at time $t = 0$:

$$0 = \frac{\partial f(x_i)}{\partial x_i} F(\widehat{\beta}_0, \nu_0, 0) + f(x_i)\frac{\partial F(\widehat{\beta}_0, \nu_0, 0)}{\partial x_i} \tag{100}$$

$$= (1-\gamma)(w - x_i - \tilde{x}_C)f(x_i)F(\widehat{\beta}_0, \nu_0, 0) + f(x_i)\frac{\partial F(\widehat{\beta}_0, \nu_0, 0)}{\partial x_i}, \tag{101}$$

and replace

$$\frac{\partial F(\widehat{\beta}_0, \nu_0, 0)}{\partial x_i} = \frac{\partial \tilde{x}_C}{\partial x_i}\frac{\partial F(\widehat{\beta}_0, \nu_0, 0)}{\partial \tilde{x}_C} \tag{102}$$

$$= (1-\gamma)\left[\widehat{\beta}_0 - \gamma k\sigma^2(1 + k\tilde{x}_C)\right] G(\widehat{\beta}_0, \nu_0, 0) + \tilde{x}_C(1-\gamma)^2\nu_0 H(\widehat{\beta}_0, \nu_0, 0). \tag{103}$$

[20] Although both functions are convex, this does not guarantee that their product is convex.

After dividing by $(1-\gamma)f(x_i)F(\widehat{\beta}_0,\nu_0,0)$, the first order condition for agent i at time $t=0$ becomes:

$$0=(w-x_i-\tilde{x}_C)+\left[\widehat{\beta}_0-\gamma k\sigma^2(1+k\tilde{x}_C)\right]\mathbb{D}_0+\tilde{x}_C(1-\gamma)\mathbb{C}_0. \tag{104}$$

The first term on the right hand side is the marginal benefit of experimentation arising from an increased consumption share. In the case of a monopolist, this term becomes $(w-2x_i)$. Therefore, if $w>2x_S^*$, then we are guaranteed that a monpolist will invest more than in the first best case. Furthermore, it can be verified from Proposition 2 that the highest level of first best experimentation is obtained in the log-utility case and equals $(\widehat{\beta}_0-k\sigma^2)/(k^2\sigma^2)$. Thus, a sufficient condition necessary to obtain more experimentation in the monopolistic case is Eq. (14) in the text:

$$w>\frac{2(\widehat{\beta}_0-k\sigma^2)}{k^2\sigma^2}. \tag{105}$$

Solving for x_i in Eq. (104) yields

$$x_i=\left[\widehat{\beta}_0-\gamma k\sigma^2(1+k\tilde{x}_C)\right]\mathbb{D}_0+(w-\tilde{x}_C)-\tilde{x}_C(\gamma-1)\nu_0\mathbb{C}_0, \quad \forall i=1,...,N. \tag{106}$$

This shows that agents' best response functions are symmetric. Aggregating across agents and dividing by N yields an equation in the total experimentation in the economy, $\tilde{x}_C$:

$$\frac{1}{N}\tilde{x}_C=\left[\widehat{\beta}_0-\gamma k\sigma^2(1+k\tilde{x}_C)\right]\mathbb{D}_0+(w-\tilde{x}_C)-\tilde{x}_C(\gamma-1)\nu_0\mathbb{C}_0, \tag{107}$$

with solution

$$\tilde{x}_C^*=\frac{\widehat{\beta}_0\mathbb{D}_0-\gamma k\sigma^2\mathbb{D}_0+w}{\gamma k^2\sigma^2\mathbb{D}_0+(\gamma-1)\nu_0\mathbb{C}_0+\frac{N+1}{N}}. \tag{108}$$

This is a fixed point problem ($\mathbb{D}_0$ and $\mathbb{C}_0$ depend on $\tilde{x}_C^*$). The function on the right hand side is continuous and starts from a strictly positive value at $\tilde{x}_C=0$. The solution is obtained when this function crosses the 45-degree line. The fixed point is unique, since we have proven in Proposition 10 that the function $J_i(\delta_t,\widehat{\beta}_t,\nu_t,t)$ admits a unique maximum and x_i is unique $\forall i=1,...,N$. To prove that $\tilde{x}_C^*$ is strictly increasing in N, consider two consecutive cases, N and $N+1$:

$$\tilde{x}_C^*(N)=\frac{\widehat{\beta}_0\mathbb{D}_0-\gamma k\sigma^2\mathbb{D}_0+w}{\gamma k^2\sigma^2\mathbb{D}_0+(\gamma-1)\nu_0\mathbb{C}_0+\frac{N+1}{N}} \tag{109}$$

$$\tilde{x}_C^*(N+1)=\frac{\widehat{\beta}_0\mathbb{D}_0-\gamma k\sigma^2\mathbb{D}_0+w}{\gamma k^2\sigma^2\mathbb{D}_0+(\gamma-1)\nu_0\mathbb{C}_0+\frac{N+1}{N}-\frac{1}{N(N+1)}}. \tag{110}$$

The two expressions on the right hand side in (109)-(110) are both functions of the experimentation level x. Naming these functions $g(x)$ and and $h(x)$ respectively, it is clear that, for any given experimentation level $x \geq 0$:

$$g(x) < h(x). \tag{111}$$

This inequality is easily verified, since for a given experimentation level x the duration $\mathbb{D}_0$ and the convexity $\mathbb{C}_0$ are the same in both functions. It then follows that the point at which the function $h(\cdot)$ crosses the 45-degree line is strictly higher than the point at which the function $g(\cdot)$ crosses the 45-degree line (see Figure 1). Since the equilibrium is unique, the quantity $\tilde{x}_C^*$ is strictly increasing in N. This completes the proof of Proposition 3. □

A.2 Implications for Asset Prices

A.2.1 Proof of Proposition 4 (Stochastic Discount Factor)

The proof follows standard results in asset pricing (Duffie, 2010). Assuming time-additive expected utility, we can define a stochastic discount factor from the optimal consumption plan of any individual as

$$\xi_t = e^{-\rho t} \frac{u'(\delta_t)}{u'(\delta_0)}. \tag{112}$$

Note that in our case agents consume fixed shares of the aggregate output and observe the economy under the same probability measure. Given the CRRA assumption, the dynamics of the stochastic discount factor can then be expressed as

$$\frac{d\xi_t}{\xi_t} = -\left[\rho + \gamma(\bar{f} + x_0\widehat{\beta}_t) - \frac{1}{2}\gamma(\gamma+1)\sigma^2(1+kx_0)^2\right] dt - \gamma\sigma(1+kx_0)d\widehat{W}_t. \tag{113}$$

The continuously compounded risk-free rate is the negative of the drift of the stochastic discount factor, whereas the market price of risk process is the negative of the diffusion of the stochastic discount factor. This yields (19) and (20) in Proposition 4. □

A.2.2 Proof of Proposition 5 (Asset Prices)

Recall that the state variables in this economy evolve according to (5)-(7). The equilibrium price of the risky asset is

$$P_t = \frac{1}{\xi_t}\mathbb{E}_t\left[\int_t^T \xi_s \delta_s ds\right] = \delta_t^\gamma \int_t^T e^{-\rho(s-t)}\mathbb{E}_t[\delta_s^{1-\gamma}]ds. \tag{114}$$

Using Proposition 9, we obtain

$$P_t = \frac{1-\gamma}{\theta^{1-\gamma}} e^{\rho t}\delta_t^\gamma J_a(\delta_t, \widehat{\beta}_t, \nu_t, t) = \delta_t F(\widehat{\beta}_t, \nu_t, t), \tag{115}$$

which proves that indeed $F(\widehat{\beta}_t, \nu_t, t)$ is the price-dividend ratio. We then obtain F_t, F_β, F_ν, and $F_{\beta\beta}$, where $F(\widehat{\beta}_t, \nu_t, t)$ is defined in (76) and $\kappa(x_0, \widehat{\beta}_t)$ is defined in (77):

$$F_t \equiv \frac{\partial F(\widehat{\beta}_t, \nu_t, t)}{\partial t} = -1 - \kappa(x_0, \widehat{\beta}_t) F(\widehat{\beta}_t, \nu_t, t) - (1-\gamma)^2 x_0^2 \nu_t G(\widehat{\beta}_t, \nu_t, t) \tag{116}$$

$$F_\beta \equiv \frac{\partial F(\widehat{\beta}_t, \nu_t, t)}{\partial \widehat{\beta}_t} = (1-\gamma) x_0 G(\widehat{\beta}_t, \nu_t, t) \tag{117}$$

$$F_{\beta\beta} \equiv \frac{\partial^2 F(\widehat{\beta}_t, \nu_t, t)}{\partial \widehat{\beta}_t^2} = (1-\gamma)^2 x_0^2 H(\widehat{\beta}_t, \nu_t, t) \tag{118}$$

$$F_\nu \equiv \frac{\partial F(\widehat{\beta}_t, \nu_t, t)}{\partial \nu_t} = \frac{(1-\gamma)^2}{2} x_0^2 H(\widehat{\beta}_t, \nu_t, t), \tag{119}$$

where $G(\widehat{\beta}_t, \nu_t, t)$ and $H(\widehat{\beta}_t, \nu_t, t)$ are defined in (85)-(86). Apply Ito's formula to P_t:

$$dP_t = \delta_t F \frac{d\delta_t}{\delta_t} + \delta_t F_\beta d\widehat{\beta}_t + \delta_t F_\nu d\nu_t + \delta_t F_t dt + \frac{1}{2}\left[\delta_t F_{\beta\beta}(d\widehat{\beta}_t)^2 + 2F_\beta (d\delta_t)(d\widehat{\beta}_t)\right], \tag{120}$$

to obtain

$$\frac{dP_t}{P_t} = \mu_{P,t} dt + \sigma_{P,t} d\widehat{W}_t, \tag{121}$$

with

$$\mu_{P,t} \equiv \bar{f} + \widehat{\beta}_t x_0 - \kappa(x_0, \widehat{\beta}_t) - \frac{1}{F(\widehat{\beta}_t, \nu_t, t)} + \gamma(1-\gamma) x_0^2 \nu_t \frac{G(\widehat{\beta}_t, \nu_t, t)}{F(\widehat{\beta}_t, \nu_t, t)} \tag{122}$$

$$\sigma_{P,t} \equiv \sigma(1 + kx_0)\left(1 + (1-\gamma)\frac{x_0^2 \nu_t}{\sigma^2(1+kx_0)^2} \frac{G(\widehat{\beta}_t, \nu_t, t)}{F(\widehat{\beta}_t, \nu_t, t)}\right). \tag{123}$$

To obtain the risk premium as in (23), multiply the market price of risk, $\theta_t = \gamma\sigma(1 + kx_0)$ (Proposition 4, Eq. 20), with the diffusion of stock returns. □

A.2.3 Discretization of the continuous-time setup (Section 3.2)

This appendix derives a discretization of our continuous-time setup. This discretization is used for simulations in Section 3.2.

$$\delta_{t+\Delta} = \delta_t e^{\left[\bar{f} + \widehat{\beta}_t x_0 - \frac{1}{2}\sigma^2(1+kx_0)^2\right]\Delta + \sigma\sqrt{\Delta}(1+kx_0) z_{t+\Delta}} \tag{124}$$

$$\widehat{\beta}_{t+\Delta} = \widehat{\beta}_t + \sqrt{\Delta} \frac{x_0}{\sigma(1+kx_0)} \nu_t z_{t+\Delta} \tag{125}$$

$$\nu_{t+\Delta} = \nu_t - \frac{x_0^2}{\sigma^2(1+kx_0)^2} \nu_t^2 \Delta, \tag{126}$$

where $z_{t+\Delta} \sim i.i.d. N(0,1)$ and Δ is the time step in years.

References

Acemoglu, D. and P. Restrepo (2017). Robots and jobs: Evidence from us labor markets. *Working Paper*.

Aghion, P., U. Akcigit, A. Bergeaud, R. Blundell, and D. Hémous (2015). Innovation and top income inequality. Technical report, National Bureau of Economic Research.

Aghion, P., N. Bloom, R. Blundell, R. Griffith, and P. Howitt (2005). Competition and innovation: An inverted-u relationship. *The Quarterly Journal of Economics 120*(2), 701–728.

Aghion, P., P. Bolton, C. Harris, and B. Jullien (1991). Optimal learning by experimentation. *The review of economic studies 58*(4), 621–654.

Aghion, P. and R. Griffith (2008). *Competition and growth: reconciling theory and evidence.* MIT press.

Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica 60*(2), 323–351.

Akcigit, U., J. Grigsby, and T. Nicholas (2017). The rise of american ingenuity: Innovation and inventors of the golden age.

Andrei, D. and M. Hasler (2014). Investor attention and stock market volatility. *The Review of Financial Studies 28*(1), 33–72.

Arrow, K. (1962). The economic implications of learning by doing. *Review of Economic Studies*, 131.

Bampoky, C., J. E. Prieger, L. R. Blanco, and A. Liu (2016). Economic growth and the optimal level of entrepreneurship. *World Development 82*, 95–109.

Beeler, J. and J. Y. Campbell (2012). The long-run risks model and aggregate asset prices: An empirical assessment. *Critical Finance Review 1*(1), 141–182.

Boldrin, M. and D. K. Levine (2008). Perfectly competitive innovation. *Journal of Monetary Economics 55*(3), 435–453.

Boyd, S. and L. Vandenberghe (2004). *Convex optimization.* Cambridge university press.

Brennan, M. J. (1998). The role of learning in dynamic portfolio decisions. *European Finance Review 1*(3), 295–306.

Bryson, A. E. and Y.-C. Ho (1975). *Applied optimal control: optimization, estimation and control.* Taylor and Francis.

Campbell, J. Y. (1999). Asset prices, consumption, and the business cycle. *Handbook of macroeconomics 1*, 1231–1303.

Carree, M. A. and A. R. Thurik (2005). The impact of entrepreneurship on economic growth. *Handbook of entrepreneurship research*, 437–471.

Cochrane, J. H. (2008). The dog that did not bark: a defense of return predictability. *Review of Financial Studies 21*(4), 1533–1575.

Cole, H. L. and M. Obstfeld (1991). Commodity trade and international risk sharing: How much do financial markets matter? *Journal of monetary economics 28*(1), 3–24.

DeMarzo, P., R. Kaniel, and I. Kremer (2007). Technological innovation and real investment booms and busts. *Journal of Financial Economics 85*(3), 735–754.

Detemple, J. (1986). Asset pricing in a production economy with incomplete information. *The Journal of Finance 41*(2), pp. 383–391.

Dothan, M. U. and D. Feldman (1986, June). Equilibrium interest rates and multiperiod bonds in a partially observable economy. *Journal of Finance 41*(2), 369–82.

Dreze, J. H. (1981). Inferring risk tolerance from deductibles in insurance contracts. *Geneva Papers on Risk and Insurance*, 48–52.

Duffie, D. (2010). *Dynamic asset pricing theory*. Princeton University Press.

Duffie, D., D. Filipović, and W. Schachermayer (2003). Affine processes and applications in finance. *Annals of applied probability*, 984–1053.

Epstein, L. G., E. Farhi, and T. Strzalecki (2014). How much would you pay to resolve long-run risk? *The American Economic Review 104*(9), 2680–2697.

Epstein, L. G. and S. E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica 57*(4), 937–969.

Fama, E. F. and K. R. French (1988). Dividend yields and expected stock returns. *Journal of financial economics 22*(1), 3–25.

Filipović, D. (2005). Time-inhomogeneous affine processes. *Stochastic Processes and their Applications 115*(4), 639–659.

Friend, I. and M. E. Blume (1975). The demand for risky assets. *The American Economic Review*, 900–922.

Futia, C. A. (1980). Schumpeterian competition. *The Quarterly Journal of Economics 94*(4), 675–695.

Gârleanu, N., L. Kogan, and S. Panageas (2012). Displacement risk and asset returns. *Journal of Financial Economics 105*(3), 491–510.

Gennotte, G. (1986, July). Optimal portfolio choice under incomplete information. *Journal of Finance 41*(3), 733–46.

Gort, M. and S. Klepper (1982). Time paths in the diffusion of product innovations. *The economic journal 92*(367), 630–653.

Grammig, J. and S. Jank (2015). Creative destruction and asset prices. *Journal of Financial and Quantitative Analysis*.

Grossman, S. J., R. E. Kihlstrom, and L. J. Mirman (1977). A bayesian approach to the production of information and learning by doing. *The Review of Economic Studies*, 533–547.

Helpman, E. and A. Razin (2014). *A theory of international trade under uncertainty.* Academic Press.

Hodrick, R. J. (1992). Dividend yields and expected stock returns: Alternative procedures for inference and measurement. *The Review of Financial Studies 5*(3), 357–386.

Johnson, T. C. (2007). Optimal learning and new technology bubbles. *Journal of Monetary Economics 54*(8), 2486–2511.

Komlos, J. (2014). Has creative destruction become more destructive? Technical report, National Bureau of Economic Research.

Kung, H. and L. Schmid (2015). Innovation, growth, and asset prices. *The Journal of Finance 70*(3), 1001–1037.

Liptser, R. and A. Shiryayev (1977). Statistics of random processes. Springer-Verlag, Berlin, New York.

Loury, G. C. (1979). Market structure and innovation. *The quarterly journal of economics*, 395–410.

Lucas, R. (1978). Asset prices in an exchange economy. *Econometrica: Journal of the Econometric Society*, 1429–1445.

Mehra, R. and E. C. Prescott (1985). The equity premium: A puzzle. *Journal of monetary Economics 15*(2), 145–161.

Newey, W. K. and K. D. West (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica 55*(3), 703–08.

Pástor, L. and P. Veronesi (2003). Stock valuation and learning about profitability. *The Journal of Finance 58*(5), 1749–1790.

Pástor, L. and P. Veronesi (2006). Was there a nasdaq bubble in the late 1990s? *Journal of Financial Economics 81*(1), 61–100.

Pastor, L. and P. Veronesi (2009). Technological revolutions and stock prices. *American Economic Review 99*(4), 1451–1483.

Pavlova, A. and R. Rigobon (2007). Asset prices and exchange rates. *Review of Financial Studies 20*(4), 1139–1180.

Reinganum, J. F. (1983). Uncertain innovation and the persistence of monopoly. *The American Economic Review 73*(4), 741–748.

Reinganum, J. F. (1985). Innovation and industry evolution. *The Quarterly Journal of Economics 100*(1), 81–99.

Rob, R. (1991). Learning and capacity expansion under demand uncertainty. *The Review of Economic Studies 58*(4), 655–675.

Romer, P. M. (1990). Endogenous technological change. *Journal of political Economy 98*(5, Part 2), S71–S102.

Schumpeter, J. A. (1934). *The theory of economic development: An inquiry into profits, capital, credit, interest, and the business cycle*, Volume 55. Transaction publishers.

Schumpeter, J. A. (1942). *Capitalism, socialism and democracy.* New York: Harper and Brothers.

Swan, P. L. (1970). Market structure and technological progress: The influence of monopoly on product innovation. *The Quarterly Journal of Economics 84*(4), 627–638.

Vissing-Jørgensen, A. (2002). Limited asset market participation and the elasticity of intertemporal substitution. *Journal of Political Economy 110*(4), 825–853.

Vissing-Jørgensen, A. and O. P. Attanasio (2003). Stock-market participation, intertemporal substitution, and risk-aversion. *The American Economic Review 93*(2), 383–391.

Williams, J. T. (1977). Capital asset prices with heterogeneous beliefs. *Journal of Financial Economics 5*(2), 219–239.

Williamson, O. E. (1965). Innovation and market structure. *Journal of political economy 73*(1), 67–73.

Witt, U. (1996). Innovations, externalities and the problem of economic progress. *Public Choice 89*(1), 113–130.

Zapatero, F. (1995). Equilibrium asset prices and exchange rates. *Journal of Economic Dynamics and Control 19*(4), 787–811.

Ziegler, A. C. (2003). *Incomplete Information and Heterogeneous Beliefs in Continuous-time Finance*. Springer-Verlag Berlin Heidelberg.