

The Lost Capital Asset Pricing Model

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Abstract

Just as the “lost city of Atlantis,” the CAPM is empirically invisible most of the time. Yet, recent evidence shows that a strong CAPM relation holds on macroeconomic announcement days. We show that these findings coexist in an economy with asymmetric information. In this context the CAPM relation holds relative to the market consensus—the average beliefs across investors—but fails for the econometrician who does not observe investors’ information nor the market portfolio. On announcement days, when investors learn about macroeconomic factors to which stocks are exposed, fundamental risk better explains variations in asset returns, clearing the “shoal of mud” that stands in the way of the CAPM.

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1 Introduction

The Capital Asset Pricing Model, one of the main pillars of modern finance theory, fails in empirical tests. A common explanation for this empirical failure is the [Roll \(1977\)](#) critique: testing the CAPM is almost axiomatically impossible because the composition of the market portfolio is unobservable. While various attempts have been undertaken to alleviate this criticism, the CAPM relation is yet to be found in the data, largely shaping the view that it does not hold. However, several recent empirical findings challenge this view. [Berk and Van Binsbergen \(2016\)](#) find that investors actually trade off risk and return using the CAPM as their model of choice. [Savor and Wilson \(2014\)](#) document a strong CAPM relation on days during which macroeconomic news are released, which vanishes immediately right after. This result is puzzling as it suggests that the CAPM behaves like a hidden “Atlantis” that unveils on particular occasions.

This paper is a theoretical attempt to understand why the CAPM fails most of the time (except on announcement days), although many investors seem to base their decisions on it. Our explanation builds on the idea that the econometrician—an *outside* observer—has a coarser information set than that of *inside* investors who trade in the marketplace ([Roll, 1978](#); [Dybvig and Ross, 1985](#); [Hansen and Richard, 1987](#)). In particular, suppose that an econometrician observes time series of realized excess returns for a large number of assets. Then, using the law of total covariance, the unconditional variance-covariance matrix of realized excess returns that the econometrician computes based on historical data,

$$\mathbb{V}[Q] = \mathbb{V} [\mathbb{E}_i^i[Q]] + \mathbb{E} [\mathbb{V}_i^i[Q]] , \tag{1}$$

can be decomposed into the observed variation “explained” by the information of an inside agent i and a remaining “unexplained” component. Our main argument is that the econometrician’s perspective implies a distortion in beta estimates relative to agents’ betas, which are forward looking and solely based on the unexplained component of the variance. Our contribution is to place economic restrictions on this distortion. Using a rational-expectations equilibrium model, we show that this distortion causes the econometrician to perceive a “flat” CAPM line, which steepens when public information is released.

We develop a dynamic model of informed trading ([Spiegel, 1998](#)), in which a continuum of mean-variance investors trade multiple assets based on private and public information. A necessity for private information to be relevant in this framework is that the market portfolio be unobservable, thus sowing the seeds of the [Roll \(1977\)](#) critique right into the building blocks of the model. To that effect the literature commonly assumes that the supply of stocks is noisy ([Grossman and Stiglitz, 1980](#)) or that some assets are privately traded ([Wang,](#)

1994); we follow the former approach. Furthermore, our main purpose requires that the model contain two additional elements. First, we assume that stock payoffs have a common, unobservable factor on which each asset loads differently. Second, we introduce periodic announcements through which public information about this common factor is released, affecting the cross section of stock returns.

Suppose now that an outside econometrician estimates a Security Market Line (SML) in this equilibrium model. Three features of the model complicate this task. First, the market portfolio is unobservable (Roll, 1977), which creates additional uncertainty. This additional uncertainty biases the beta estimates. Second, the information set on which investors condition their expectations is unobservable (Roll, 1978; Hansen and Richard, 1987; Dybvig and Ross, 1985). Conditioning down on a coarser information set further biases beta estimates. Third, aggregating investors' dispersed private information results in a violation of the law of iterated expectations (Allen, Morris, and Shin, 2006), which introduces another source of distortion in beta estimates. In general, without an equilibrium specification of stock excess returns, the bias resulting from these three features is merely a statistical effect that distorts the estimated security market line in an arbitrary way—"anything goes." In contrast, our equilibrium model places an endogenous structure on excess returns, thus providing economic restrictions on this statistical effect.

The main argument is that the econometrician, who holds unconditional beliefs and thus faces more uncertainty than the "average investor" (a fictitious agent whose beliefs define the market consensus), perceives the comovement among assets differently. While the average investor sees the *true* SML—a line that crosses the origin with a slope given by the market risk premium—the econometrician's SML rotates clockwise around the market portfolio, which flattens its slope and creates a positive intercept. Specifically, the beta of the market becomes the "center of gravity" around which econometrician's betas "inflate": assets with a beta higher than one display a higher beta than the true beta, whereas assets with a beta lower than one display a lower beta than the true beta.¹ The magnitude of this distortion in beta estimates is proportional to the informational advantage of investors relative the econometrician.

Suppose further that we allow the econometrician to estimate the CAPM relation conditioning on days when investors observe public announcements, as in Savor and Wilson (2014). Two effects arise during announcement days. First, due to uncertainty about the public announcement, investors require a high risk premium for holding the stocks right before announcements. Keeping betas of assets constant, this effect raises expected returns

¹Another way to see this is that the average of betas is one for both the agent and the econometrician. Thus, the average bias in econometrician's betas has to be zero and therefore betas cannot be all biased in the same direction.

on all stocks, thus causing the SML to appear steeper. Second, and most importantly, because announcements provide additional information about macroeconomic fundamentals that drive the cross section of asset payoffs, the covariance matrix of asset returns changes in a way that the market variance becomes more informative about fundamental risk on announcement days. That is, the common factor better explains variations in asset returns on announcement days, causing the CAPM relation to look distinctly stronger.

On announcement days, a stronger CAPM relation arises along with several other phenomena. In the model, the Sharpe ratio increases on the announcement day and macroeconomic uncertainty increases over a typical announcement cycle, two patterns that have been documented by Savor and Wilson (2014, 2016). We also extend the model to allow for multiple factors. We find that the econometrician’s problem of estimating a security market line worsens in the multiple factor case: not only does the CAPM look flat, but the relation between average excess returns and betas is no longer linear. Finally, we allow for private information diffusion over the announcement cycle (Cieslak, Morse, and Vissing-Jorgensen, 2015), and find that the transmission of information has to take place exactly during the announcement day to be consistent with Savor and Wilson (2014).

The overall purpose of this paper is twofold. First, this paper attempts to explain why the CAPM fails for an outside observer, while it still remains a valuable model used by many investors to make investment decisions (Graham and Harvey, 2001; Berk and Van Binsbergen, 2016; Barber, Huang, and Odean, 2016). Second, this paper proposes that a flat SML should be the rule rather than the exception, but that the CAPM should become distinctly stronger on announcement days (Savor and Wilson, 2014). That the econometrician sees a flat SML originates from two well-known critiques. First, the market portfolio is unobservable, which precludes a test of the CAPM (the Roll (1977) critique). Second, empiricists do not observe investors’ beliefs and thus cannot include them in asset-pricing specifications (the Hansen and Richard (1987) critique).² Adding to these critiques, the aggregation of dispersed private information further distorts the CAPM relation perceived by the econometrician. We acknowledge that other possible explanations for a flat CAPM exist.³ However, none try to jointly explain why the CAPM fails most of the time, why it holds on announcement days and why many investors keep basing their investment decisions on it.

²The issue of not being able to observe information sets and its implications for the SML has been first analyzed by Roll (1978). The “Hansen-Richard critique” designation belongs to Cochrane (2009), by analogy to the “Roll critique.” Along the same vein, Dybvig and Ross (1985) show that conditionally mean-variance returns from a managed portfolio might appear unconditionally mean-variance inefficient.

³Other reasons include leverage constraints (Black, 1972; Frazzini and Pedersen, 2014), inflation (Cohen, Polk, and Vuolteenaho, 2005), disagreement (Hong and Sraer, 2016), preference for volatile, skewed returns (Kumar, 2009; Bali, Cakici, and Whitelaw, 2011), market sentiment (Antoniou, Doukas, and Subrahmanyam, 2015), stochastic volatility (Campbell, Giglio, Polk, and Turley, 2012), and benchmarking of institutional investors (Baker, Bradley, and Wurgler, 2011; Buffa, Vayanos, and Woolley, 2014).

Section 2 approaches the problem of conditioning down on a coarser information set in a simple model and provides intuition into the distortion in betas. Sections 3 and 4 present the dynamic model. Section 5 elaborates on the distortion of the SML in the dynamic model and describes the effect of a periodic public announcement, Section 6 studies extensions of the model, and Section 7 concludes. All proofs are relegated to the Appendix.

2 Background: The Effect of Conditioning Down

Before building a full-fledged asset pricing model, we provide economic intuition for the distortion of unconditional beta estimates in a simpler context. Consider N random variables (e.g., realized returns, or payoffs), the values of which are driven by a common “factor” F plus noise:

$$D \equiv \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_N \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \end{bmatrix} F + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{bmatrix} \equiv \Phi F + \epsilon, \quad (2)$$

where we assume that all random variables are independently normally distributed with zero means and precisions τ_F and τ_ϵ : $F \sim \mathcal{N}(0, \tau_F^{-1}) \perp \epsilon \sim \mathcal{N}(0_{N \times 1}, \tau_\epsilon^{-1} \mathbb{I}_N)$.

Suppose that a continuum of economic agents know the structure of realized payoffs in Eq. (2), but do not observe the single common factor (later in the paper, we extend this model to allow for multiple factors). Each agent i forms expectations about F based on both a private signal V_i and a public signal G :

$$V_i = F + v_i \quad (3)$$

$$G = F + v, \quad (4)$$

where the noise terms v and v_i are independently normally distributed: $v_i \sim \mathcal{N}(0, \tau_v^{-1})$, $\forall i$; $v \sim \mathcal{N}(0, \tau_G^{-1})$; $v_i \perp v_j$, $\forall i \neq j$ and $v \perp v_i$, $\forall i$.

Denoting by $\tau \equiv \tau_F + \tau_v + \tau_G$ the total precision based on all available information, an agent i 's expectations and posterior variances satisfy:

$$\mathbb{E}_i[D] = \frac{\tau_v}{\tau} \Phi V_i + \frac{\tau_G}{\tau} \Phi G \quad (5)$$

$$\mathbb{V}_i[D] = \frac{1}{\tau} \Phi \Phi' + \frac{1}{\tau_\epsilon} \mathbb{I}_N. \quad (6)$$

Consider now an “outside econometrician” who does not observe F nor signals about it, but only observes the payoff realizations D ex-post. Because the econometrician’s information set is coarser than that of inside agents, her assessment of the covariance of realized payoffs is different. In particular, the law of total covariance implies

$$\mathbb{V}[D] = \mathbb{V}^i[D] + \mathbb{V} [\mathbb{E}^i[D]], \quad (7)$$

where the left-hand side represents the variance of the econometrician based on realizations of D , whereas the conditional expectation and conditional variance on the right-hand side are based on the information set of any agent i in the economy.

In this Gaussian framework, the conditional variance $\mathbb{V}^i[D]$ is identical and constant for all agents, and represents the conditional variance of the “average agent” in the economy. We denote it by $\bar{\mathbb{V}}[D]$ hereafter. Aggregating over the continuum of agents further yields

$$\mathbb{V}[D] = \bar{\mathbb{V}}[D] + \int_i \mathbb{V} [\mathbb{E}_i[D]] di. \quad (8)$$

Moreover, because the expectation of the average agent and that of an agent i differ by the noise contained in agent i ’s private signal,

$$\mathbb{E}_i[D] = \bar{\mathbb{E}}[D] + \Phi \frac{\tau_v}{\tau} v_i, \quad (9)$$

the law of total covariance must incorporate an adjustment term:

$$\mathbb{V}[D] = \bar{\mathbb{V}}[D] + \mathbb{V} [\bar{\mathbb{E}}[D]] + \frac{\tau_v}{\tau^2} \Phi \Phi'. \quad (10)$$

The “adjusted” law of total covariance in Eq. (10) shows that the average agent’s covariance and that of the econometrician’s differ for two reasons. First, because the econometrician conditions on a coarser information set, she faces higher uncertainty. She thus obtains noisier estimates, an effect that is reflected in the second term on the right-hand side of Eq. (10). Second, the variance of individual expectations, $\mathbb{V} [\mathbb{E}_i[D]]$, differs from the variance of the average expectation, $\mathbb{V} [\bar{\mathbb{E}}[D]]$. The third term on the right-hand side accounts for this wedge, which is an additional source of covariation across payoffs in the eyes of the econometrician.

Using the covariance matrix of the average agent and that of the econometrician, we can compute their respective beta estimates based on exogenous payoffs (we will compute beta estimates based on endogenous equilibrium returns shortly). Assuming for simplicity that

the “market portfolio” has equal weights in both assets, betas satisfy

$$\beta = N \frac{\bar{\mathbb{V}}[D]\mathbb{1}}{\mathbb{1}'\bar{\mathbb{V}}[D]\mathbb{1}} \text{ and } \tilde{\beta} = N \frac{\mathbb{V}[D]\mathbb{1}}{\mathbb{1}'\mathbb{V}[D]\mathbb{1}}, \quad (11)$$

where $\mathbb{1}$ denotes a vector of ones of dimension N and the tilde stands for econometrician’s estimates. We are interested in the difference between these betas, which precisely represents the econometrician’s “distortion” of betas relative to the average agent.

Proposition 1. *Conditioning on a coarser information set magnifies the distance between econometrician’s betas and their average (which equals one) with respect to the distance between agent’s betas and their average:*

$$\tilde{\beta} - \mathbb{1} = (\beta - \mathbb{1}) \left(1 + \frac{\Delta_\tau}{\tau_F + \frac{\tau_\epsilon}{N} \left(\sum_{i=1}^N \phi_i \right)^2} \right), \quad (12)$$

where $\Delta_\tau \equiv \tau - \tau_F > 0$ represents the “informational advantage” of the average agent relative to the econometrician.

Proof. See Appendix A.1. □

Proposition 1 shows that conditioning on a coarser information set increases the dispersion of betas measured by the econometrician. Because the econometrician faces higher uncertainty about the common factor relative to economic agents ($\Delta_\tau > 0$), she overestimates betas on stocks with high factor loadings (i.e., betas that are higher than one) and underestimates betas on stocks with low factors loadings (betas that are lower than one). The key implication is that betas are distorted differently depending on whether they are lower or higher than one. Because betas result from regressing stock returns against market returns, all betas inflate around the beta of the market, which is one by definition.

We illustrate this effect in panel (a) of Figure 1, where we consider a model with two assets in which $\phi_1 = 1 + \phi$, $\phi_2 = 1 - \phi$, and $\phi > 0$ (i.e., asset one has a high beta and asset two has a low beta). The solid lines represent the distortion in betas, $\tilde{\beta}_i - \beta_i$, for the first asset (upward sloping line) and for the second asset (downward sloping line), as functions of the parameter ϕ . In the same panel, we show that dispersed private information plays an additional, nontrivial role in distorting betas. To illustrate this effect, we simply remove the third term from Eq. (10) and plot the resulting distortions in betas (the dashed lines). Clearly, removing the adjustment resulting from the aggregation of dispersed information attenuates the distortion in betas.

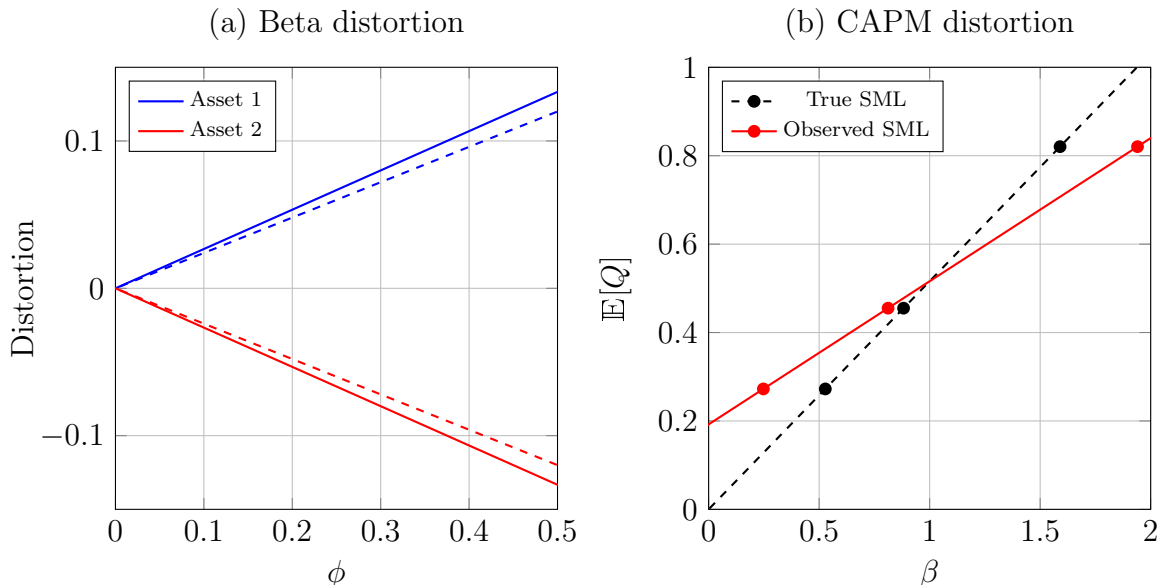


Figure 1: Beta distortion due to a coarser information set. Panel (a) results from Proposition 1 and shows the distortion in betas of payoffs, i.e. $\tilde{\beta}_i - \beta_i$, in a model with two assets with exposures $\phi_1 = 1 + \phi$ and $\phi_2 = 1 - \phi$, where $\phi > 0$. The blue solid line corresponds to asset 1 and the red solid line corresponds to asset 2. Dashed lines are obtained by removing the higher order wedge (third term in Eq. 10). Panel (b) results from Proposition 2 and shows the distortion of the SML in an equilibrium model with three assets with exposures $\phi_1 = -1/4$, $\phi_2 = 1/2$, and $\phi_3 = 2$. The black dashed line shows the true SML, whereas the red solid line shows the SML measured by the econometrician. The rest of the parameters used for this figure are: $\gamma = 1$ and $\tau_F = \tau_v = \tau_G = \tau_\epsilon = \tau_X = 1$.

Our previous discussion relies on exogenous payoffs, as opposed to equilibrium returns. We now solve for the linear equilibrium of this simple, static economy (Admati, 1985). For generality we consider the N assets case and let the elements $\{\phi_i\}_{i=1}^N$ of the vector of loadings Φ be arbitrary. We assume that the noise ϵ is independent across assets. To allow prices to play an informational role in equilibrium, we make the customary assumption that the supply $X \equiv [X_1 \dots X_N]'$ of assets is noisy: on average each asset still has equal weight $1/N$, but their weights are now normally distributed with covariance matrix $\tau_X^{-1}\mathbb{I}_N$. We further assume that agents have utility over terminal wealth with constant absolute risk aversion γ and we normalize the risk-free rate to 0. All equilibrium computations are in Appendix A.2.

Denoting excess returns by $Q \equiv D - P$, the market-clearing condition creates a relation between the average agent's expectations $\bar{\mathbb{E}}[Q]$ and posterior variance $\bar{\mathbb{V}}[Q]$:

$$\bar{\mathbb{E}}[Q] = \gamma \bar{\mathbb{V}}[Q] X. \quad (13)$$

Based on this endogenous relation, we can rewrite the law of total covariance in Eq. (10) in

terms of excess returns:

$$\mathbb{V}[Q] = \bar{\mathbb{V}}[Q] \left(\mathbb{I}_N + \frac{\gamma^2}{\tau_X} \bar{\mathbb{V}}[Q] \right) + \frac{\tau_v}{\tau^2} \Phi \Phi'. \quad (14)$$

The customary assumption of noisy supply—necessary for prices to play an informational role—sows the seeds of the Roll (1977) critique right into the assumptions of the model. In particular, it makes the market portfolio unobservable both to economic agents and to the econometrician. As a result, supply shocks generate fluctuations in expected returns, and the covariance matrix of realized returns measured by the econometrician becomes an “inflated” version of the posterior covariance matrix conditioned on investors’ information. The scale of this amplification effect is given by the term in brackets in Eq. (14). Moreover, the last term in Eq. (14), which results from the assumption that information is dispersed, further magnifies variations observed by the econometrician.⁴

We now measure the *equilibrium* distortion in the econometrician’s betas (relative to the average agent) based on endogenous excess returns, as opposed to exogenous payoffs.

Proposition 2. *In equilibrium, the vector β measured by the average agent and the vector $\tilde{\beta}$ measured by the econometrician, both net of their average—which is the beta on the market portfolio and equals one—are proportional:*

$$\tilde{\beta} - \mathbb{1} = (\beta - \mathbb{1})(1 + \delta). \quad (15)$$

Furthermore, if the informational advantage of agents, $\Delta_\tau \equiv \tau - \tilde{\tau}$, satisfies

$$\Delta_\tau > \frac{\gamma^2 \tilde{\tau}}{\tau_X \tau_\epsilon}, \quad (16)$$

then $\delta > 0$, meaning that the econometrician overestimates betas that are higher than one and underestimates betas that are lower than one.

Proof. See Appendix A.3. □

Similar to Proposition 1, Proposition 2 implies that the the econometrician obtains betas that inflate around their average. Using the endogenous excess returns implied by the model, we can now study the consequences of this SML distortion in equilibrium. We illustrate this effect in Panel (b) of Figure 1, where we consider a simple model with three assets (all

⁴A customary result in models with dispersed information is that the law of iterated expectations fails (Allen et al., 2006; Bacchetta and Wincoop, 2008). In this example and throughout the paper, the law of iterated expectations fails from the viewpoint of the econometrician, in that the variance of individual expectations differs from the variance of first-order expectations, introducing an additional “wedge” in the law of total covariance with respect to the average agent.

parameters are provided in the description of the figure). Because betas higher than one are over-estimated and betas lower than one are under-estimated, the econometrician’s SML rotates clockwise around the market portfolio and becomes significantly flatter than the actual SML (which results from the market clearing condition (13)). The econometrician thus concludes that the CAPM fails, although a true CAPM relation genuinely holds for the average agent in equilibrium.

In this example, it may seem that the “econometrician” is overly unsophisticated, since her information set contains nothing but prices. As it turns out, allowing the econometrician to condition her estimates on public information does not change the result. Specifically, in Appendix A.2 we show that equilibrium excess returns take the following linear form:

$$Q = D - P = \Phi \frac{\tau_F}{\tau} F - \Phi \frac{\tau_G}{\tau} v + \epsilon - \xi X, \quad (17)$$

from which it follows that

$$\text{Cov}[Q, G] = \mathbf{0}_{N \times 1}. \quad (18)$$

Thus, allowing the econometrician to observe the public signal cannot possibly improve her estimates of the covariance matrix of returns—prices already incorporate all public information available in the economy.

Although public announcements do not help the econometrician improve her estimates in a static model, they do in a dynamic model. Intuitively, because periods with and without public announcements alternate in a dynamic model, the price coefficients in Eq. (17) differ whether a scheduled announcement is imminent or whether it takes place in a distant future. In this context, the econometrician can benefit from these changes in price coefficients by estimating a CAPM relation separately on announcement and non-announcement days (Savor and Wilson, 2014). Making this point requires a dynamic framework, which is the purpose of the next section.

3 Model

We now introduce periodic public announcements and a factor structure within a standard, dynamic stationary rational-expectations equilibrium model with multiple assets (e.g. Spiegel, 1998; Watanabe, 2008).

3.1 The Economy

We consider a discrete-time economy that goes on forever. The economy is populated with a continuum of investors indexed by $i \in [0, 1]$, who have CARA utility with common risk aversion γ . Each investor i lives for two periods, entering period t with wealth W_t^i and consuming W_{t+1}^i next period. There are N risky assets (stocks) and an exogenous riskless bond with constant gross interest rate $R > 1$. At each period t the N stocks pay a vector D of dividends with a *common factor structure*:

$$D_t = \bar{D}\mathbb{1}_N + \Phi F_t + \epsilon_t^D, \quad (19)$$

where $\mathbb{1}_N$ a vector with all N components being 1, $\epsilon_t^D \sim N(\mathbf{0}_{N \times 1}, \sigma_D^2 \mathbb{I}_N)$ is an i.i.d. asset-specific innovation, \mathbb{I}_N is the identity matrix of dimension N , and F denotes a factor that commonly affects all dividends on all stocks. The extent to which dividends of a given stock load on this factor is determined by the vector Φ of loadings. Without loss of generality and for ease of interpretation, we assume that the ordering of the vector of loadings is strictly ascending so that stock $n = 1$ has the smallest loading and stock $n = N$ has the highest loading. These loadings are common knowledge to all agents in the economy.⁵

The factor F_t is assumed to mean-revert over time around zero according to

$$F_t = \kappa_F F_{t-1} + \epsilon_t^F, \quad \epsilon_t^F \sim N(0, \sigma_F^2), \quad \text{i.i.d.}, \quad (20)$$

with $0 \leq \kappa_F \leq 1$. Furthermore, as is customary in the noisy rational expectations literature, the per-capita supply of stocks is stochastic and evolves according to:

$$X_t = (1 - \kappa_X)\bar{X} + \kappa_X X_{t-1} + \epsilon_t^X, \quad \epsilon_t^X \sim N(0_N, \sigma_X^2 \mathbb{I}_N), \quad \text{i.i.d.}, \quad (21)$$

where \bar{X} is a vector of dimension N with equal elements that sum up to one. Supply shocks prevent prices from fully revealing investors' private information.

We assume that there are overlapping generations of investors who live for two periods. Investors maximize their expected utility over their next period wealth by optimally building a portfolio with the N risky assets and the riskless bond. Specifically, investor i chooses a vector x_t^i of holdings in the N stocks at time t to maximize

$$\max_{x_t^i} \mathbb{E}_t^i \left[-\exp(-\gamma W_{t+1}^i) \right] \quad (22)$$

⁵We consider a single factor for simplicity of exposition. The model can accommodate a vector of multiple factors F_t .

subject to

$$W_{t+1}^i = (W_t^i - (x_t^i)' P_t) R + (x_t^i)' (P_{t+1} + D_{t+1}), \quad (23)$$

where P_t denotes the vector of ex-dividend prices. We follow the literature and focus on linear equilibria in which the price P is a linear function of the state variables of the economy.⁶ Because these state variables are Gaussian and because investors are myopic and have CARA utility, their demands take the following standard form:

$$x_t^i = \frac{1}{\gamma} (\mathbb{V}_t^i [P_{t+1} + D_{t+1}])^{-1} \mathbb{E}_t^i [P_{t+1} + D_{t+1} - R P_t]. \quad (24)$$

A linear equilibrium then results from imposing the market-clearing condition

$$\int_{i \in [0,1]} x_t^i di = X_t. \quad (25)$$

3.2 Periodic Public Announcements

We introduce periodic public announcements in the model by assuming that every T periods a new public signal centered on the fundamental F is available to all investors, according to the sequence illustrated in Figure 2.

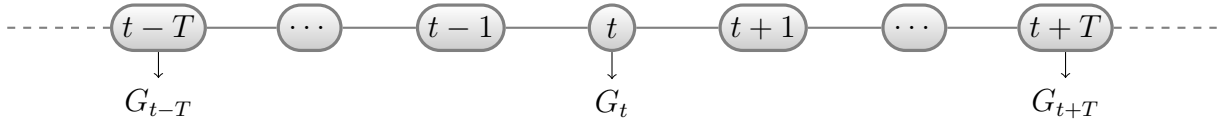


Figure 2: Timeline of periodic public announcements.

Without loss of generality, we adopt the convention that a public announcement is made at time t . Then for any date $t - k$, with $k \in \{-T + 1, \dots, T\}$ the public announcements available to investors take the form:

$$G_{t-k} = \begin{cases} F_{t-T} + \epsilon_{t-T}^G, & \forall k \in \{1, \dots, T\} \\ F_t + \epsilon_t^G, & \forall k \in \{-T + 1, \dots, 0\}, \end{cases} \quad (26)$$

with $\epsilon_{t-T}^G, \epsilon_t^G \sim N(0, \sigma_G^2)$ and independent of each other.

⁶With N assets, there are 2^N linear equilibria in this model. We focus on the low-volatility equilibrium, as this is the only stable equilibrium to which a finite horizon economy would converge. [Bacchetta and Wincoop \(2008\)](#), [Banerjee \(2010\)](#), and [Watanabe \(2008\)](#) discuss the multiplicity of equilibria in infinite horizon models.

In addition to public announcements, we assume that all investors commonly observe the fundamental at lag T and beyond (Townsend, 1983; Singleton, 1987). We make this assumption for tractability—without it the information structure would introduce an infinite-regress inference problem whereby investors would need to infer unobservables shocks over infinitely many periods back in time. This assumption serves to eliminate this infinite-regress problem: at any time s , the fundamental F_{s-T} becomes public information and thus investors only need to infer unobservable shocks up to lag $T - 1$.

Investors observe private information at each trading date. Formally, at any date s , each investor i receives a private signal v^i about the current factor innovation:

$$v_s^i = \epsilon_s^F + \epsilon_s^i, \quad \epsilon_s^i \sim N(0, \sigma_v^2), \quad \epsilon_s^i \perp \epsilon_s^k, \quad \forall k \neq i. \quad (27)$$

Finally, as is customary in the literature, all investors observe current and past dividends, along with current and past prices. Specifically, we conjecture that prices at time $t - k$, with $k \in \{0, \dots, T - 1\}$, take the following linear form:

$$P_{t-k} = \bar{\alpha}_k \bar{D} + \alpha_k F_{t-k-T} + \bar{\xi}_k \bar{X} + \xi_k X_{t-k-T} + d_k \tilde{D}_{t-k} + g_k G_{t-k} + a_k \tilde{\epsilon}_{t-k}^F + b_k \tilde{\epsilon}_{t-k}^X, \quad (28)$$

where we use the following notation⁷:

- \tilde{D}_{t-k} is a stacked vector of dimension NT which includes the payoffs of all N assets from time $t - k - T + 1$ to time $t - k$.
- $\tilde{\epsilon}_{t-k}^F$ is a vector of dimension T containing all the fundamental shocks ϵ^F from time $t - k - T + 1$ to time $t - k$.
- $\tilde{\epsilon}_{t-k}^X$ is a stacked vector of dimension NT containing the supply shocks for all assets from time $t - k - T + 1$ to time $t - k$.
- The price coefficients $\bar{\alpha}_k$, α_k , $\bar{\xi}_k$, ξ_k , d_k , g_k , a_k and b_k are scalars/vectors/matrices of conformable dimension.

To understand the price structure (28), notice that any investor i observes at date $t - k$, $k \in \{0, \dots, T - 1\}$:

$$\{ \{P_s\}_{s \leq t-k}, \{D_s\}_{s \leq t-k}, \{F_s\}_{s \leq t-k-T}, \{v_s^i\}_{s \leq t-k} \}. \quad (29)$$

It follows that an investor i also observes $\{\epsilon_s^F\}_{s \leq t-k-T}$, hence the vector of innovations $\tilde{\epsilon}_{t-k}^F$ in the common factor. Furthermore, in this dynamic setup the sequence of past

⁷A detailed representation of the linear form (28) can be found in equation (97) of Appendix A.4.

dividends $\{D_{t-k-s}\}_{s=0}^{T-1}$ reveals information regarding past, unobservable factor innovations $\{\epsilon_{t-k-s}^F\}_{s=0}^{T-1}$, hence the vector \tilde{D}_{t-k} . Finally, to understand the structure of the vector of past supply innovations, consider the price at time $t - k - T$. This price reveals a linear combination of $\{\epsilon_{t-k-s}^F\}_{s=0}^{T-1}$ and X_{t-k-T} . Since these factor innovations are observable, so is X_{t-k-T} , hence the vector $\tilde{\epsilon}_{t-k}^X$ and the presence of F_{t-k-T} and X_{t-k-T} in the equilibrium price.

4 Equilibrium

In this section we solve for an equilibrium of the model. We start by solving investors' learning problem (Section 4.1) and then clear the market to obtain the undetermined price coefficients (Section 4.2).

4.1 Learning

When building her portfolio position, an investor i must form expectations about future excess returns next period, as apparent from equation (24). For the purpose of forecasting future excess returns, it is sufficient to form expectations about the vector $\tilde{\epsilon}^F$ of innovations in the common factor. We thus first compute an investor i ' conditional expectations $\mathbb{E}_t^i[\tilde{\epsilon}_t^F]$ and conditional variance $\mathbb{V}_t^i[\tilde{\epsilon}_t^F]$ at each trading date during the periodic announcement cycle.

At time $t - k$, with $k \in \{0, \dots, T - 1\}$, an investor i 's information set contains four sources of information regarding the vector $\tilde{\epsilon}_{t-k}^F$: past and current prices, private signals, dividends, and public announcements. Starting with past and current prices, notice from equation (28) that they reveal a combination of observable and unobservable information; accordingly, an investor i can isolate the informational part of prices P_{t-k}^a that only contains unobservables:

$$P_{t-k}^a = a_k \tilde{\epsilon}_{t-k}^F + b_k \tilde{\epsilon}_{t-k}^X, \quad k \in \{0, \dots, T - 1\}, \quad (30)$$

which we stack into a single vector \tilde{P}_{t-k}^a of dimension NT :

$$\tilde{P}_{t-k}^a \equiv \left(P_{t-k}^a \quad P_{t-k-1}^a \quad \cdots \quad P_{t-k-T+2}^a \quad P_{t-k-T+1}^a \right)'. \quad (31)$$

Similarly, we collect all past and current private signals that investor i has gathered by time $t - k$ into a vector V_{t-k}^i of dimension T :

$$V_{t-k}^i \equiv \left(v_{t-k}^i \quad v_{t-k-1}^i \quad \cdots \quad v_{t-k-T+1}^i \right)'. \quad (32)$$

Furthermore, because we assume that the common factor F becomes public information after T periods, dividends and public announcements also reveal a combination of observable and unobservable information. In particular, the dividend at time $t - k$ can be written as

$$D_{t-k} = \bar{D}\mathbb{1}_N + \Phi \left(\kappa_F^T F_{t-k-T} + \sum_{\tau=0}^{T-1} \kappa_F^\tau \epsilon_{t-k-\tau}^F \right) + \epsilon_{t-k}^D \quad (33)$$

$$= \bar{D}\mathbb{1}_N + \Phi \kappa_F^T F_{t-k-T} + D_{t-k}^a, \quad (34)$$

where D_{t-k}^a represents the informational part of dividends that only contains unobservables, which we stack into a single vector \tilde{D}_{t-k}^a of dimension NT :

$$\tilde{D}_{t-k}^a \equiv \left(D_{t-k-T+1}^a \quad D_{t-k-T+2}^a \quad \cdots \quad D_{t-k}^a \right)'. \quad (35)$$

Public announcements G_{t-k} have a similar structure:

$$G_{t-k} = \kappa_F^{k+j(k)} F_{t-k-T} + \sum_{\tau=0}^{k+j(k)-1} \kappa_F^\tau \epsilon_{t+j(k)-T-\tau}^F + \epsilon_{t+j(k)-T}^G \quad (36)$$

$$= \kappa_F^{k+j(k)} F_{t-k-T} + G_{t-k}^a \quad (37)$$

where G_{t-k}^a represents the part of public signals that only contains unobservables and the indexing function $j(k)$ equals T if $k = 0$ and 0 otherwise.

Overall, an investor i 's information at time $t - k$ is fully summarized by \tilde{P}_{t-k}^a , V_{t-k}^i , \tilde{D}_{t-k}^a , and G_{t-k}^a . Since the vector grouping this information and the vector $\tilde{\epsilon}_{t-k}^F$ are jointly normally distributed, investor i forms her conditional expectations $\mathbb{E}_t^i[\tilde{\epsilon}_t^F]$ and conditional variance $\mathbb{V}_t^i[\tilde{\epsilon}_t^F]$ by projecting the former vector on the latter. We write the conditional expectation and conditional variance in Proposition 3.

Proposition 3. *At time $t - k$ an investor i 's conditional expectation and variance of the fundamental innovations $\tilde{\epsilon}_{t-k}^F$ satisfy*

$$\mathbb{E}_{t-k}^i [\tilde{\epsilon}_{t-k}^F] = \underbrace{\sigma_F^2 H_k' (\sigma_F^2 H_k H_k' + R_k)^{-1}}_{\equiv M_k} \begin{pmatrix} \tilde{P}_{t-k}^a \\ V_{t-k}^i \\ \tilde{D}_{t-k}^a \\ G_{t-k}^a \end{pmatrix} \quad (38)$$

and

$$\mathbb{V}_{t-k}^i [\tilde{\epsilon}_{t-k}^F] = \sigma_F^2 (\mathbb{I}_T - M_k H_k) \quad (39)$$

where the matrices H_k and R_k are defined in Appendix A.5.

Proof. See Appendix A.5. □

Equation (38) shows how investors form expectations about the current and past innovations in the common factor F . When forecasting fundamental innovations, investors use all the public information available (including current and past prices, current and past dividends, and the periodic public signal), but also their private information gathered up to time $t - k$. The posterior variance of $\tilde{\epsilon}_{t-k}^F$ equals $\sigma_F^2 \mathbb{I}_T$ when investors have no information, but changes as soon as investors learn from private and public signals, as shown in equation (39).

Based on the conditional expectations of Proposition 3, investor i can forecast future excess returns next period, $P_{t-k+1} + D_{t-k+1}$. The conditional expectation and the conditional variance of future excess returns, $\mathbb{E}_{t-k}^i [P_{t-k+1} + D_{t-k+1}]$ and $\mathbb{V}_{t-k}^i [P_{t-k+1} + D_{t-k+1}]$, completely determine investor i 's portfolio strategy, as described in equation (24). We relegate these expressions to Appendix A.5, and now aggregate investment strategies over the population of investors to clear the market and obtain equilibrium prices.

4.2 Equilibrium Pricing

We first rewrite individual demands in (24) at each lag k in the public announcement cycle:

$$x_{t-k}^i = \frac{1}{\gamma} \mathbb{V}_{t-k}^i [P_{t-k+1} + D_{t-k+1}]^{-1} (\mathbb{E}_{t-k}^i [P_{t-k+1} + D_{t-k+1}] - RP_{t-k}), \quad (40)$$

and define average market expectations at time $t - k$ as

$$\bar{\mathbb{E}}_{t-k} [P_{t-k+1} + D_{t-k+1} - RP_{t-k}] \equiv \int_{i \in [0,1]} \mathbb{E}_{t-k}^i [P_{t-k+1} + D_{t-k+1} - RP_{t-k}] di. \quad (41)$$

Furthermore, the precision of information (public and private) is the same across investors and thus we can write

$$\mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] \equiv \mathbb{V}_{t-k}^i [P_{t-k+1} + D_{t-k+1} - RP_{t-k}]. \quad (42)$$

The pair $(\bar{\mathbb{E}}, \mathbb{V})$ can be interpreted as the beliefs of a fictitious agent—the average investor—who determines the average market consensus in the economy. Using the average beliefs from (41) and (42), the market-clearing condition yields:

$$\bar{\mathbb{E}}_{t-k} [P_{t-k+1} + D_{t-k+1} - RP_{t-k}] = \gamma \mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] X_{t-k}, \quad (43)$$

which defines the conditional pricing relation that prevails in the model at each date during the public announcement cycle. Stock prices thus satisfy

$$P_{t-k} = \frac{1}{R} \bar{\mathbb{E}}_{t-k} [P_{t-k+1} + D_{t-k+1}] - \frac{\gamma}{R} \mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] X_{t-k}. \quad (44)$$

Stock prices equal the present value of the future prices and payoffs, discounted at the risk free rate, minus a risk premium term which is proportional to the supply of assets. Clearly, investors demand a higher risk premium to hold assets that are in large supply, and the magnitude of this risk premium term is dictated by the risk aversion coefficient γ .

We conclude the equilibrium derivation by verifying that the pricing relation in equation (43) is consistent with the price conjecture in equation (28).

Proposition 4. *In equilibrium prices take the linear form in equation (28) where the pricing relation in equation (43) implies that the coefficients $\bar{\alpha}$, α , $\bar{\xi}$, ξ , d , g , a and b solve a nonlinear system of equation that we provide in Appendix A.6.*

Proof. See Appendix A.6. □

5 Results

5.1 Distortion of the Security Market Line

In this section we adopt the approach of an empiricist and estimate a security market line in the dynamic equilibrium model of the previous section. Defining excess returns as

$$Q_{t+1} \equiv P_{t+1} + D_{t+1} - RP_t. \quad (45)$$

and following the same reasoning as in Section 2, we start by writing the law of total covariance:

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}[\mathbb{V}_t[Q_{t+1}]] + \mathbb{V}[\bar{\mathbb{E}}_t[Q_{t+1}]] + \sigma_v^2 \Pi \Pi', \quad (46)$$

where Π is a matrix of conformable dimensions that multiplies the vector of private signals in the individual expectation of each agent i , $\mathbb{E}_t^i[Q_{t+1}]$.

We briefly repeat here some of the arguments we made in the context of Section 2. Eq. (46) shows that three terms, none of which the econometrician can directly measure, account for the observed covariation in excess returns: the average agent's conditional covariance matrix, the variance of expected excess returns, and the adjustment associated with

the aggregation of private information. The market-clearing condition (Eq. 43) imposes a relation between the conditional expectation and the conditional variance of excess returns. This relation can be replaced in Eq. (46), which yields

$$\mathbb{V}[Q_{t+1}] = \mathbb{E}[\mathbb{V}_t[Q_{t+1}]] + \gamma^2 \mathbb{V}[\mathbb{V}_t[Q_{t+1}]X_t] + \sigma_v^2 \mathbb{I}\mathbb{I}' \quad (47)$$

This relation has several implications for the econometrician's estimate of the covariance matrix. First, the market portfolio X_t is unobservable both to the econometrician and investors—the Roll critique—generating additional variations through the second term on the right-hand side. Second, investors' private information further increases the total variation in excess returns measured by the econometrician, the last term on the right-hand side. Finally, as explained in Section 4, in a dynamic environment the conditional covariance matrix of the average agent, $\mathbb{V}_t[Q_{t+1}]$, changes over the announcement cycle (although the model is stationary, it varies with each lag k within a typical announcement cycle), adding to the total variation measured by the econometrician.

To understand the impact of these effects on the SML, notice first that the equilibrium relation (43) implies that the market portfolio X_t is mean-variance efficient for the “average investor,” a fictitious agent with beliefs $(\bar{\mathbb{E}}_t[Q_{t+1}], \mathbb{V}_t[Q_{t+1}])$ and risk aversion γ . After taking the unconditional expectation of the pricing relation in equation (43) and performing a few standard manipulations, we obtain an *unconditional* CAPM relation from the perspective of an average investor who holds the *average market portfolio*, \bar{X} :

$$\mathbb{E}[Q_{t+1}] = \frac{\bar{\mathbb{V}}[Q_{t+1}] \bar{X}}{\bar{\mathbb{V}}[M_{t+1}]} \mathbb{E}[M_{t+1}], \quad (48)$$

where $M_{t+1} \equiv \bar{X}'Q_{t+1}$ is the realized return of the market portfolio and $\bar{\mathbb{V}}[\cdot]$ represents the unconditional average over all the $\mathbb{V}_{t-k}[\cdot]$ for all lags k in the announcement cycle. The ratio

$$\beta \equiv \frac{\bar{\mathbb{V}}[Q_{t+1}] \bar{X}}{\bar{\mathbb{V}}[M_{t+1}]} \quad (49)$$

represents the vector of betas as measured by the average investor in this economy. It then follows from equation (48) that the CAPM holds unconditionally for the average investor, who observes a security market line that crosses the origin with a slope given by the market risk premium.

The econometrician, instead, can only measure the unconditional variance of returns and

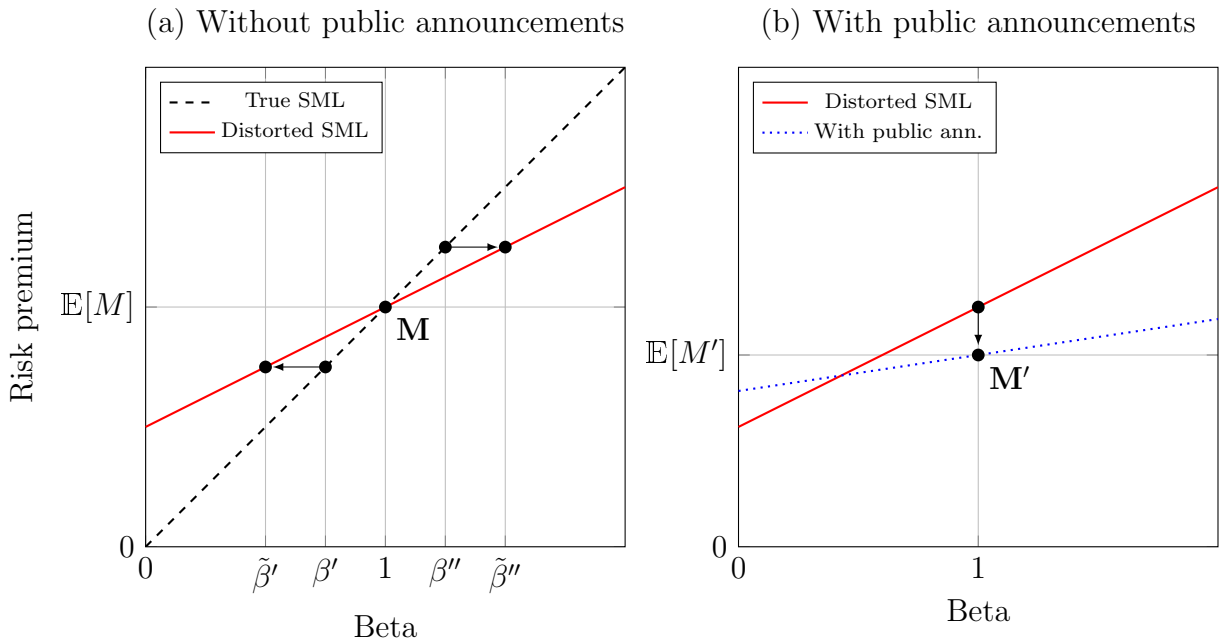


Figure 3: Distortion of the Security Market Line. The left panel illustrates the true SML (dashed black line) and the SML perceived by the econometrician (solid red line) in an economy without public announcements. The econometrician overestimates betas higher than one and underestimates betas lower than one. The right panel depicts the additional distortion of the SML induced by a periodic public announcement. While the overall unconditional market excess return is lower with public announcements, fluctuations in the conditional variance of asset returns further flattens the SML (dotted blue line).

thus obtains the following vector of betas:

$$\tilde{\beta} = \frac{\mathbb{V}[Q_{t+1}] \bar{X}}{\mathbb{V}[M_{t+1}]}, \quad (50)$$

which, in light of our discussion of the law of total covariance in Eq. (46), differs from the vector of true betas in equation (49). The effect of this difference on the security market line is illustrated in Figure 3, where the left panel describes an economy without public announcements. The dashed line is the true CAPM relation (48) that holds for the average agent. A direct implication of Proposition 2, which remains valid in this dynamic setup, is that the SML that the econometrician perceives is flatter than the actual SML—it rotates clockwise around the market portfolio, denoted by the point $\mathbf{M} = (1, \mathbb{E}[M])$ on the graph.

The right panel of Figure 3 is specific to the dynamic setup and shows the additional effect on the SML implied by changes in the conditional covariance matrix $\mathbb{V}_t[Q_{t+1}]$ over the announcement cycle. The red solid line represents the “distorted” Security Market Line in an economy without public announcements, whereas the blue dotted line shows the additional distortion when public announcements are introduced. In an economy with

a periodic public announcement, more informed agents require a smaller risk premium to hold the assets, which lowers the *unconditional* risk premium in the economy. The market portfolio therefore moves down towards the point $\mathbf{M}' = (1, \mathbb{E}[M'])$ on the graph. However, periodic public announcements generate time variation in investors' conditional volatility of future excess returns. This additional variation, which materializes in the second term on the right hand side of Eq. (47), further distorts the SML measured by the econometrician, resulting in additional flattening.

5.2 Announcement Day Effects

We now focus on the behaviour of the CAPM relationship during announcement cycles. From now onwards the analysis involves solving our theoretical model. We choose to show results in a model where we fix the number of assets at $N = 4$ and we assume that a public announcement arises every 4 periods.⁸ This will cause the SML to move depending on whether the econometrician measures it during announcement days or during non-announcement days.

Figure 4 illustrates two effects. First, the left panel shows the unconditional CAPM during all days. The dashed line represents the true SML, whereas the solid line is distorted, in line with our analysis from the previous Section. The SML flattens and has an intercept greater than zero—in other words, the SML is pivoting clockwise around the point $(1, \mathbb{E}[M_{t+1}])$, which leads the econometrician to conclude that the CAPM does not hold. The panel also shows the true betas (four dots on the dashed line) and econometrician's betas (four triangles on the solid line). As explained in Section 5.1, the econometrician underestimates betas lower than one and overestimates betas higher than one, and thus low-beta assets appear to deliver high average returns and low risk relative to high-beta assets.

The second effect is illustrated in the right panel and shows the SML estimated by the econometrician during three types of days: all days (solid blue line—this is the same line as the solid line in the left panel), announcement days (dashed line), and non-announcement days (dotted line). We follow the methodology in [Savor and Wilson \(2014\)](#) and estimate the betas over the whole sample. The three lines pivot around a common point with positive beta and positive expected returns. This point corresponds to a portfolio whose exposure to the factor F is $\phi = 0$ and thus is not affected by any macroeconomic information. As the plot shows, on announcement days the SML slope steepens and as a result the econometrician observes a stronger CAPM relationship ([Savor and Wilson, 2014](#)).

The steepening of the CAPM during *A-days* can be understood as follows. Due to uncertainty about the public announcement, agents require a high risk premium to hold the

⁸Other parameters are: $\gamma = 2$, $\kappa_F = \kappa_X = 0.99$, $\phi_1 = 1$, $\phi_2 = 1/2$, $\phi_3 = 1/3$, $\phi_4 = 1/4$, $\sigma_F = \sigma_X = 0.06$, $\sigma_D = 0.32$, $\sigma_G = 0.001$, $\bar{X} = 1$, $\bar{D} = 0$, and $R = 1.22$.

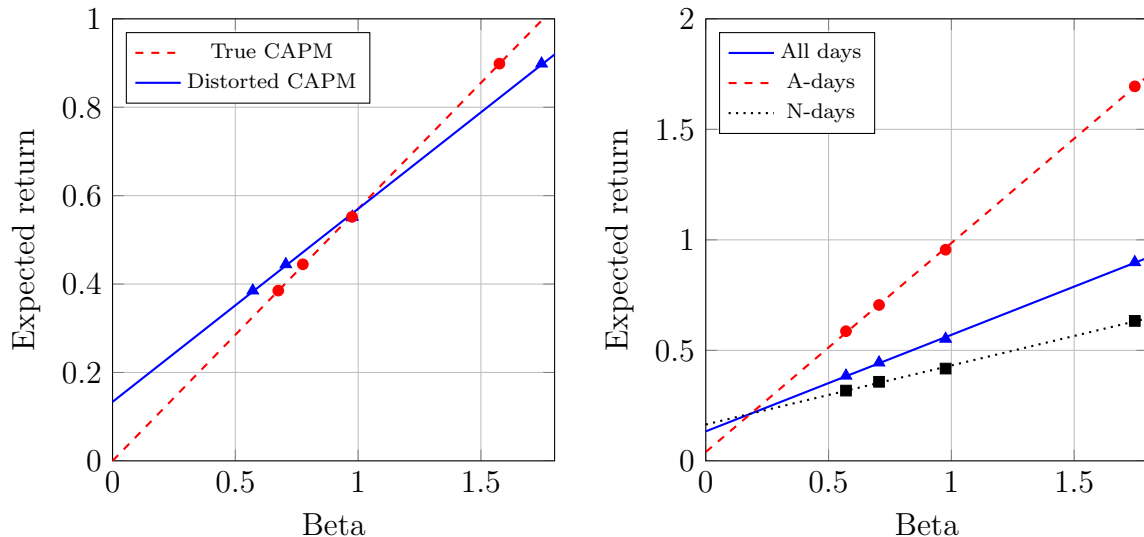


Figure 4: CAPM during announcement and non-announcement days. The left panel shows the SML distortion explained in Section 5.1, in line with Figure 4. The right panel shows the SML during all days (solid), announcement days (dashed), and non-announcement days (dotted).

	<i>All days</i>	<i>A-days</i>	<i>N-days</i>
Average market excess return	0.57	0.98	0.43
Market volatility	0.39	0.42	0.26
Sharpe ratio	1.46	2.36	1.64

Table 1: Average market excess returns, volatility and Sharpe ratio on three different days.

asset at the beginning of the announcement day. Thus, both the market expected return and realized return are high during *a-days*. Holding betas of assets constant, this makes all expected returns higher and thus the overall SML appears steeper (and closer to the true SML).⁹ On the contrary, on *n-days*, the agents face less uncertainty about upcoming announcements and thus all assets' expected returns are lower, which further flattens the SML (dotted line in the left panel of Figure 4) and increases its intercept.

Table 1 further confirms this. The first line shows the average market excess returns during the three types of days. As expected, average excess returns are higher during *A-days*. Although the market volatility follows a similar pattern and increases during announcement days, it does so less than the average market excess returns. This results in much higher Sharpe ratios during announcement days, as documented by Savor and Wilson (2016).

⁹This result does not depend on holding the betas constant. In separate calculations, we estimate different betas on different types of days. Although there is variation in betas across types of days, the same steepening/flattening obtains during *A-days/N-days*. See Savor and Wilson (2014) for a discussion of variation of betas across types of days.

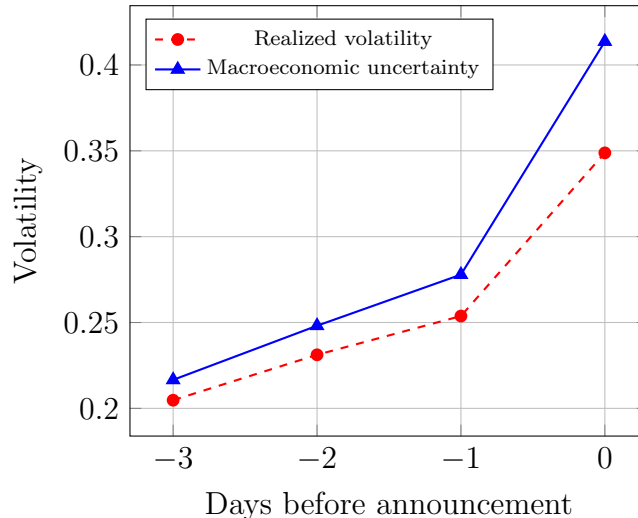


Figure 5: Market volatility during the announcement cycle. Evolution of realized volatility of excess returns (solid line) and macroeconomic uncertainty (dashed line) over public announcement cycles. Date 0 represent the date of the announcement.

Figure 5 shows the evolution of the realized return volatility over the announcement cycle. Dates on the right axis represent trading days before the announcement, which takes place at date 0. Realized volatility is highest during the announcement day, but also increases before the announcement. This increase in volatility before the actual public announcement is generated by changes in investor’s expectations about the future value of stocks (He and Wang, 1995). That is, although at time $t - 2$ investors do not expect a public announcement next period, they know that prices at $t - 1$ will change in order to anticipate the upcoming public announcement. This expectation channel changes prices at $t - 2$ and thus volatility increases before the public announcement.

Figure 5 also shows the evolution of the macroeconomic uncertainty over the announcement cycle, which follows the same pattern with stock market volatility. We define macroeconomic uncertainty as the uncertainty perceived by agents when trying to forecast market returns (note that this variable is not observed by the econometrician), $\sqrt{\mathbb{V}_t[M_{t+1}]}$.

6 Additional Results

6.1 Multiple Factors

In this section we investigate how the presence of multiple factors may affect our results. The main implication is that, in contrast to the one-factor case, expected returns and econometrician’s betas do not plot on a straight line—the econometrician now faces a “broken” SML, although the CAPM holds for the average agent. However, our result that high betas

are inflated and low betas are deflated in the eyes of the econometrician remains valid.

To show how the presence of multiple factors alters the linearity of the econometrician's perceived SML, we consider the simplest example, which builds on the static setting of Section 2 and illustrates this point. Suppose that there are 2 factors and 3 assets and that factor loadings satisfy:¹⁰

$$\begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} 1 + \phi_1 & 0 \\ 1 - \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix} \underbrace{\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}}_{\mathbf{F}} + \epsilon, \quad (51)$$

with $\phi_1 > 0$ and $\phi_2 \geq 0$. With respect to the setting of Section 2, the above structure now accommodates a second factor, F_2 . We examine the effect of this additional factor on the SML when $\phi_2 > 0$.

The above structure implies that each stock depends on one independent factor or none, and thus factors are independent under each agent i 's information set:

$$\tau \equiv \mathbb{V}_i[\mathbf{F}]^{-1} = \begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix}, \quad (52)$$

where the factor precisions τ have closed-form solutions that we relegate to Appendix A.8.

We now want to construct the SML perceived by the econometrician. To do so we proceed as in previous sections and write the covariance matrix of the average agent as

$$\bar{\mathbb{V}}[Q] = \begin{pmatrix} \frac{(1+\phi_1^2)}{\tau_1} + \frac{1}{\tau_\epsilon} & \frac{(1-\phi_1)^2}{\tau_1} & 0 \\ \frac{(1-\phi_1)^2}{\tau_1} & \frac{(-1+\phi_1^2)}{\tau_1} + \frac{1}{\tau_\epsilon} & 0 \\ 0 & 0 & \frac{\phi_2^2}{\tau_2} + \frac{1}{\tau_\epsilon} \end{pmatrix}, \quad (53)$$

then we recover the covariance matrix of the econometrician from the law of total covariance:

$$\mathbb{V}[Q] = \bar{\mathbb{V}}[Q] + \frac{\gamma^2}{\tau_X^2} \bar{\mathbb{V}}[Q] \bar{\mathbb{V}}[Q] + \tau_v \Phi \tau^{-1} \tau^{-1} \Phi'. \quad (54)$$

From the previous sections we also know that, in the eyes of the average agent, the SML plots on a straight line that crosses the origin and has a slope equal to the market risk premium. In particular, using the matrix $\bar{\mathbb{V}}[Q]$ we can write risk premia as

$$\mathbb{E}[Q] = \gamma \bar{\mathbb{V}}[M] \frac{\bar{\mathbb{V}}[Q] \bar{X}_t}{\bar{\mathbb{V}}[M]} \equiv \gamma \bar{\mathbb{V}}[M] \beta, \quad (55)$$

¹⁰In Appendix A.8 we solve a more general case with N assets and K factors.

where M represents the return of an equally weighted market portfolio with weights $\bar{X} = 1/3$, and where β denotes the vector of betas of the average agent.

We now want to measure the distortion in econometrician’s betas, $\tilde{\beta} = \mathbb{V}[M]^{-1}\mathbb{V}[Q]\bar{X}\mathbb{1}$. To determine whether econometrician’s betas will plot on a straight line, we examine whether

$$\beta_e - \mathbb{1} \propto \beta_a - \mathbb{1}, \quad (56)$$

where we subtract a vector of ones on both sides since the SML rotates around the point $(1, E[M])$. Clearly, unless the proportionality relation in Eq. (56) is satisfied, the pair of expected excess returns and econometrician’s betas will not plot on a straight line. In Appendix A.8, we compute the ratio of the two betas element by element

$$\frac{\beta_e - 1}{\beta_a - 1} = \frac{\bar{\mathbb{V}}[M]}{\mathbb{V}[M]} \left(\delta \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \phi_2^2 \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} \right), \quad (57)$$

and show that $\delta > 1$ and that $\Delta_i, i = 1, 2, 3$ take distinct values as soon as $\phi_2 > 0$. We conclude that the proportionality relation (56) is preserved only when $\phi_2 = 0$, i.e, when there is only one factor. Furthermore, tedious computations show that

$$\frac{\bar{\mathbb{V}}[M]}{\mathbb{V}[M]} \delta > 1 \quad (58)$$

so that we recover the result of previous sections that betas inflate around their average of one. However, in general, the average agent’s betas and those of the econometrician are not proportional. As a result, the econometrician’s betas and excess returns will not plot a on straight line—the econometrician contemplates a broken SML. In other words, although the presence of multiple common factors breaks the linearity of the SML in the eyes of the econometrician, the broken SML still rotates clockwise around the market portfolio.

6.2 Diffusion of Private Information

A question arising at this point is whether the nature of information (public or private) matters for these results. In particular, if agents hold private information about the fundamental, it is natural to assume that this information might be diffused through social networks in-between announcements (Cieslak et al., 2015).

To address this question, we modify our setup and allow for information diffusion (Andrei and Cujean, 2016) with time-varying intensity. The details of the model are in Appendix A.7. We implement the following experiment. We assume that a public announcement is

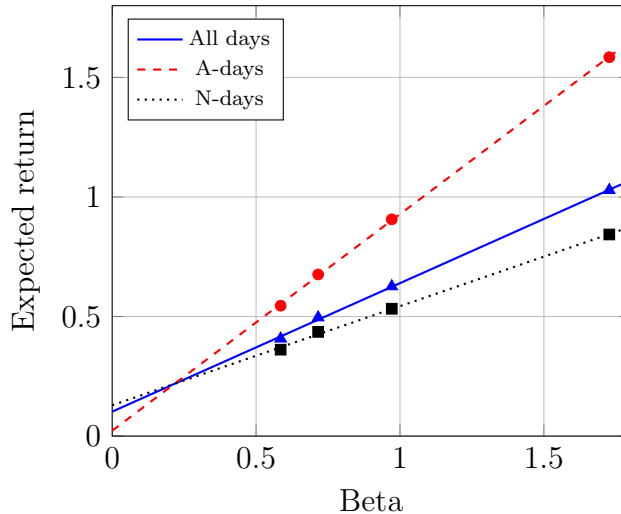


Figure 6: Information diffusion and the CAPM. CAPM during all days (solid), announcement days (dashed), and non-announcement days (dotted), when the public signal is not precise, but the information diffusion λ is intense during the announcement day only ($\lambda = 5$ on announcement days and $\lambda = 0$ otherwise).

scheduled as usual every $T = 4$ periods, but that the informativeness of that signal is very small (large σ_G). However, because announcements are scheduled events anticipated by market participants, it is natural to assume that a lot of private information is exchanged around announcement days. In the context of our model, this can be done by assuming that the meeting intensity is higher during announcement days than on all other days.

Figure 6 depicts the results and shows that similar pattern arises with private information. If investors exchange a lot of information about the fundamental *during* announcement days, then the SML gets steeper and the CAPM again appears stronger. The nature of information (public or private) is therefore not particularly important for our results, although the timing of information exchange is important: the diffusion of information has to take place during announcement days, and not before. In the latter case, this will generate a steepening of the CAPM before the announcement day. This result suggests that the private information transmission during the announcement cycle is weak.

Further questions to be addressed here pertain to the trading volume around macroeconomic announcements, but also to the trading strategies of investors based on their precision of information relative to the market average precision. More specifically, in the presence of information diffusion, investors end up with different amounts of information and thus trade differently not only because they hold different signals but also because their signals are more/less precise than others' signals. Consequently, some of the investors will find it optimal to engage in "beta-arbitrage" (buy low beta stocks and sell high-beta stocks) whereas other investors will find it optimal to take the opposite bet.

7 Conclusion

We build an economy with periodic public announcements in which two major critiques regarding standard tests of the CAPM apply—the market portfolio and investors’ information are unobservable. In this context, one of the main result is that the CAPM relationship looks flatter in the eyes of the econometrician, although it holds with respect to the average beliefs across investors. Furthermore, we show that public announcements can alleviate the above critiques and make the CAPM look stronger, as empirically documented by [Savor and Wilson \(2014\)](#).

Several aspects have yet to be investigated. First, with equally weighted stocks, the worst-case scenario for the econometrician is to find a perfectly flat security market line when the CAPM relation actually holds. However, heterogeneity in asset “size” might cause the security market line to “break” and even to slope downwards, as we observe in the data. Second, it would be interesting to relax the assumption of homogeneous variance of specific shocks across assets. This would open up possibilities for exploring the effect of idiosyncratic risk on stock returns. Third, [Berk and Van Binsbergen \(2016\)](#) note that much of the observed flows in and out of mutual funds remain unexplained. An extension of our model to allow for the formation of mutual funds ([Garcia and Vanden, 2009](#)) would offer the possibility to understand why we observe so much variation in flows. Finally, on the empirical side, our model suggests that measuring individual expected returns, as [Martin and Wagner \(2016\)](#) do, might help us get closer to the true variance-covariance matrix and thus to improve tests of the CAPM.

A Appendix

A.1 Proof of Proposition 1

Econometrician's covariance matrix is given by

$$\mathbb{V}[D] = \frac{1}{\tau_F} \Phi \Phi' + \frac{1}{\tau_\epsilon} \mathbb{I}_N, \quad (59)$$

and thus the following relationship holds between econometrician's matrix and the average agent's matrix

$$\frac{\tau}{\tau_F} \bar{\mathbb{V}}[D] = \mathbb{V}[D] + \frac{\tau - \tau_F}{\tau_F \tau_\epsilon} \mathbb{I}. \quad (60)$$

Starting from the definition of betas in Eq. (11), write

$$\beta \frac{\mathbb{1}' \bar{\mathbb{V}}[D] \mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}} \frac{\tau}{\tau_F} = N \frac{\bar{\mathbb{V}}[D] \mathbb{1}}{\mathbb{1}' \bar{\mathbb{V}}[D] \mathbb{1}} \frac{\mathbb{1}' \bar{\mathbb{V}}[D] \mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}} \frac{\tau}{\tau_F} = N \frac{\frac{\tau}{\tau_F} \bar{\mathbb{V}}[D] \mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}}, \quad (61)$$

and observe the $\frac{\tau}{\tau_F} \bar{\mathbb{V}}[Q]$ in the numerator. Use then Eq. (60) to obtain:

$$\beta(1 + \delta) = N \frac{\mathbb{V}[D] \mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}} + N \frac{\tau - \tau_F}{\tau_F \tau_\epsilon} \frac{\mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}} \quad (62)$$

$$= \tilde{\beta} + N \frac{\tau - \tau_F}{\tau_F \tau_\epsilon} \frac{\mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}}. \quad (63)$$

where we have defined

$$1 + \delta \equiv \frac{\mathbb{1}' \bar{\mathbb{V}}[D] \mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}} \frac{\tau}{\tau_F}. \quad (64)$$

Multiply the relation above with $\mathbb{1}'$ and then divide by N :

$$1 + \delta = 1 + N \frac{\tau - \tau_F}{\tau_F \tau_\epsilon} \frac{\mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}}, \quad (65)$$

where we have used the result that the average of betas is equal to one (for both the econometrician and the average agent). It thus follows that:

$$\delta = N \frac{\tau - \tau_F}{\tau_F \tau_\epsilon} \frac{\mathbb{1}}{\mathbb{1}' \mathbb{V}[D] \mathbb{1}} = \frac{\Delta_\tau}{\tau_F + \frac{\tau_\epsilon}{N} \left(\sum_{i=1}^N \phi_i \right)^2}, \quad (66)$$

with $\Delta_\tau \equiv \tau - \tau_F \geq 0$. We can then write:

$$\beta(1 + \delta) = \tilde{\beta} + \delta \mathbb{1}, \quad (67)$$

and subtract $(1 + \delta) \mathbb{1}$ on both sides:

$$(\beta - \mathbb{1})(1 + \delta) = \tilde{\beta} - \mathbb{1}, \quad (68)$$

which is the result of Proposition 1. \square

A.2 Equilibrium in a static setting

As is customary in the literature, we start by conjecturing the following form for prices:

$$\underbrace{\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix}}_P = \underbrace{\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix}}_\alpha F + \underbrace{\begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_N \end{bmatrix}}_g v + \underbrace{\begin{bmatrix} \xi_{11} & \xi_{12} & \cdots & \xi_{1N} \\ \xi_{21} & \xi_{22} & \cdots & \xi_{2N} \\ \vdots & \vdots & \ddots & \\ \xi_{N1} & \xi_{N2} & \cdots & \xi_{NN} \end{bmatrix}}_\xi \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}}_X, \quad (69)$$

where the undetermined coefficients multiplying the random variables F , v , and X_j will be pinned down by the market clearing condition.

Any investor i has three sources of information: (i) one private signal V_i , (ii) one public signal G , and (iii) N public prices. We denote the total precision of these sources of information by τ (an endogenous quantity to be determined in equilibrium):

$$\tau = \tau_F + \tau_v + \tau_G + \sum_{j=1}^N \tau_{P_j}, \quad (70)$$

with τ_{P_j} , $j = 1, \dots, N$ denoting the (endogenous) precisions of prices as public signals. Denote by P_j^a , $j = 1, \dots, N$, the “normalized adjusted” prices from which the public signal G has been subtracted:

$$P_j^a \equiv \frac{P_j - g_j G}{\alpha_j - g_j} = F + \begin{bmatrix} \frac{\xi_{j1}}{\alpha_j - g_j} & \frac{\xi_{j2}}{\alpha_j - g_j} & \cdots & \frac{\xi_{jN}}{\alpha_j - g_j} \end{bmatrix} X. \quad (71)$$

Learning then implies:

$$\mathbb{E}_i[F] = \frac{\tau_v}{\tau} V_i + \frac{\tau_G}{\tau} G + \sum_{j=1}^N \frac{\tau_{P_j}}{\tau} P_j^a \quad (72)$$

Notice that we can use the definition of the total precision (70) and the fact that all signals are normalized to re-write Eq. (72) as follows:

$$\mathbb{E}_i[F] = \frac{\tau - \tau_F}{\tau} F + \frac{\tau_v}{\tau} v_i + \frac{\tau_G}{\tau} v + \frac{1}{\tau} \underbrace{\left[\sum_{j=1}^N \frac{\tau_{P_j} \xi_{j1}}{\alpha_j - g_j} \quad \sum_{j=1}^N \frac{\tau_{P_j} \xi_{j2}}{\alpha_j - g_j} \quad \cdots \quad \sum_{j=1}^N \frac{\tau_{P_j} \xi_{jN}}{\alpha_j - g_j} \right]}_{\zeta'} X, \quad (73)$$

and therefore we obtain

$$\mathbb{E}_i[D] = \Phi \frac{\tau - \tau_F}{\tau} F + \Phi \frac{\tau_v}{\tau} v_i + \Phi \frac{\tau_G}{\tau} v + \Phi \zeta' X \quad (74)$$

$$\mathbb{V}_i[D] = \frac{1}{\tau} \Phi \Phi' + \frac{1}{\tau_\epsilon} \mathbb{I}_N. \quad (75)$$

The conditional variance is the same across investors, thus we denote it by $\bar{\mathbb{V}}[D]$. Investor i 's optimal demand writes

$$x_i = \frac{1}{\gamma} (\bar{\mathbb{V}}[D])^{-1} (\mathbb{E}_i[D] - P), \quad (76)$$

which we aggregate for market clearing:

$$\bar{\mathbb{E}}[D] - P = \gamma \bar{\mathbb{V}}[D]X, \quad (77)$$

or

$$P = \Phi \frac{\tau - \tau_F}{\tau} F + \Phi \frac{\tau_G}{\tau} v + \left(\Phi \zeta' - \frac{\gamma}{\tau} \Phi \Phi' - \frac{\gamma}{\tau_\epsilon} \mathbb{I}_N \right) X, \quad (78)$$

and therefore the undetermined coefficients in Eq. (69) are given by:

$$\alpha = \Phi \frac{\tau - \tau_F}{\tau} \quad (79)$$

$$g = \Phi \frac{\tau_G}{\tau} \quad (80)$$

$$\xi = \Phi \zeta' - \frac{\gamma}{\tau} \Phi \Phi' - \frac{\gamma}{\tau_\epsilon} \mathbb{I}_N. \quad (81)$$

It then follows that excess returns can be written:

$$Q = D - P = \Phi \frac{\tau_F}{\tau} F - \Phi \frac{\tau_G}{\tau} v - \xi X + \epsilon, \quad (82)$$

from which we obtain

$$\text{Cov}[Q, G] = \text{Cov} \left[\Phi \frac{\tau_F}{\tau} F - \Phi \frac{\tau_G}{\tau} v, F + v \right] = \Phi \frac{\tau_F}{\tau} \text{Cov}[F, F + v] - \Phi \frac{\tau_G}{\tau} \text{Cov}[v, F + v] = 0. \quad (83)$$

The solution to (79)-(81) requires solving numerically for τ_{Pj} , $j = 1, \dots, N$. This further determines τ and then all the coefficients in the conjecture (69) follow from (79)-(81). The equations that pin-down τ_{Pj} , $j = 1, \dots, N$ result from the learning solution (72), which provides N equations with N unknowns:

$$\tau_{Pj} = \frac{\tau_X \tau_\epsilon^2 \tau_v \left[\left(\tau_v + \sum_{k \neq j} \tau_{Pk} \right) \phi_j^2 - \tau_{Pj} \sum_{k \neq j} \phi_k^2 \right]}{\gamma^2 \left(\tau + \tau_\epsilon \sum_k \phi_k^2 \right)^2}, \quad \text{for } j = 1, \dots, N, \quad (84)$$

which yields

$$\tau_{Pj} = \kappa \phi_j^2, \quad \text{where } \kappa \equiv \frac{\tau_X \tau_\epsilon^2 \tau_v (\tau - \tau_F - \tau_G)}{\tau_X \tau_\epsilon^2 \tau_v \sum_{k=1}^N \phi_k^2 + \gamma^2 \left(\tau + \tau_\epsilon \sum_k \phi_k^2 \right)^2}, \quad \text{for } j = 1, \dots, N. \quad (85)$$

Thus, the only unknown is now κ , which solves the following equation:

$$\kappa = \frac{\tau_X \tau_\epsilon^2 \tau_v \left(\tau_v + \kappa \sum_{k=1}^N \phi_k^2 \right)}{\tau_X \tau_\epsilon^2 \tau_v \sum_{k=1}^N \phi_k^2 + \gamma^2 \left[\tau_F + \tau_G + \tau_v + (\kappa + \tau_\epsilon) \sum_{k=1}^N \phi_k^2 \right]^2}, \quad (86)$$

which has a unique real solution.

A.3 Proof of Proposition 2

Using (85), we can re-write the vector ζ (which is defined in Eq. 73) as a function of the initial vector of loadings Φ :

$$\zeta = -\frac{[\kappa\tau + \tau_\epsilon(\tau - \tau_F - \tau_G - \tau_v)]}{\tau\tau_\epsilon\tau_v}\Phi. \quad (87)$$

Compute then the variance-covariance matrix as perceived by the econometrician:

$$\mathbb{V}[Q] = \left(\frac{\tau_F}{\tau^2} + \frac{\tau_G}{\tau^2}\right)\Phi\Phi' + \frac{1}{\tau_X}\left(\Phi\zeta' - \frac{\gamma}{\tau}\Phi\Phi' - \frac{\gamma}{\tau_\epsilon}\mathbb{I}_N\right)\left(\Phi\zeta' - \frac{\gamma}{\tau}\Phi\Phi' - \frac{\gamma}{\tau_\epsilon}\mathbb{I}_N\right)' + \frac{1}{\tau_\epsilon}\mathbb{I}_N. \quad (88)$$

Using the proportionality relation (87), we obtain

$$\mathbb{V}[Q] = \frac{1}{\tilde{\tau}}\Phi\Phi' + \left(\frac{\gamma^2}{\tau_X\tau_\epsilon^2} + \frac{1}{\tau_\epsilon}\right)\mathbb{I}_N \quad (89)$$

where $\tilde{\tau}$ stands for “the precision of the econometrician,” and we have $\tilde{\tau} < \tau$, because the econometrician has an informational disadvantage with respect to the average agent. Using the definition (75), we obtain the following relationship between the two covariance matrices:

$$\frac{\tau}{\tilde{\tau}}\bar{\mathbb{V}}[Q] = \mathbb{V}[Q] + \left(\frac{\tau}{\tilde{\tau}\tau_\epsilon} - \frac{\gamma^2}{\tau_X\tau_\epsilon^2} - \frac{1}{\tau_\epsilon}\right)\mathbb{I}_N. \quad (90)$$

To get the relationship between betas, start with

$$\beta \frac{\mathbb{1}'\bar{\mathbb{V}}[Q]\mathbb{1}}{\mathbb{1}'\mathbb{V}[Q]\mathbb{1}} \frac{\tau}{\tilde{\tau}} = N \frac{\frac{\tau}{\tilde{\tau}}\bar{\mathbb{V}}[Q]\mathbb{1}}{\mathbb{1}'\mathbb{V}[Q]\mathbb{1}} = N \frac{\left[\mathbb{V}[Q] + \left(\frac{\tau}{\tilde{\tau}\tau_\epsilon} - \frac{\gamma^2}{\tau_X\tau_\epsilon^2} - \frac{1}{\tau_\epsilon}\right)\mathbb{I}_N\right]\mathbb{1}}{\mathbb{1}'\mathbb{V}[Q]\mathbb{1}}, \quad (91)$$

which we can write

$$\beta(1 + \delta) = \tilde{\beta} + N \frac{\left(\frac{\tau - \tilde{\tau}}{\tilde{\tau}\tau_\epsilon} - \frac{\gamma^2}{\tau_X\tau_\epsilon^2}\right)\mathbb{1}}{\mathbb{1}'\mathbb{V}[Q]\mathbb{1}}. \quad (92)$$

Multiply both sides with $\mathbb{1}'$ and divide by N , then use the result that the average of betas is one for both the agent and the econometrician, which provides the “distortion” δ :

$$\delta = \frac{N}{\mathbb{1}'\mathbb{V}[Q]\mathbb{1}} \left(\frac{\tau - \tilde{\tau}}{\tilde{\tau}\tau_\epsilon} - \frac{\gamma^2}{\tau_X\tau_\epsilon^2}\right), \quad (93)$$

and thus the following relation holds between betas

$$\tilde{\beta} - \mathbb{1} = (1 + \delta)(\beta - \mathbb{1}). \quad (94)$$

Furthermore, if the informational advantage of agents, $\Delta_\tau \equiv \tau - \tilde{\tau}$, satisfies

$$\Delta_\tau > \frac{\gamma^2\tilde{\tau}}{\tau_X\tau_\epsilon}, \quad (95)$$

then $\delta > 0$ and econometrician's betas are "inflated" with respect to their average of one. This inequality is satisfied even when there is no private or public information. In this case we have

$$\frac{\tau - \tilde{\tau}}{\tilde{\tau}\tau_\epsilon} - \frac{\gamma^2}{\tau_X\tau_\epsilon^2} = \frac{\gamma^2 [\tau_F + \tau_\epsilon(\phi_1^2 + \phi_2^2 + \phi_3^2)]}{\tau_F\tau_X\tau_\epsilon^2} > 0, \quad (96)$$

and thus Eq. (95) is verified. This is because the market portfolio is unobservable, and thus supply shocks distorts econometrician's view. \square

A.4 Detailed Price Form

$$\begin{aligned} P_{t-k} = & \underbrace{\bar{\alpha}_k}_{N \times 1} \bar{D} + \underbrace{\alpha_k}_{N \times 1} F_{t-k-T} + \underbrace{\bar{\xi}_k}_{N \times 1} \bar{X} + \underbrace{\xi_k}_{N \times N} X_{t-k-T} + g_k G_{t-k} \\ & + \underbrace{\begin{pmatrix} d_{k,-T+1} & d_{k,-T+2} & \dots & d_{k,0} \end{pmatrix}}_{N \times NT} \begin{pmatrix} D_{t-T-k+1} \\ D_{t-T-k+2} \\ \vdots \\ D_{t-k} \end{pmatrix} \\ & + \underbrace{\begin{pmatrix} a_{k,-T+1} & a_{k,-T+2} & \dots & a_{k,0} \end{pmatrix}}_{N \times T} \begin{pmatrix} \epsilon_{t-k-T+1}^F \\ \epsilon_{t-k-T+2}^F \\ \vdots \\ \epsilon_{t-k}^F \end{pmatrix} \\ & + \underbrace{\begin{pmatrix} b_{k,-T+1} & b_{k,-T+2} & \dots & b_{k,0} \end{pmatrix}}_{N \times NT} \begin{pmatrix} \epsilon_{t-k-T+1}^X \\ \epsilon_{t-k-T+2}^X \\ \vdots \\ \epsilon_{t-k}^X \end{pmatrix}. \end{aligned} \quad (97)$$

A.5 Proof of Proposition 3 (Learning)

We start by stacking all observable information in a vector. An investor i observes current and past prices, dividends, public announcements and her set of private signals. Starting with prices

we first write the vector p_{t-k} in (31) as

$$p_{t-k} = \underbrace{\begin{pmatrix} a_{i(k),-T+1} & a_{i(k),-T+2} & \cdots & a_{i(k),-1} & a_{i(k),0} \\ a_{i(k+1),-T+2} & a_{i(k+1),-T+3} & \cdots & a_{i(k+1),0} & \mathbf{0}_{N \times 1} \\ a_{i(k+2),-T+3} & a_{i(k+2),-T+4} & \cdots & \mathbf{0}_{N \times 1} & \mathbf{0}_{N \times 1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i(k+T-1),0} & \mathbf{0}_{N \times 1} & \cdots & \mathbf{0}_{N \times 1} & \mathbf{0}_{N \times 1} \end{pmatrix}}_{A_k \ (NT \times T)} \bar{\epsilon}_{t-k}^F \quad (98)$$

$$+ \underbrace{\begin{pmatrix} b_{i(k),-T+1} & b_{i(k),-T+2} & \cdots & b_{i(k),-1} & b_{i(k),0} \\ b_{i(k+1),-T+2} & b_{i(k+1),-T+3} & \cdots & b_{i(k+1),0} & \mathbf{0}_{N \times N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{i(k+T-1),0} & \mathbf{0}_{N \times N} & \mathbf{0}_{N \times N} & \mathbf{0}_{N \times N} & \mathbf{0}_{N \times N} \end{pmatrix}}_{B_k \ (NT \times NT)} \bar{\epsilon}_{t-k}^X \quad (99)$$

$$\equiv A_k \bar{\epsilon}_{t-k}^F + B_k \bar{\epsilon}_{t-k}^X. \quad (100)$$

Similarly, we express the vector of past and current private signals in (32) as

$$V_{t-k}^i = \underbrace{\begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{pmatrix}}_{\Omega \ (T \times T)} \bar{\epsilon}_{t-k}^F + \underbrace{\begin{pmatrix} \tilde{\epsilon}_{t-k}^i \\ \tilde{\epsilon}_{t-k-1}^i \\ \vdots \\ \tilde{\epsilon}_{t-k-T+2}^i \\ \tilde{\epsilon}_{t-k-T+1}^i \end{pmatrix}}_{\bar{\epsilon}_{t-k}^i \ T \times 1} \quad (101)$$

$$\equiv \Omega \bar{\epsilon}_{t-k}^F + \bar{\epsilon}_{t-k}^i \quad (102)$$

where the vector of investor-specific noise is distributed as

$$\bar{\epsilon}_{t-k}^i \sim N \left(\mathbf{0}_{T \times 1}, \underbrace{\begin{pmatrix} \sigma_v^2 & 0 & \cdots & 0 \\ 0 & \sigma_v^2 (n_{t-k,1})^{-1} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \sigma_v^2 (n_{t-k,T-1})^{-1} \end{pmatrix}}_{S_k \ (T \times T)} \right) \quad (103)$$

To express the informational parts of dividends in vector form, notice that the dividend at time $t-k$ can be written as

$$D_{t-k} = \bar{D} \mathbf{1}_{N \times 1} + \Phi \left(\kappa_F^T F_{t-k-T} + \sum_{\tau=0}^{T-1} \kappa_F^\tau \epsilon_{t-k-\tau}^F \right) + \epsilon_{t-k}^D \quad (104)$$

$$\equiv \Phi \kappa_F^T F_{t-k-T} + D_{t-k}^a \quad (105)$$

where D_{t-k}^a represents the part of dividends that only contains unobservables. Likewise, the dividends at time $t-k-1$ satisfies

$$D_{t-k-1} = \Phi \left(\kappa_F^{T-1} F_{t-k-T} + \sum_{\tau=0}^{T-2} \kappa_F^\tau \epsilon_{t-k-\tau-1}^F \right) + \epsilon_{t-k-1}^D \quad (106)$$

$$\equiv \Phi \kappa_F^{T-1} F_{t-k-T} + D_{t-k-1}^a. \quad (107)$$

Proceeding iteratively, the dividend at time $t-k-T+1$ satisfies

$$D_{t-k-T+1} = \Phi \left(\kappa_F F_{t-k-T} + \epsilon_{t-k-T+1}^F \right) + \epsilon_{t-k-T+1}^D \quad (108)$$

$$\equiv \Phi \kappa_F F_{t-k-T} + D_{t-k-T+1}^a. \quad (109)$$

Accordingly, we can express the vector of informational dividends in (35) as

$$\bar{D}_{t-k}^a = \underbrace{\begin{pmatrix} \Phi & \mathbf{0}_{N \times 1} & \dots & \mathbf{0}_{N \times 1} & \mathbf{0}_{N \times 1} \\ \Phi \kappa_F & \Phi & \dots & \mathbf{0}_{N \times 1} & \mathbf{0}_{N \times 1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \Phi \kappa_F^{T-2} & \Phi \kappa_F^{T-3} & \dots & \Phi & \mathbf{0}_{N \times 1} \\ \Phi \kappa_F^{T-1} & \Phi \kappa_F^{T-2} & \dots & \Phi \kappa_F & \Phi \end{pmatrix}}_{\Lambda \ (NT \times T)} \bar{\epsilon}_{t-k}^F + \underbrace{\begin{pmatrix} \epsilon_{t-k-T+1}^D \\ \epsilon_{t-k-T+2}^D \\ \vdots \\ \epsilon_{t-k-1}^D \\ \epsilon_{t-k}^D \end{pmatrix}}_{\bar{\epsilon}_{t-k}^D \ (NT \times 1)} \quad (110)$$

$$\equiv \Lambda \bar{\epsilon}_{t-k}^F + \bar{\epsilon}_{t-k}^D. \quad (111)$$

Finally, we can write the informational part of the public signal in (36) as

$$G_{t-k}^a = \underbrace{\begin{pmatrix} \kappa_F^{k+j(k)-1} & \kappa_F^{k+j(k)-2} \mathbf{1}_{k+j(k)-2 \geq 0} & \dots & \kappa_F^{k+j(k)-T} \mathbf{1}_{k+j(k)-T \geq 0} \end{pmatrix}}_{\Xi_k \ (1 \times T)} \bar{\epsilon}_{t-k}^F + \epsilon_{t+j(k)-T}^G \quad (112)$$

$$\equiv \Xi_k \bar{\epsilon}_{t-k}^F + \epsilon_{t+j(k)-T}^G. \quad (113)$$

We now stack all observables into a single vector according to:

$$\begin{pmatrix} p_{t-k} \\ V_{t-k}^i \\ \bar{D}_{t-k}^a \\ G_{t-k}^a \end{pmatrix} = \underbrace{\begin{pmatrix} A_k \\ \Omega \\ \Lambda \\ \Xi_k \end{pmatrix}}_{H_k \ (((2N+1)T+1) \times T)} \bar{\epsilon}_{t-k}^F + \underbrace{\begin{pmatrix} B_k & \mathbf{0}_{NT \times T} & \mathbf{0}_{NT \times NT} & \mathbf{0}_{NT \times 1} \\ \mathbf{0}_{T \times NT} & \mathbf{I}_T & \mathbf{0}_{T \times NT} & \mathbf{0}_{T \times 1} \\ \mathbf{0}_{NT \times NT} & \mathbf{0}_{NT \times T} & \mathbf{I}_{NT} & \mathbf{0}_{NT \times 1} \\ \mathbf{0}_{1 \times NT} & \mathbf{0}_{1 \times T} & \mathbf{0}_{1 \times NT} & 1 \end{pmatrix}}_{\Theta_k \ (((2N+1)T+1) \times ((2N+1)T+1))} \begin{pmatrix} \bar{\epsilon}_{t-k}^X \\ \bar{\epsilon}_{t-k}^i \\ \bar{\epsilon}_{t-k}^D \\ \epsilon_{t+j(k)-T}^G \end{pmatrix} \quad (114)$$

where the vector of noise is distributed as

$$\begin{pmatrix} \bar{\epsilon}_{t-k}^X \\ \bar{\epsilon}_{t-k}^i \\ \bar{\epsilon}_{t-k}^D \\ \epsilon_{t+j(k)-T}^G \end{pmatrix} \sim N \left(\mathbf{0}_{((2N+1)T+1) \times 1}, \underbrace{\begin{pmatrix} \sigma_X^2 \mathbf{I}_{NT} & \mathbf{0}_{NT \times T} & \mathbf{0}_{NT \times NT} & \mathbf{0}_{NT \times 1} \\ \mathbf{0}_{T \times NT} & S_k & \mathbf{0}_{T \times NT} & \mathbf{0}_{T \times 1} \\ \mathbf{0}_{NT \times NT} & \mathbf{0}_{NT \times T} & \sigma_D^2 \mathbf{I}_{NT} & \mathbf{0}_{NT \times 1} \\ \mathbf{0}_{1 \times NT} & \mathbf{0}_{1 \times T} & \mathbf{0}_{1 \times NT} & \sigma_G^2 \end{pmatrix}}_{\Sigma_k \ (((2N+1)T+1) \times ((2N+1)T+1))} \right). \quad (115)$$

Using these matrices, define

$$R_k = \Theta_k \Sigma_k \Theta_k^\top, \quad (116)$$

which we can use to apply the projection theorem. In particular, notice that the vector:

$$\begin{pmatrix} \bar{\epsilon}_{t-k}^F \\ p_{t-k} \\ V_{t-k}^i \\ \bar{D}_{t-k}^a \\ G_{t-k}^a \end{pmatrix} \sim N \left(\mathbf{0}_{((2N+2)T+1) \times 1}, \left(\begin{array}{c|c} \sigma_F^2 \mathbf{I}_T & \sigma_F^2 H_k^\top \\ \hline \sigma_F^2 H_k & \sigma_F^2 H_k H_k^\top + R_k \end{array} \right) \right) \quad (117)$$

is jointly normally distributed. The projection theorem then implies that:

$$\mathbb{E}_{t-k}^i [\bar{\epsilon}_{t-k}^F] = \underbrace{\sigma_F^2 H_k^\top (\sigma_F^2 H_k H_k^\top + R_k)^{-1}}_{M_k \quad (T \times ((2N+1)T+1))} \begin{pmatrix} p_{t-k} \\ V_{t-k}^i \\ \bar{D}_{t-k}^a \\ G_{t-k}^a \end{pmatrix} \quad (118)$$

and

$$\underbrace{\mathbb{V}_{t-k}^i [\bar{\epsilon}_{t-k}^F]}_{(T \times T)} = \sigma_F^2 (\mathbf{I}_T - M_k H_k). \quad (119)$$

Using this result, we can now compute $\mathbb{E}_{t-k}^i [P_{t-k+1} + D_{t-k+1}]$. We first rewrite $P_{t-k+1} + D_{t-k+1}$ using the price conjecture as

$$\begin{aligned} P_{t-k+1} + D_{t-k+1} &= D_{t-k+1} + \bar{\alpha}_{i(k+T-1)} \bar{D} + \alpha_{i(k+T-1)} F_{t-k-T+1} + \bar{\xi}_{i(k+T-1)} \bar{X} + g_{i(k+T-1)} G_{t-k+1} \\ &\quad + \xi_{i(k+T-1)} X_{t-k-T+1} + d_{i(k+T-1)} \bar{D}_{t-k+1} + a_{i(k+T-1)} \bar{\epsilon}_{t-k+1}^F + b_{i(k+T-1)} \bar{\epsilon}_{t-k+1}^X \\ &= \bar{\alpha}_{i(k+T-1)} \bar{D} + \alpha_{i(k+T-1)} \kappa_F F_{t-k-T} + (\bar{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)} \mathbf{1}_{N \times 1} (1 - \kappa_X)) \bar{X} \\ &\quad + \xi_{i(k+T-1)} \kappa_X X_{t-k-T} + a_{i(k+T-1)} \bar{\epsilon}_{t-k+1}^F + b_{i(k+T-1)} \bar{\epsilon}_{t-k+1}^X + g_{i(k+T-1)} G_{t-k+1} \\ &\quad + \alpha_{i(k+T-1)} \epsilon_{t-k-T+1}^F + \xi_{i(k+T-1)} \epsilon_{t-k-T+1}^X + D_{t-k+1} + d_{i(k+T-1)} \bar{D}_{t-k+1}. \end{aligned} \quad (120)$$

Furthermore, defining a new vector d^* as

$$d_{i(k+T-1)} \bar{D}_{t-k+1} = d_{i(k+T-1),0} D_{t-k+1} + \underbrace{\left(\mathbf{0}_{N \times N} \quad d_{i(k+T-1),-T+1} \quad d_{i(k+T-1),-T+2} \quad \cdots \quad d_{i(k+T-1),-1} \right)}_{d_{i(k+T-1)}^* \quad (N \times NT)} \bar{D}_{t-k} \quad (121)$$

$$\equiv d_{i(k+T-1),0} \left(\bar{D} \mathbf{1}_{N \times 1} + \Phi \left(\kappa_F^{T+1} F_{t-k-T} + \sum_{\tau=0}^T \kappa_F^\tau \epsilon_{t-k-\tau+1}^F \right) + \epsilon_{t-k+1}^D \right) + d_{i(k+T-1)}^* \bar{D}_{t-k}, \quad (122)$$

we obtain

$$\begin{aligned}
P_{t-k+1} + D_{t-k+1} &= (\bar{\alpha}_{i(k+T-1)} + (d_{i(k+T-1),0} + \mathbf{I}_N)\mathbf{1}_{N \times 1})\bar{D} + (\bar{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)}\mathbf{1}_{N \times 1}(1 - \kappa_X))\bar{X} \\
&\quad + \kappa_F (\alpha_{i(k+T-1)} + \kappa_F^T(d_{i(k+T-1),0} + \mathbf{I}_N)\Phi) F_{t-k-T} + \xi_{i(k+T-1)}\kappa_X X_{t-k-T} \\
&\quad + g_{i(k+T-1)}G_{t-k+1} + d_{i(k+T-1)}^* \bar{D}_{t-k} + b_{i(k+T-1)} \bar{c}_{t-k+1}^X + \xi_{i(k+T-1)} \epsilon_{t-k-T+1}^X \\
&\quad + a_{i(k+T-1)} \bar{c}_{t-k+1}^F + \alpha_{i(k+T-1)} \epsilon_{t-k-T+1}^F + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi \sum_{\tau=0}^T \kappa_F^\tau \epsilon_{t-k-\tau+1}^F \\
&\quad + (d_{i(k+T-1),0} + \mathbf{I}_N) \epsilon_{t-k+1}^D.
\end{aligned} \tag{123}$$

Now, importantly, the only time G_{t-k+1} is unobservable is at date $t-1$, right before the announcement. To see this, consider that at time $t-2$ investors attempt to forecast P_{t-1} , which is a function of G_{t-T} that they observe. At time t investors attempt to forecast P_{t+1} , which is a function of G_t that they observe. The only time this property is violated is at time $t-1$, at which investors attempt to forecast P_t , which is a function of G_t that they *do not observe yet*. In this case, we must further decompose G_t as

$$G_t = F_t + \epsilon_t^G = \kappa_F^{T+1} F_{t-1-T} + \sum_{\tau=0}^T \kappa_F^\tau \epsilon_{t-\tau}^F + \epsilon_t^G; \tag{124}$$

accordingly, we write

$$\begin{aligned}
P_{t-k+1} + D_{t-k+1} &= (\bar{\alpha}_{i(k+T-1)} + (d_{i(k+T-1),0} + \mathbf{I}_N)\mathbf{1}_{N \times 1})\bar{D} + (\bar{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)}\mathbf{1}_{N \times 1}(1 - \kappa_X))\bar{X} \\
&\quad + \kappa_F \left(\alpha_{i(k+T-1)} + \kappa_F^T(d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\kappa_F^{T+1}\mathbf{1}_{k=1} \right) F_{t-k-T} + \xi_{i(k+T-1)}\kappa_X X_{t-k-T} \\
&\quad + g_{i(k+T-1)}G_{t-k+1}\mathbf{1}_{k \neq 1} + d_{i(k+T-1)}^* \bar{D}_{t-k} + b_{i(k+T-1)} \bar{c}_{t-k+1}^X + \xi_{i(k+T-1)} \epsilon_{t-k-T+1}^X \\
&\quad + a_{i(k+T-1)} \bar{c}_{t-k+1}^F + \alpha_{i(k+T-1)} \epsilon_{t-k-T+1}^F + ((d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}) \sum_{\tau=0}^T \kappa_F^\tau \epsilon_{t-k-\tau+1}^F \\
&\quad + (d_{i(k+T-1),0} + \mathbf{I}_N) \epsilon_{t-k+1}^D + g_{i(T)} \epsilon_t^G \mathbf{1}_{k=1}.
\end{aligned} \tag{125}$$

Using this expression we can define a new vector a^* as

$$\begin{aligned}
&a_{i(k+T-1)} \bar{c}_{t-k+1}^F + \alpha_{i(k+T-1)} \epsilon_{t-k-T+1}^F + ((d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}) \sum_{\tau=0}^T \kappa_F^\tau \epsilon_{t-k-\tau+1}^F \\
&= \underbrace{\begin{pmatrix} \alpha_{i(k+T-1)} + \kappa_F^T((d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}) \\ a_{i(k+T-1),-T+1} + \kappa_F^{T-1}((d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}) \\ \vdots \\ a_{i(k+T-1),-1} + \kappa_F((d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}) \end{pmatrix}^\top}_{a_{i(k+T-1)}^* (N \times T)} \bar{c}_{t-k+1}^F + (a_0 + (d_0 + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}) \epsilon_{t-k+1}^F \\
&\equiv a_{i(k+T-1)}^* \bar{c}_{t-k+1}^F + (a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1}) \epsilon_{t-k+1}^F
\end{aligned} \tag{126}$$

and a new vector b^* as

$$\begin{aligned}
b_{i(k+T-1)}\bar{\epsilon}_{t-k+1}^X + \xi_{i(k+T-1)}\epsilon_{t-k-T+1}^X &= \underbrace{\left(\xi_{i(k+T-1)} \quad b_{i(k+T-1),-T+1} \quad b_{i(k+T-1),-T+2} \quad \cdots \quad b_{i(k+T-1),-1} \right)}_{b_{i(k+T-1)}^* \quad (N \times NT)} \bar{\epsilon}_{t-k}^X \\
&+ b_{i(k+T-1),0}\epsilon_{t-k+1}^X \\
&= b_{i(k+T-1)}^*\bar{\epsilon}_{t-k}^X + b_{i(k+T-1),0}\epsilon_{t-k+1}^X
\end{aligned} \tag{127}$$

to finally write

$$\begin{aligned}
P_{t-k+1} + D_{t-k+1} &= f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k}\mathbf{1}_{k \neq 1}) + a_{i(k+T-1)}^*\bar{\epsilon}_{t-k}^F + b_{i(k+T-1)}^*\bar{\epsilon}_{t-k}^X \\
&+ (a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1})\epsilon_{t-k+1}^F + b_{i(k+T-1),0}\epsilon_{t-k+1}^X \\
&+ (d_{i(k+T-1),0} + \mathbf{I}_N)\epsilon_{t-k+1}^D + g_{i(T)}\epsilon_t^G\mathbf{1}_{k=1}.
\end{aligned} \tag{128}$$

where

$$\begin{aligned}
f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k}\mathbf{1}_{k \neq 1}) &:= (\bar{\alpha}_{i(k+T-1)} + (d_{i(k+T-1),0} + \mathbf{I}_N)\mathbf{1}_{N \times 1})\bar{D} \\
&+ (\bar{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)}\mathbf{1}_{N \times 1}(1 - \kappa_X))\bar{X} + \xi_{i(k+T-1)}\kappa_X X_{t-k-T} + g_{i(k+T-1)}G_{t-k+1}\mathbf{1}_{k \neq 1} \\
&+ \kappa_F \left(\alpha_{i(k+T-1)} + \kappa_F^T(d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\kappa_F^{T+1}\mathbf{1}_{k=1} \right) F_{t-k-T} + d_{i(k+T-1)}^*\bar{D}_{t-k}.
\end{aligned} \tag{129}$$

To compute investor i 's conditional expectation and posterior variance, it is convenient to use

$$\bar{\epsilon}_{t-k}^X = B_k^{-1}p_{t-k} - B_k^{-1}A_k\bar{\epsilon}_{t-k}^F \tag{130}$$

to rewrite this expression as

$$\begin{aligned}
P_{t-k+1} + D_{t-k+1} &= f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k}\mathbf{1}_{k \neq 1}) + b_{i(k+T-1)}^*B_k^{-1}p_{t-k} \\
&+ \underbrace{\left(a_{i(k+T-1)}^* - b_{i(k+T-1)}^*B_k^{-1}A_k \right)}_{\psi_k \quad (N \times T)} \bar{\epsilon}_{t-k}^F + (d_{i(k+T-1),0} + \mathbf{I}_N)\epsilon_{t-k+1}^D + g_{i(T)}\epsilon_t^G\mathbf{1}_{k=1} \\
&+ (a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1})\epsilon_{t-k+1}^F + b_{i(k+T-1),0}\epsilon_{t-k+1}^X \\
&\equiv f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k}\mathbf{1}_{k \neq 1}) + b_{i(k+T-1)}^*B_k^{-1}p_{t-k} + \psi_k\bar{\epsilon}_{t-k}^F \\
&+ (d_{i(k+T-1),0} + \mathbf{I}_N)\epsilon_{t-k+1}^D + g_{i(T)}\epsilon_t^G\mathbf{1}_{k=1} \\
&+ (a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N)\Phi + g_{i(T)}\mathbf{1}_{k=1})\epsilon_{t-k+1}^F + b_{i(k+T-1),0}\epsilon_{t-k+1}^X.
\end{aligned} \tag{131}$$

Hence, using our previous projection results we obtain

$$\mathbb{E}_{t-k}^i [P_{t-k+1} + D_{t-k+1}] = f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k}\mathbf{1}_{k \neq 1}) + b_{i(k+T-1)}^*B_k^{-1}p_{t-k} + \psi_k \mathbb{E}_{t-k}^i [\bar{\epsilon}_{t-k}^F] \tag{132}$$

and

$$\mathbb{V}_{t-k}^i [P_{t-k+1} + D_{t-k+1}] = \sigma_F^2 \psi_k (\mathbf{I}_T - M_k H_k) \psi_k^\top + (d_{i(k+T-1),0} + \mathbf{I}_N)(d_{i(k+T-1),0} + \mathbf{I}_N)^\top \sigma_D^2 \quad (133)$$

$$+ \sigma_G^2 g_{i(T)} g_{i(T)}^\top \mathbf{1}_{k=1} + b_{i(k+T-1),0} b_{i(k+T-1),0}^\top \sigma_X^2 \quad (134)$$

$$+ (a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N) \Phi + g_{i(T)} \mathbf{1}_{k=1}) (a_{i(k+T-1),0} + (d_{i(k+T-1),0} + \mathbf{I}_N) \Phi + g_{i(T)} \mathbf{1}_{k=1})^\top \sigma_F^2. \quad (135)$$

A.6 Proof of Proposition 4 (Equilibrium)

For the purpose of aggregating individual demands, it is important to characterize their dependence on the vector N_k of number of signals that a given investor holds at time $t - k$:

$$N_k = \begin{pmatrix} 1 & n_{t-k,1} & \dots & n_{t-k,T-1} \end{pmatrix}^\top. \quad (136)$$

In particular, we can rewrite the matrix M_k as

$$M_k = \mathbb{V}_{t-k}^i [\bar{\epsilon}_{t-k}^F] H_k^\top R_k^{-1}. \quad (137)$$

Using this expression we can write the first term involved in individual demands as

$$\mathbb{V}_{t-k}^i [P_{t-k+1} + D_{t-k+1}]^{-1} \mathbb{E}_{t-k}^i [P_{t-k+1} + D_{t-k+1}] = \quad (138)$$

we denote a general linear relation by $\mathbf{L}(\cdot)$

To setup the system of equations for equilibrium coefficients we use the law of large numbers whereby $\int_{i \in I} \bar{\epsilon}_{t-k}^i di = \mathbf{0}_{T \times 1}$, and obtain that the average expectation in (41) satisfies

$$\begin{aligned} \bar{\mathbb{E}}_{t-k} [P_{t-k+1} + D_{t-k+1}] &= f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k} \mathbf{1}_{k \neq 1}) \\ &+ b_{i(k+T-1)}^* B_k^{-1} p_{t-k} + \psi_k M_k \begin{pmatrix} p_{t-k} \\ \Omega \bar{\epsilon}_{t-k}^F \\ \bar{D}_{t-k}^a \\ G_{t-k}^a \end{pmatrix}. \end{aligned} \quad (139)$$

Observing that

$$\bar{D}_{t-k}^a = \begin{pmatrix} D_{t-k-T+1} \\ D_{t-k-T+2} \\ \vdots \\ D_{t-k} \end{pmatrix} - \mathbf{1}_{N \times 1} \bar{D} - \underbrace{\begin{pmatrix} \Phi \kappa_F \\ \Phi \kappa_F^2 \\ \vdots \\ \Phi \kappa_F^T \end{pmatrix}}_{L \ (NT \times 1)} F_{t-k-T} \quad (140)$$

$$\equiv \bar{D}_{t-k} - \mathbf{1}_{N \times 1} \bar{D} - L F_{t-k-T} \quad (141)$$

and that

$$G_{t-k}^a = G_{t-k} - \kappa_F^{k+j(k)} F_{t-k-T} \quad (142)$$

we can express the vector above as

$$\begin{aligned}
\begin{pmatrix} p_{t-k} \\ \Omega \bar{\epsilon}_{t-k}^F \\ \bar{D}_{t-k}^a \\ G_{t-k}^a \end{pmatrix} &= \begin{pmatrix} A_k \bar{\epsilon}_{t-k}^F + B_k \bar{\eta}_{t-k} \\ \Omega \bar{\epsilon}_{t-k}^F \\ \bar{D}_{t-k} - \mathbf{1}_{N \times 1} \bar{D} - L F_{t-k-T} \\ G_{t-k} - \kappa_F^{k+j(k)} F_{t-k-T} \end{pmatrix} \\
&= \underbrace{\begin{pmatrix} A_k \\ \Omega \\ \mathbf{0}_{NT \times T} \\ \mathbf{0}_{1 \times T} \end{pmatrix}}_{H_k^* \ ((2N+1)T+1) \times T} \bar{\epsilon}_{t-k}^F + \underbrace{\begin{pmatrix} B_k \\ \mathbf{0}_{T \times NT} \\ \mathbf{0}_{NT \times NT} \\ \mathbf{0}_{1 \times NT} \end{pmatrix}}_{B_k^* \ ((2N+1)T+1) \times NT} \bar{\epsilon}_{t-k}^X + \underbrace{\begin{pmatrix} \mathbf{0}_{NT \times NT} \\ \mathbf{0}_{T \times NT} \\ \mathbf{I}_{NT} \\ \mathbf{0}_{1 \times NT} \end{pmatrix}}_C \ ((2N+1)T+1) \times NT \bar{D}_{t-k} \\
&\quad - \underbrace{\begin{pmatrix} \mathbf{0}_{NT \times 1} \\ \mathbf{0}_{T \times 1} \\ \mathbf{1}_{N \times 1} \\ 0 \end{pmatrix}}_K \ ((2N+1)T+1) \times 1 \bar{D} + \underbrace{\begin{pmatrix} \mathbf{0}_{NT \times 1} \\ \mathbf{0}_{T \times 1} \\ \mathbf{0}_{NT \times 1} \\ 1 \end{pmatrix}}_E \ ((2N+1)T+1) \times 1 G_{t-k} - \underbrace{\begin{pmatrix} \mathbf{0}_{NT \times 1} \\ \mathbf{0}_{T \times 1} \\ L \\ \kappa_F^{k+j(k)} \end{pmatrix}}_{L_k^* \ ((2N+1)T+1) \times 1} F_{t-k-T} \\
&\equiv H_k^* \bar{\epsilon}_{t-k}^F + B_k^* \bar{\epsilon}_{t-k}^X + C \bar{D}_{t-k} - K \bar{D} + E G_{t-k} - L^* F_{t-k-T}.
\end{aligned} \tag{143}$$

Substituting this expression in average market expectations and using the pricing relation in (43), we can write:

$$\begin{aligned}
RP_{t-k} &= f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k} \mathbf{1}_{k \neq 1}) + b_{i(k+T-1)}^* \bar{\epsilon}_{t-k}^X + b_{i(k+T-1)}^* B_k^{-1} A_k \bar{\epsilon}_{t-k}^F \\
&\quad + \psi_k M_k (H_k^* \bar{\epsilon}_{t-k}^F + B_k^* \bar{\epsilon}_{t-k}^X + C \bar{D}_{t-k} - K \bar{D} + E G_{t-k} - L_k^* F_{t-k-T}) - \gamma \mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] X_{t-k} \\
&= f(\bar{D}, \bar{X}, X_{t-k-T}, F_{t-k-T}, \bar{D}_{t-k}, G_{t-k} \mathbf{1}_{k \neq 1}) + \psi_k M_k (C \bar{D}_{t-k} - K \bar{D} + E G_{t-k} - L_k^* F_{t-k-T}) \\
&\quad + (b_{i(k+T-1)}^* B_k^{-1} A_k + \psi_k M_k H_k^*) \bar{\epsilon}_{t-k}^F + (b_{i(k+T-1)}^* + \psi_k M_k B_k^*) \bar{\epsilon}_{t-k}^X \\
&\quad - \gamma \mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] \left(\kappa_X^T X_{t-k-T} + \bar{X} (1 - \kappa_X) \sum_{\tau=0}^{T-1} \kappa_X^\tau \mathbf{1}_{N \times 1} + \sum_{\tau=0}^{T-1} \kappa_X^\tau \epsilon_{t-k-\tau}^X \right),
\end{aligned} \tag{144}$$

which confirms our initial price conjecture. We finally obtain the equilibrium fixed point by matching this expression with the price conjecture, which yields:

$$\begin{pmatrix} \bar{\alpha}_k \\ \alpha_k \\ \bar{\xi}_k \\ \xi_k \\ d_k \\ g_k \\ a_k \\ b_k \end{pmatrix} = \frac{1}{R} \begin{pmatrix} (\bar{\alpha}_{i(k+T-1)} + (d_{i(k+T-1),0} + \mathbf{I}_N) \mathbf{1}_{N \times 1}) - \psi_k M_k K \\ \kappa_F (\alpha_{i(k+T-1)} + \kappa_F^T (d_{i(k+T-1),0} + \mathbf{I}_N) \Phi + g_{i(T)} \kappa_F^{T+1} \mathbf{1}_{k=1}) - \psi_k M_k L_k^* \\ (\bar{\xi}_{i(k+T-1)} + \xi_{i(k+T-1)} \mathbf{1}_{N \times 1} (1 - \kappa_X)) - \gamma \mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] \mathbf{1}_{N \times 1} (1 - \kappa_X) \sum_{\tau=0}^{T-1} \kappa_X^\tau \\ \xi_{i(k+T-1)} \kappa_X - \gamma \mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] \kappa_X^T \\ d_{i(k+T-1)}^* + \psi_k M_k C \\ g_{i(k+T-1)} \mathbf{1}_{k \neq 1} + \psi_k M_k E \\ b_{i(k+T-1)}^* B_k^{-1} A_k + \psi_k M_k H_k^* \\ b_{i(k+T-1)}^* + \psi_k M_k B_k^* - \gamma \mathbb{V}_{t-k} [P_{t-k+1} + D_{t-k+1}] \begin{pmatrix} \kappa_X^{T-1} \mathbf{I}_N & \kappa_X^{T-2} \mathbf{I}_N & \dots & \kappa_X \mathbf{I}_N & \mathbf{I}_N \end{pmatrix} \end{pmatrix}. \tag{145}$$

for all $k \in \{0, \dots, T-1\}$.

A.7 Information Percolation

We describe here the private information of investors and how it diffuses among the population of investors. At any date s each investor i receives a private signal v^i about the current factor innovation:

$$v_s^i = \epsilon_s^F + \epsilon_s^i, \quad \epsilon_s^i \sim N(0, \sigma_v^2), \quad \epsilon_s^i \perp \epsilon_s^k, \quad \forall k \neq i \quad (146)$$

Following [Andrei and Cujean \(2016\)](#) we allow investors to exchange these signals in private, bilateral meetings through the information percolation theory ([Duffie, Malamud, and Manso, 2009](#)) whereby meetings take place continuously at Poisson arrival times. Unlike [Andrei and Cujean \(2016\)](#), however, we allow meetings to occur with an intensity λ_s that varies over time. This meeting intensity is common to all investors. When two investors meet at time s , they exchange all the signals they have gathered about factor innovations at any relevant lag. This assumption along with the normality of individual signals imply that an agent's private information is completely summarized by her number $n_{s,k}$ of signals regarding the factor innovation at lag k for all lags k and her posterior expectation of the vector $\bar{\epsilon}_s^F$. These numbers are sufficient statistics for the information investors exchange when they meet and "talk."

We assume that the meeting intensity varies over the public signal cycles. That is, the meeting intensity between date $t - k$ and $t - k + 1$ exhibits the periodic pattern highlighted in [Figure 7](#):

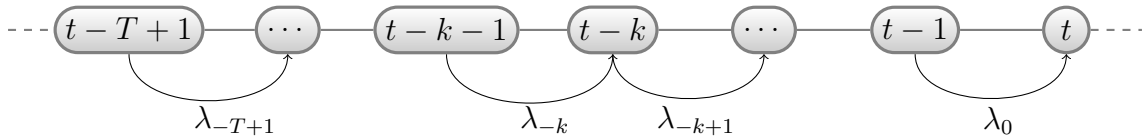


Figure 7: Information time line of private signals exchange for any $k \geq 2$.

Except from the fact that this periodic pattern repeats itself during the public announcement cycles, we do not impose any structure within the cycle. In other words, we can assume any pattern for the vector of meeting intensities (which is common knowledge to all agents in the economy):

$$\lambda_k, \forall k \in \{-T + 1, \dots, 0\}. \quad (147)$$

This will allow us to understand the effect of information diffusion on asset prices, especially during announcement days, where we expect investors to exchange more information than usual, for instance by chatting on Bloomberg terminals.

Following [Figure 7](#), suppose that today $s = t - k$ is located k periods away from the announcement date. Today investor i receives one signal regarding the current factor innovation ϵ_s^F and looks for other investors with intensity λ_{-k+1} . Yesterday she was looking for investors to talk to with intensity λ_{-k} and so on and so forth until the previous announcement date at time $t - T$ at which she was looking for investors to talk to with intensity λ_{-T+1} . Right before the last announcement date she looked for other investors with intensity λ_0 and so on and so forth until the last relevant date $s - T + 1$ at which she looks for someone with intensity λ_{-k+2} , consistent with the periodic pattern in λ . Note that, as of today (date s), date $s - T + 1$ is the last relevant date, since all innovations prior to this date are perfectly known. Moreover, as of today, investor i has 1 signal regarding ϵ_s^F , $n_{s,1}$ signals regarding ϵ_{s-1}^F and so on.

The number of signals $n_{s,k}$ that an arbitrary investor possesses at time s regarding the factor

innovation ϵ_{s-k}^F are distributed according to a certain cross-sectional distribution $\mu_{s,k}$. Adapting the computations in Duffie et al. (2009), this cross-sectional distribution can be defined recursively. In particular, the cross-sectional distribution of the number $n_{s,k}$ of signals regarding the factor innovation at lag k gathered by date s evolves according to the dynamics

$$\frac{d}{du}\mu_{u,k}(n) = \lambda_{s-l}\mu_{u,k} * \mu_{u,k} - \lambda_{s-l}\mu_{u,k} \quad (148)$$

$$= \lambda_{s-l} \sum_{m=1}^{n-1} \mu_{u,k}(n-m)\mu_{u,k}(m) - \lambda_{s-l}\mu_{u,k}(n), \quad (149)$$

with $u \in [s-l, s-l+1]$, $l \in 1, \dots, k$, $\mu_{s-k} = \delta_{n=1}$, and where “*” denotes the discrete convolution product. The summation term on the right hand side in (148) represents the rate at which new agents of a given type are created, whereas the second term in (148) captures the rate at which agents leave a given type.

A key statistic in the equilibrium construction is the first moment of this distribution. In particular, we denote the cross-sectional average number of signals regarding factor innovations at lag k gathered by time s by $\Omega_{s,k} = \sum_{n \in \mathbb{N}} n\mu_{s,k}(n)$. Using the dynamics of the cross-sectional distribution of number of signals, the cross-sectional average must satisfy

$$\frac{d}{du}\Omega_{u,k} = \lambda_{s-l}\Omega_{u,k}, \quad \forall u \in [s-l, s-l+1], \quad \forall l \in 1, \dots, k, \quad \Omega_{s-k} = 1. \quad (150)$$

Solving this equation recursively, the cross-sectional average number of signals regarding information at lag l gathered by date $t-k$ satisfies

$$\Omega_{t-k,l} = \exp\left(\mathbb{1}_{l \geq 1} \sum_{m=k+1}^{k+l} \lambda_{-i(m-1)}\right), \quad \forall k \in \{0, \dots, T-1\}, \quad l \in \{0, \dots, T-1\}. \quad (151)$$

Finally, because information is Gaussian and investors share all their information, the collection $\{\epsilon_{s-k,j}^F\}_{j=1}^{n_{s,k}}$ of private signals about innovations at lag k gathered by date s can be summarized with a single private signal with variance $\sigma_v^2(n_{s,k})^{-1}$, which we denote by \hat{v}_{s-k}^i :

$$\hat{v}_{s-k}^i = \epsilon_{s-k}^F + \hat{\epsilon}_{s-k}^i, \quad \text{where } \hat{\epsilon}_{s-k}^i \equiv \left(\frac{1}{n_{s,k}} \sum_{j=1}^{n_{s,k}} \tilde{\epsilon}^j\right) \sim N(0, \sigma_v^2(n_{s,k})^{-1}). \quad (152)$$

A.8 Multiple Factors Structure

Denote a vector of K factors by

$$\mathbf{F} = (F_1 \quad F_2 \quad \dots \quad F_K)' \sim \mathbb{N}(\mathbf{0}_{K \times 1}, \tau_F^{-1} \mathbf{I}_K) \quad (153)$$

and a vector of $N \geq K$ realized dividends by

$$\mathbf{D} = \underbrace{\begin{pmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,K} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ \phi_{N,1} & \phi_{N,2} & \cdots & \phi_{N,K} \end{pmatrix}}_{\Phi \ (N \times K)} \mathbf{F} + \underbrace{\begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_N \end{pmatrix}}_{\epsilon \ (N \times 1)} \equiv \Phi \mathbf{F} + \epsilon \quad (154)$$

with $\epsilon \sim \mathbb{N} (N \times 1, \tau_\epsilon^{-1} \mathbf{I}_N)$. Each asset is in supply $X \sim \mathbb{N} (\mathbf{0}_{N \times 1}, \tau_X^{-1} \mathbf{I}_N)$. Each agent i observes a vector of private signals about the K factors

$$V^i = \mathbf{F} + v^i, \quad v^i \sim \mathbb{N} (\mathbf{0}_{K \times 1}, \tau_v^{-1} \mathbf{I}_K), \quad (155)$$

as well as a common public signal

$$G = \mathbf{F} + v, \quad v \sim \mathbb{N} (\mathbf{0}_{K \times 1}, \tau_G^{-1} \mathbf{I}_K). \quad (156)$$

In addition they observe prices, which we conjecture satisfy

$$P = \underbrace{\begin{pmatrix} \alpha_{1,1} & \alpha_{1,2} & \cdots & \alpha_{1,K} \\ \alpha_{2,1} & \alpha_{2,2} & \cdots & \alpha_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{N,1} & \alpha_{N,2} & \cdots & \alpha_{N,K} \end{pmatrix}}_{\alpha \ (N \times K)} \mathbf{F} + \underbrace{\begin{pmatrix} g_{1,1} & g_{1,2} & \cdots & g_{1,K} \\ g_{2,1} & g_{2,2} & \cdots & g_{2,K} \\ \vdots & \vdots & \vdots & \vdots \\ g_{N,1} & g_{N,2} & \cdots & g_{N,K} \end{pmatrix}}_{g \ (N \times K)} G + \underbrace{\begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,N} \\ \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,N} \\ \vdots & \vdots & \cdots & \vdots \\ \beta_{N,1} & \beta_{N,2} & \cdots & \beta_{N,N} \end{pmatrix}}_{\beta \ (N \times N)} X. \quad (157)$$

Since agents observe the public signal G , the effective price signal is

$$P^a \equiv P - gG = \alpha \mathbf{F} + \beta X. \quad (158)$$

Regrouping all signals in a vector we obtain

$$S^i = \begin{pmatrix} P^a \\ V^i \\ G \end{pmatrix} = \underbrace{\begin{pmatrix} \alpha \\ \mathbf{I}_K \\ \mathbf{I}_K \end{pmatrix}}_{H \ (N+2K) \times K} \mathbf{F} + \underbrace{\begin{pmatrix} \beta & \mathbf{0}_{N \times K} & \mathbf{0}_{N \times K} \\ \mathbf{0}_{K \times N} & \mathbf{I}_K & \mathbf{0}_{K \times K} \\ \mathbf{0}_{K \times N} & \mathbf{0}_{K \times K} & \mathbf{I}_K \end{pmatrix}}_{\Theta \ (N+2K) \times (N+2K)} \begin{pmatrix} X \\ v^i \\ v \end{pmatrix} \quad (159)$$

with

$$\begin{pmatrix} X \\ v^i \\ v \end{pmatrix} \sim \mathbb{N} \left(\mathbf{0}_{(N+2K) \times 1}, \underbrace{\begin{pmatrix} \tau_X^{-1} \mathbf{I}_N & \mathbf{0}_{N \times K} & \mathbf{0}_{N \times K} \\ \mathbf{0}_{K \times N} & \tau_v^{-1} \mathbf{I}_K & \mathbf{0}_{K \times K} \\ \mathbf{0}_{K \times N} & \mathbf{0}_{K \times K} & \tau_G^{-1} \mathbf{I}_K \end{pmatrix}}_{\Sigma \ (N+2K) \times (N+2K)} \right). \quad (160)$$

Using these matrices we now define a $(N + 2K) \times (N + 2K)$ -matrix:

$$R \equiv (\Theta \Sigma \Theta')^{-1} = \begin{pmatrix} (\beta \beta')^{-1} \tau_X & \mathbf{0}_{N \times K} & \mathbf{0}_{N \times K} \\ \mathbf{0}_{K \times N} & \mathbf{I}_K \tau_v & \mathbf{0}_{K \times K} \\ \mathbf{0}_{K \times N} & \mathbf{0}_{K \times K} & \mathbf{I}_K \tau_G \end{pmatrix}. \quad (161)$$

The projection theorem then implies that

$$\mathbb{V}_i[\mathbf{F}] = (\mathbf{I}_K \tau_F + H' R H)^{-1} \equiv \bar{\mathbb{V}}[\mathbf{F}], \quad (162)$$

from which we define average precision τ as

$$\underbrace{\tau}_{K \times K} \equiv \bar{\mathbb{V}}[\mathbf{F}]^{-1} = \mathbf{I}_K \tau_F + H' R H. \quad (163)$$

The projection theorem also yields

$$\mathbb{E}_i[\mathbf{F}] = \tau^{-1} H' R S_i. \quad (164)$$

Aggregating over the vector of signals we can write

$$\bar{S} = \int_{i \in [0,1]} S_i di = \underbrace{\begin{pmatrix} \alpha \\ \mathbf{0}_{K \times K} \\ \mathbf{I}_K \end{pmatrix}}_{s_F \ (N+2K) \times K} \mathbf{F} + \underbrace{\begin{pmatrix} \mathbf{0}_{N \times K} \\ \mathbf{0}_{K \times K} \\ \mathbf{I}_K \end{pmatrix}}_{s_G \ (N+2K) \times K} G + \underbrace{\begin{pmatrix} \beta \\ \mathbf{0}_{K \times N} \\ \mathbf{0}_{K \times N} \end{pmatrix}}_{s_X \ (N+2K) \times N} X. \quad (165)$$

It follows that average expectations satisfy

$$\bar{\mathbb{E}}[\mathbf{F}] = \tau^{-1} H' R (s_F \mathbf{F} + s_G G + s_X X). \quad (166)$$

The market-clearing condition then requires that

$$\alpha \mathbf{F} + g G + \beta X = \bar{\mathbb{E}}[D] - \gamma \bar{\mathbb{V}}[D] X \quad (167)$$

$$= \Phi \tau^{-1} H' R (s_F \mathbf{F} + s_G G + s_X X) - \gamma (\Phi \tau \Phi' + \tau_\epsilon^{-1} \mathbf{I}_N) X \quad (168)$$

$$= \Phi \tau^{-1} H' R s_F \mathbf{F} + \Phi \tau^{-1} H' R s_G G + (\Phi \tau^{-1} H' R s_X - \gamma (\Phi \tau^{-1} \Phi' + \tau_\epsilon^{-1} \mathbf{I}_N)) X. \quad (169)$$

Separating variables we obtain the following system of equations:

$$\alpha = \Phi \tau^{-1} H' R s_F \quad (170)$$

$$g = \Phi \tau^{-1} H' R s_G \quad (171)$$

$$\beta = \Phi \tau^{-1} H' R s_X - \gamma (\Phi \tau^{-1} \Phi' + \tau_\epsilon^{-1} \mathbf{I}_N). \quad (172)$$

We can reduce the size of this system of equations. We first define

$$\underbrace{\tau_P}_{K \times N} \equiv \alpha' (\beta \beta')^{-1} \quad (173)$$

and write accordingly

$$\underbrace{H'R}_{K \times (N+2K)} = \begin{pmatrix} \tau_P \tau_X & \mathbf{I}_K \tau_v & \mathbf{I}_K \tau_G \end{pmatrix}. \quad (174)$$

Substituting into the system of equations we obtain g as a function of the unknown matrix τ :

$$g = \Phi \tau^{-1} \tau_G. \quad (175)$$

Similarly, rewriting total precision we can further write

$$\tau - \mathbf{I}_K \tau_F = H'RH = H'Rs_F + \mathbf{I}_K \tau_G, \quad (176)$$

from which we obtain an expression for α as a function of the unknown matrix τ :

$$\alpha = \Phi \tau^{-1} (\tau - \mathbf{I}_K \tau_F - \mathbf{I}_K \tau_G). \quad (177)$$

Finally, substituting into the equation for β we obtain an expression for β as a function of the unknown matrices τ and τ_P :

$$\beta = \gamma (\Phi \tau^{-1} \tau_P \tau_X - \mathbf{I}_N)^{-1} (\Phi \tau^{-1} \Phi' + \tau_\epsilon^{-1} \mathbf{I}_N). \quad (178)$$

Hence, we need to pin down the matrices τ and τ_P . We just use the filter equation for τ :

$$\tau = \mathbf{I}_K (\tau_F + \tau_v + \tau_G) + \tau_P \tau_X \alpha \quad (179)$$

$$= \mathbf{I}_K (\tau_F + \tau_v + \tau_G) + \tau_P \tau_X \Phi \tau^{-1} (\tau - \mathbf{I}_K \tau_F - \mathbf{I}_K \tau_G), \quad (180)$$

along with the definition of the matrix τ_P :

$$\tau_P = \frac{1}{\gamma^2} (\Phi \tau^{-1} (\tau - \mathbf{I}_K \tau_F - \mathbf{I}_K \tau_G))' (\Phi \tau^{-1} \tau_P \tau_X - \mathbf{I}_N) X (\Phi \tau^{-1} \tau_P \tau_X - \mathbf{I}_N) \quad (181)$$

where

$$X = \left((\Phi \tau^{-1} \Phi' + \tau_\epsilon^{-1} \mathbf{I}_N) (\Phi \tau^{-1} \Phi' + \tau_\epsilon^{-1} \mathbf{I}_N)' \right)^{-1}. \quad (182)$$

Regarding the simple example of Section 6.1, the above equations collapse into a system of five equations for the five unknowns τ_1 , τ_2 , $\tau_{P,1}$, $\tau_{P,2}$ and $\tau_{P,3}$:

$$\frac{\tau_X (-\tau_F - \tau_G + \tau_1) (\phi_1 \tau_{P,1} - \phi_1 \tau_{P,2} + \tau_{P,1} + \tau_{P,2})}{\tau_1} + \tau_F + \tau_G + \tau_v = \tau_1 \quad (183)$$

$$\phi_2 \tau_X \tau_{P,3} + \tau_F + \tau_G + \tau_v - \frac{\phi_2 \tau_X \tau_{P,3} (\tau_F + \tau_G)}{\tau_2} = \tau_2 \quad (184)$$

$$\gamma^2 \tau_1 \tau_{P,1} (\tau_1 + 2 (\phi_1^2 + 1) \tau_\epsilon)^2 - \tau_\epsilon^2 (-\tau_F - \tau_G + \tau_1) (\tau_1 (\phi_1 + 1) - 2 (\phi_1^2 + 1) \tau_X \tau_{P,1}) \quad (185)$$

$$\times (\tau_1 - \tau_X (\phi_1 \tau_{P,1} - \phi_1 \tau_{P,2} + \tau_{P,1} + \tau_{P,2})) = 0 \quad (186)$$

$$\tau_\epsilon^2 (-\tau_F - \tau_G + \tau_1) (2 (\phi_1^2 + 1) \tau_X \tau_{P,2} + \tau_1 (\phi_1 - 1)) \quad (187)$$

$$\times (\tau_1 - \tau_X (\phi_1 \tau_{P,1} - \phi_1 \tau_{P,2} + \tau_{P,1} + \tau_{P,2})) + \gamma^2 \tau_1 \tau_{P,2} (\tau_1 + 2 (\phi_1^2 + 1) \tau_\epsilon)^2 = 0 \quad (188)$$

$$\phi_2 \tau_\epsilon^2 (-\tau_F - \tau_G + \tau_2) (\tau_2 - \phi_2 \tau_X \tau_{P,3})^2 - \gamma^2 \tau_2 \tau_{P,3} (\tau_2 + \phi_2^2 \tau_\epsilon)^2 = 0. \quad (189)$$

This system has a unique, real solution, which is analytical in this case. In particular, defining a function

$$\varphi_i = \begin{cases} 2(1 + \phi_1^2) & i = 1 \\ \phi_2^2 & i = 2 \end{cases} \quad (190)$$

along with

$$\bar{\tau}_i \equiv \tau_G + \tau_F + \tau_v + \tau_\epsilon \varphi_i, \quad i = 1, 2 \quad (191)$$

and

$$\delta_i = 4\gamma^2 \bar{\tau}_i^3 + 27\tau_v^2 \tau_X \tau_\epsilon^2 \varphi_i, \quad i = 1, 2. \quad (192)$$

We can then write the solution for τ_i with $i = 1, 2$ as:

$$\tau_i = \frac{1}{6} \left(\begin{array}{l} 2\bar{\tau}_i + \frac{2\gamma^2 \bar{\tau}_i^2}{\sqrt[3]{\gamma^6 \bar{\tau}_i^3 + \frac{3}{2} (\sqrt{3} \sqrt{\gamma^8 \delta_i \varphi_i \tau_v^2 \tau_X \tau_\epsilon^2 + 9\gamma^4 \varphi_i \tau_v^2 \tau_X \tau_\epsilon^2})}} \\ + \frac{2^{2/3} \sqrt[3]{2\gamma^6 \bar{\tau}_i^3 + 3\sqrt{3} \sqrt{\gamma^8 \delta_i \varphi_i \tau_v^2 \tau_X \tau_\epsilon^2 + 27\gamma^4 \varphi_i \tau_v^2 \tau_X \tau_\epsilon^2}}}{\gamma^2} - 6\varphi_i \tau_\epsilon \end{array} \right). \quad (193)$$

The solutions for $\tau_{P,1}$ and $\tau_{P,2}$ are analytical, but irrelevant for the example since they do not intervene in the characterization of the SML.

When comparing agent's betas versus econometrician's betas (Equation (57) in the text), we find the following values for δ and Δ_i , $i = 1, 2, 3$:

$$\delta = \frac{2\gamma^2 (\tau_1 + \phi_1^2 \tau_\epsilon + \tau_\epsilon) + (\tau_1 + \tau_v) \tau_X \tau_\epsilon}{\tau_1 \tau_X \tau_\epsilon} > 1 \quad (194)$$

and

$$\begin{pmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{pmatrix} = \begin{pmatrix} \frac{(2\tau_2(\phi_1^2+1) - \tau_1\phi_2^2)\gamma^2 + (\tau_2 - \tau_1)\tau_v\tau_X}{\tau_2\tau_X(\tau_2(6\phi_1+2) - \tau_1\phi_2^2)} \\ \frac{(\tau_1\phi_2^2 - 2\tau_2(\phi_1^2+1))\gamma^2 + (\tau_1 - \tau_2)\tau_v\tau_X}{\tau_2\tau_X(\tau_1\phi_2^2 + \tau_2(6\phi_1-2))} \\ \frac{(2\tau_2(\phi_1^2+1) - \tau_1\phi_2^2)\gamma^2 + (\tau_2 - \tau_1)\tau_v\tau_X}{\tau_2\tau_X(2\tau_2 - \tau_1\phi_2^2)} \end{pmatrix}. \quad (195)$$

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